Announcements

- Midterm 2 – **Thursday, April 2, 8:00 – 9:30 PM, WTHR 200**
- Covers lectures 11 to 26 – from Moving Reference Frames (2D and 3D) to particle kinetics (all methods)
- Review sessions:
    1. Pi-Tau-Sigma: Tuesday, March 31, 6:30 – 7:30 PM, WTHR 104
    2. Prof. Krousgrill: Wednesday, April 1, 7:00 PM, on Zoom (link on website)
- No class on Friday, April 3 (after exam)

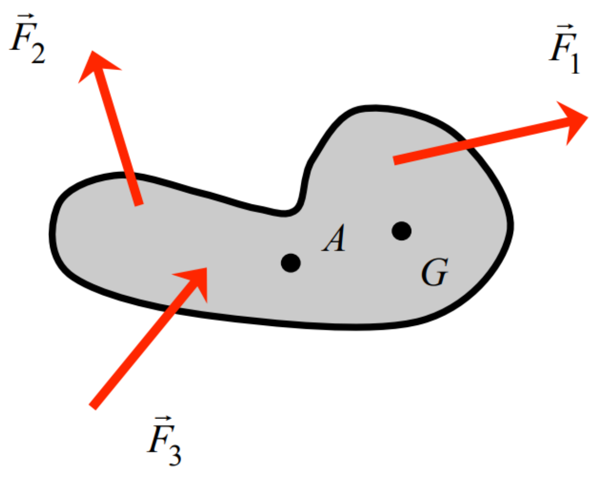
ME 274: Basic Mechanics II

Week 11 – *Monday, March 30*

Newton-Euler Equations

Instructor: Manuel Salmerón

Newton-Euler Equations



$$\sum \vec{F} = m\vec{a}_G$$

$$\sum \overrightarrow{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$$

- If you choose A to be the center of mass, G, then $\vec{r}_{G/A} = \vec{0}$. For this case, the above equation reduces to:

$$\sum \vec{M}_G = I_G\vec{\alpha}$$

- If you choose A to be a fixed point on the body, O, then $\vec{a}_O = \vec{0}$. For this case, the above equation reduces to:

$$\sum \vec{M}_O = I_O\vec{\alpha}$$

- If you choose a point A which has an acceleration vector that is parallel to $\vec{r}_{G/A}$ (and therefore, $\vec{r}_{G/A} \times \vec{a}_A = \vec{0}$), then the above equation reduces to:

$$\sum \vec{M}_A = I_A\vec{\alpha}$$

Not using the center of mass?
Use the parallel axis theorem:

$$I_A = I_G + md_{AG}^2$$

Given radius of gyration?
Convert to mass moment of inertia:

$$I_A = mk_A^2$$

Activity

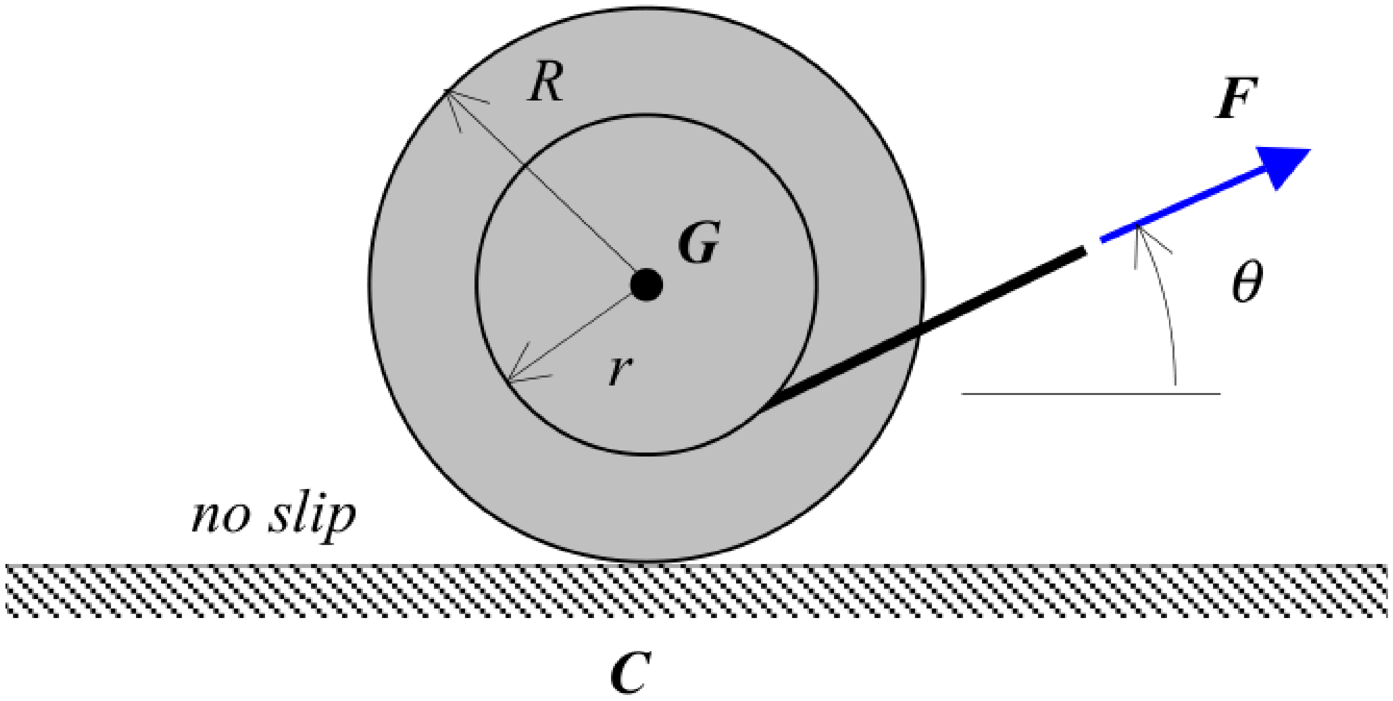
This activity has several stages. Read them carefully. You will upload all your work to Gradescope. Do your work on a piece a paper or your tablet.

1. You will see a series of examples from the lecture book in the screen.
2. You will have **2 minutes to draw the FBD** of each example. Then, the correct FBD will be shown in the screen.
3. You will have **2 minutes to write down the Newton/Euler equations** that you consider helpful for solving what the problem asks.
4. After all the examples are shown, you must **choose one example and finish it** (kinematics, solve).
5. Finally, explain how you solved your problem to a peer and hear your peer’s explanation on how they solved their problem. After hearing the explanation, **solve the problem**.
6. Upload your work to Gradescope. Make sure to include: all FBDs, all Newton/Euler eqs., your problem, and the problem explained by your peer.

Example 5.A.12

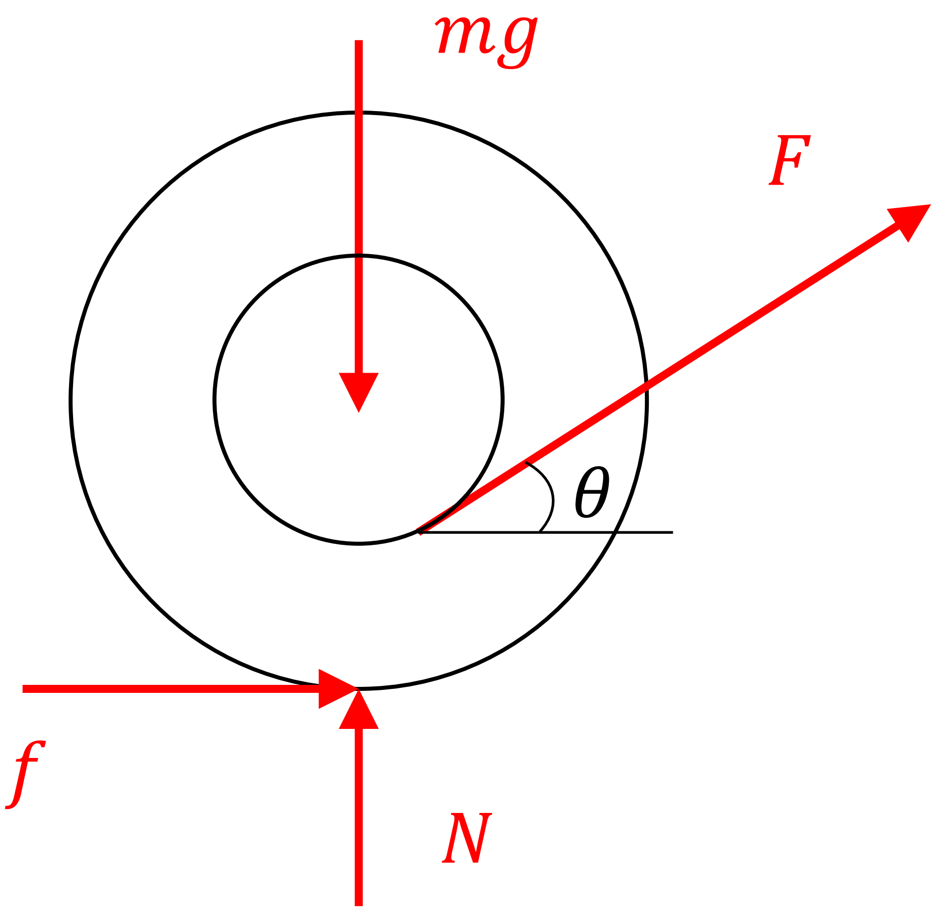
Given: A spool has a mass of $m = 30$ kg, a centroidal radius of gyration $k_G = 0.25$ m, an outer radius of $R = 0.3$ m and an inner radius of $r = 0.1$ m. A constant force $F = 60$ N is applied at an angle of θ by a cord that is wrapped around the inner radius of the spool. The spool rolls without slipping on the rough horizontal surface.

Find: Determine the angular acceleration of the spool as a function of the angle θ.



Friendly reminder:
$I_A = mk_A^2$

1. FBD



2. Kinetics

$$\sum F_x = f + F\cos\theta = ma_{Gx}$$

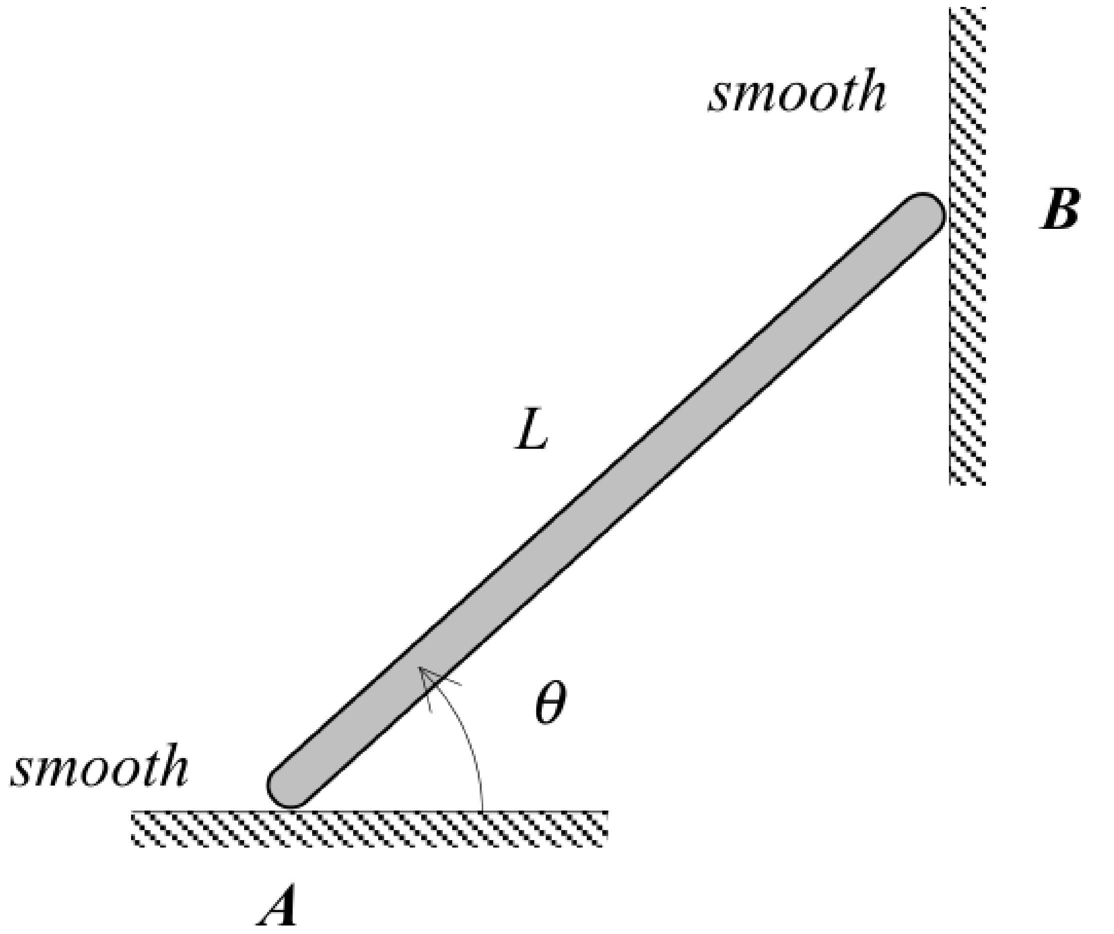
$$\sum F_y = N + F\sin\theta - mg = 0$$

$$\sum M_G = fR + Fr = I_G\alpha = mk_G^2\alpha$$

Example 5.A.13

Given: The thin, homogeneous bar AB (having a mass of $m = 30$ kg and length $L = 4$ m) is released from rest at the position shown where $\theta = 36.87°$. Assume all surfaces to be smooth.

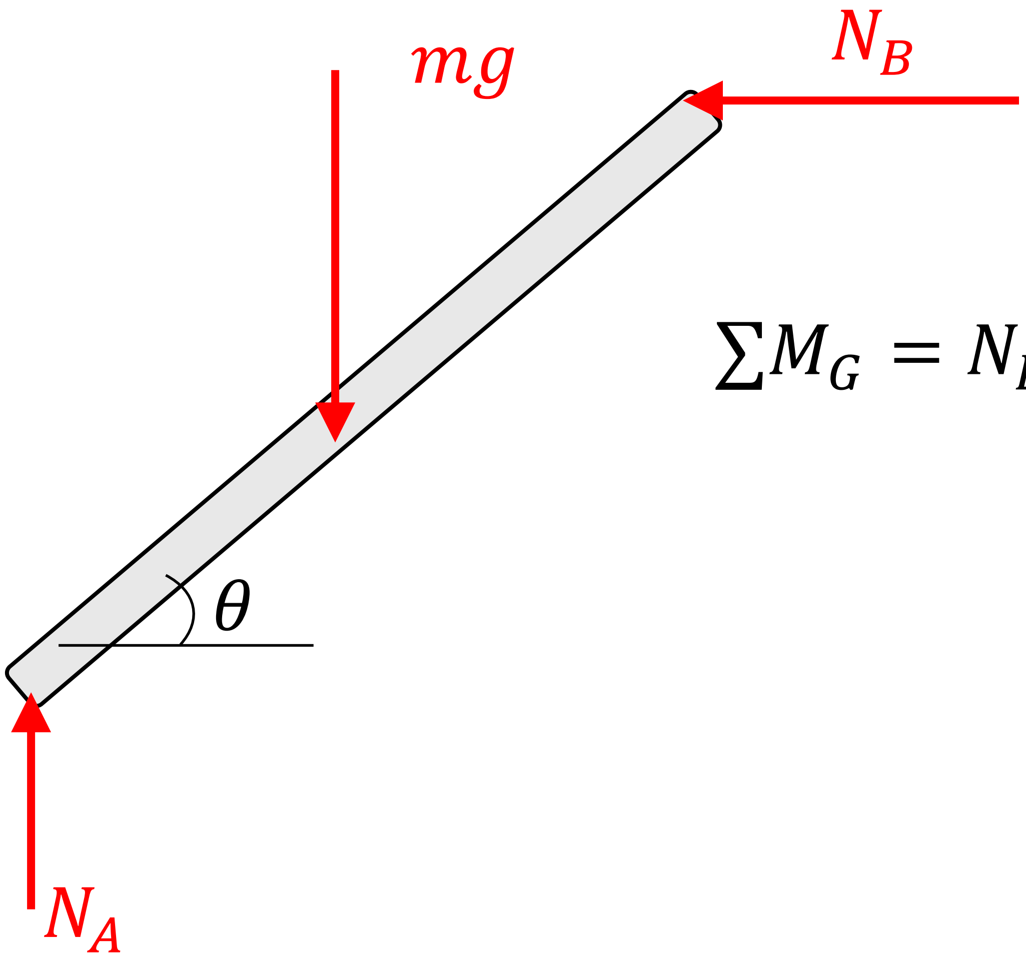
Find: Determine the angular acceleration of the bar on release.



Friendly reminder: for a bar,

$$I_G = \frac{1}{12} mL^2$$

1. FBD



2. Kinetics

$$\sum F_x = -N_B = ma_{Gx}$$

$$\sum F_y = N_A - mg = ma_{Gy}$$

$$\sum M_G = N_B \frac{L}{2} \sin\theta - N_A \frac{L}{2} \cos\theta = I_G \alpha = \frac{1}{12} mL^2 \alpha$$

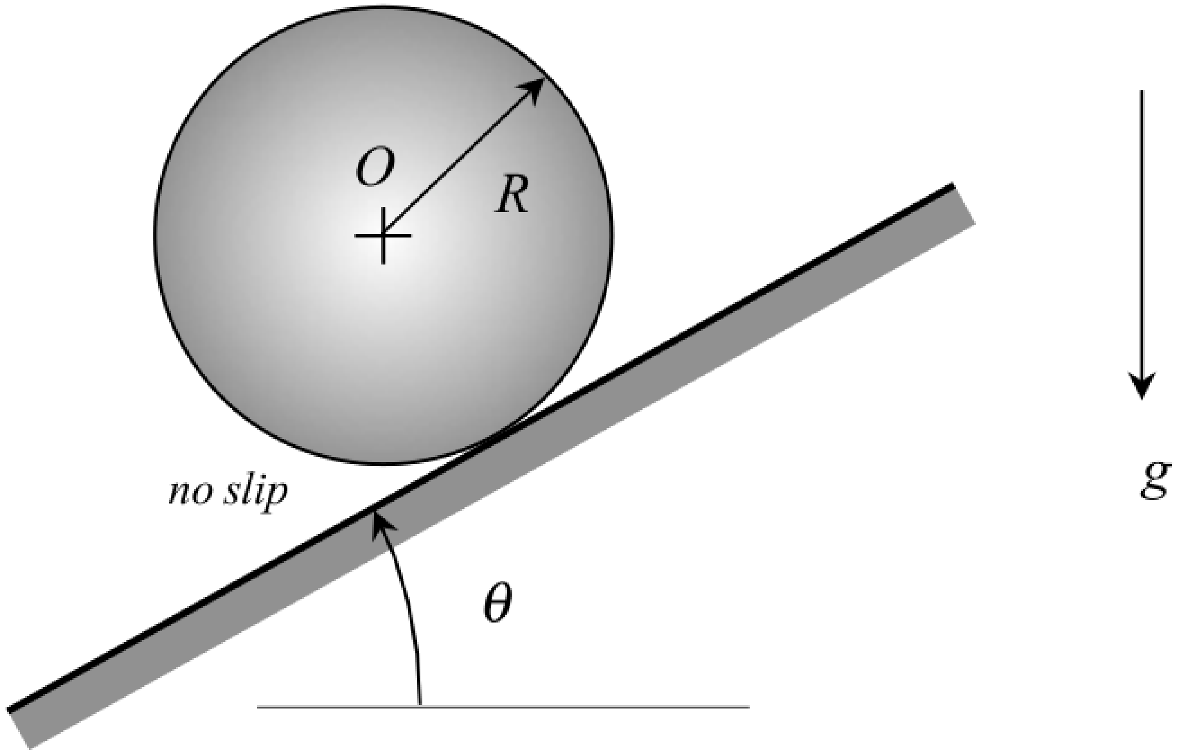
Example 5.A.15

Given: The homogeneous sphere (having a mass of m and radius R) is released from rest on a rough incline at an angle of θ. After release, the sphere rolls without slipping on the incline.

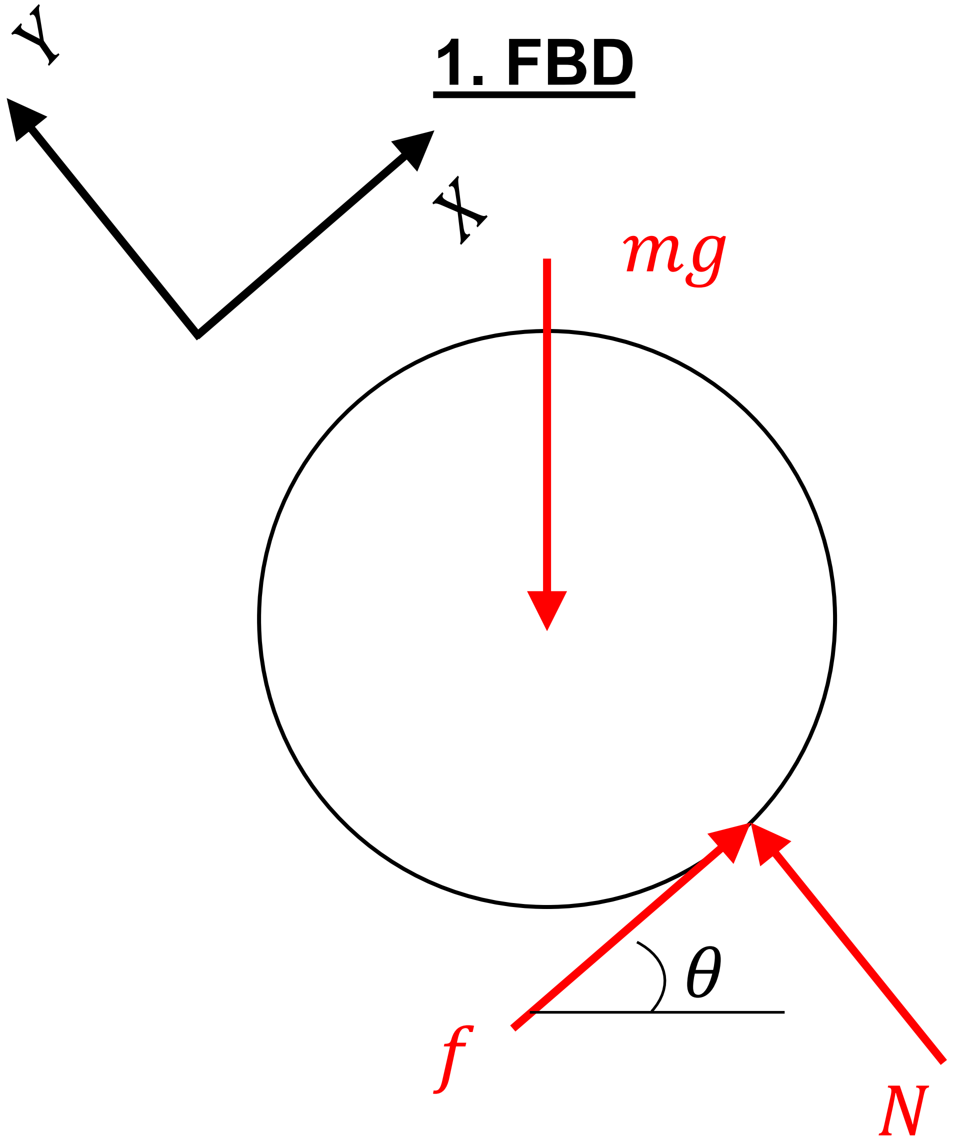
Find: Determine:

(a) The acceleration of the center of mass G of the sphere; and

(b) The force of friction acting on the sphere on release.



1. FBD



2. Kinetics

$$\sum F_x = f - mg\sin\theta = ma_{Gx}$$

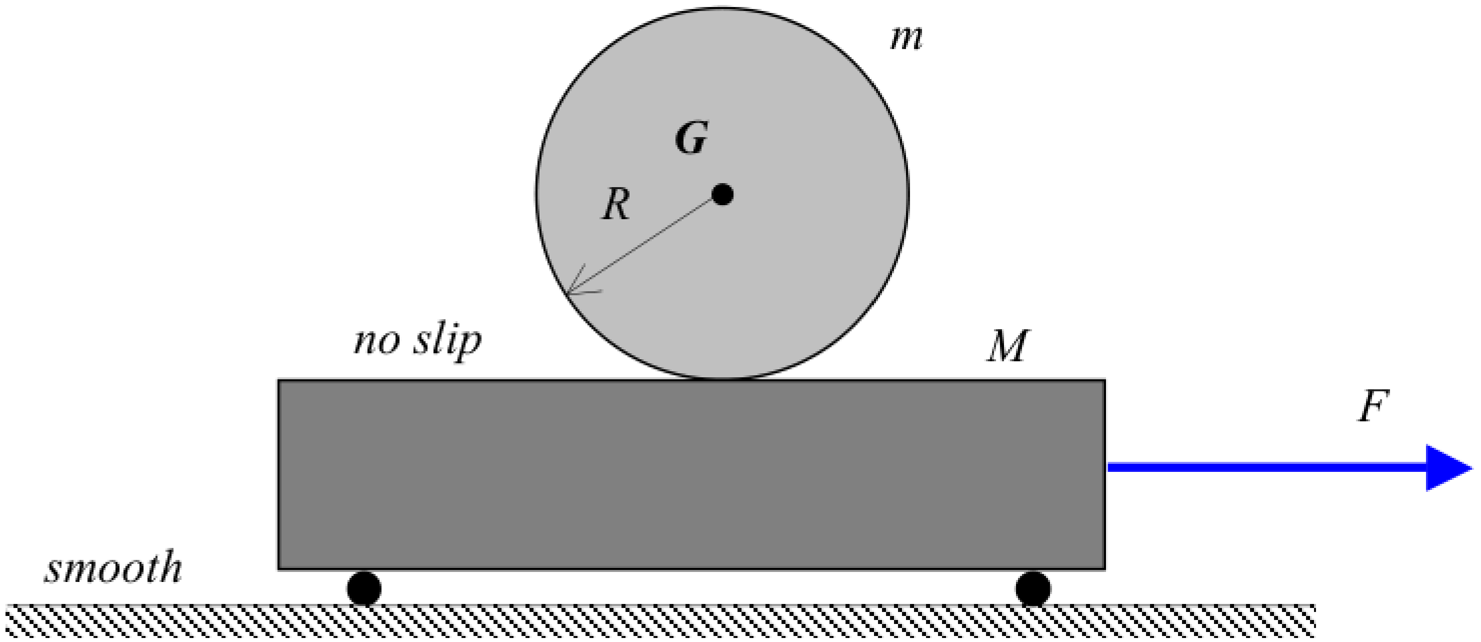
$$\sum F_y = N - mg\cos\theta = 0$$

$$\sum M_G = fR = I_G\alpha$$

Example 5.A.16

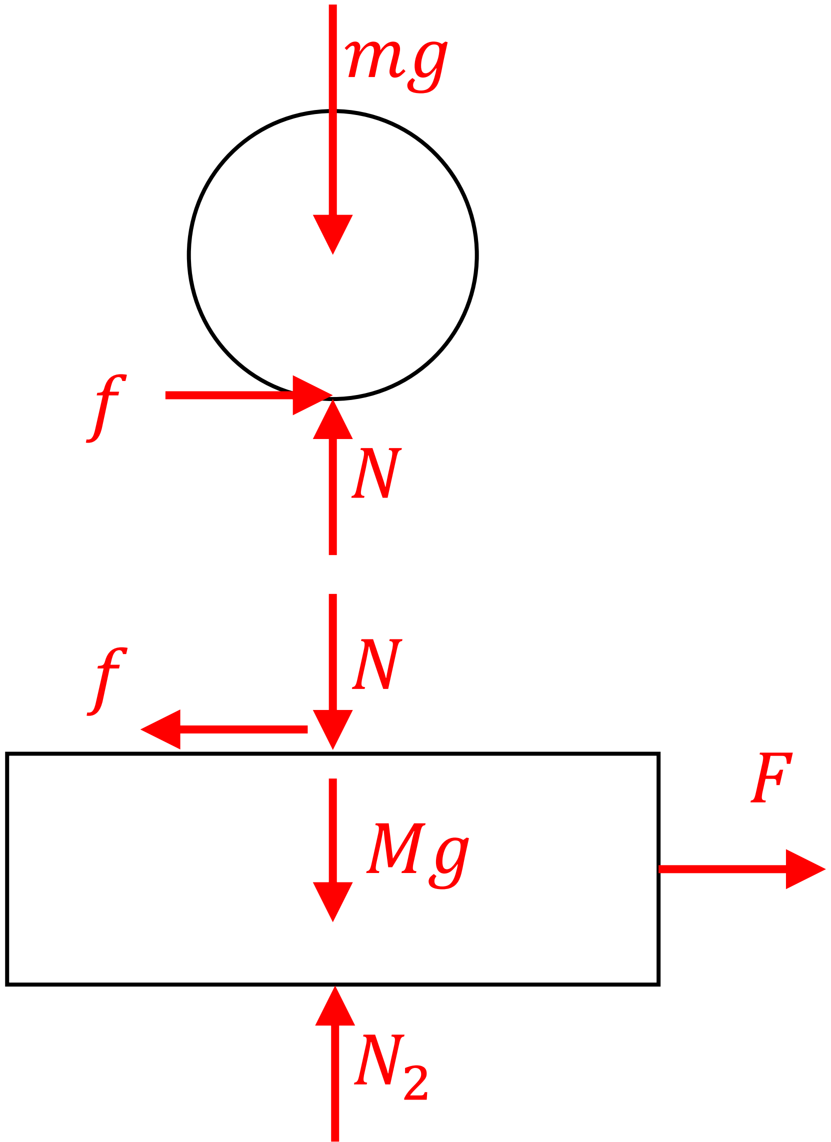
Given: The system is initially at rest.

Find: Determine the acceleration of G when force F acts on the cart.



Friendly reminder: for a disk, $I_G = \frac{1}{2}mR^2$

1. FBD



2. Kinetics

For the disk:

$$\sum F_x = f = ma_{Gx}$$

$$\sum F_y = N - mg = 0$$

$$\sum M_G = fR = I_G\alpha = \frac{1}{2}mR^2\alpha$$

For the block:

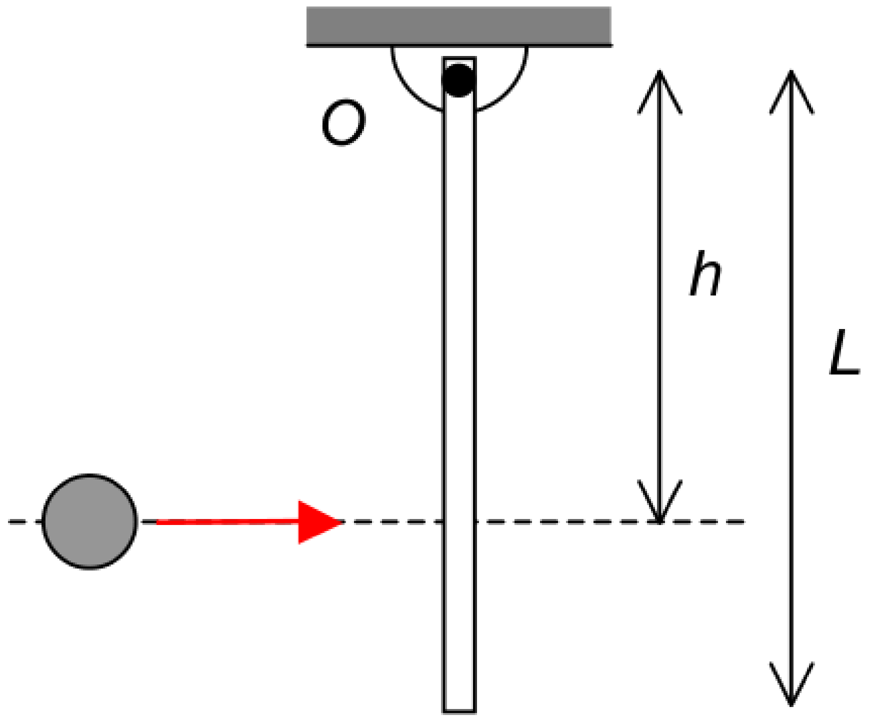
$$\sum F_x = -f + F = Ma_x$$

$$\sum F_y = -N + N_2 - Mg = 0$$

Example 5.A.17

Given: A ball of mass m strikes a homogeneous bar of length L and mass M at a distance h from the pivot point O.

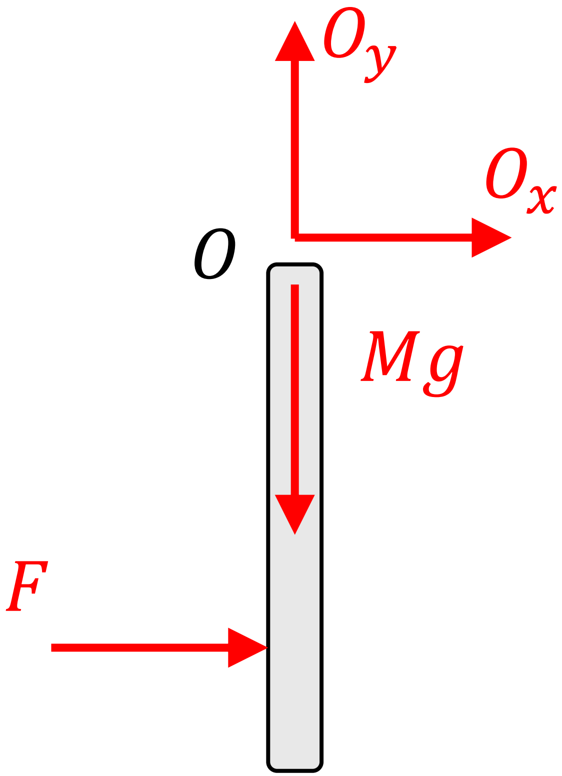
Find: Determine the distance h such that the horizontal reaction force at O on the bar is zero. (This location on the bar is known as the body's "center of percussion". Can you think of a practical application of knowing the location of the center of percussion of a body?)



Friendly reminder: for a bar,

$$I_G = \frac{1}{12} mL^2$$

1. FBD



2. Kinetics

$$\sum F_x = F + O_x = Ma_{Gx}$$

$$\sum F_y = O_y - Mg = Ma_{Gy}$$

$$\sum M_O = Fh = \left[I_G + M\left(\frac{L}{2}\right)^2 \right] \alpha = \frac{1}{3} ML^2 \alpha$$

Activity

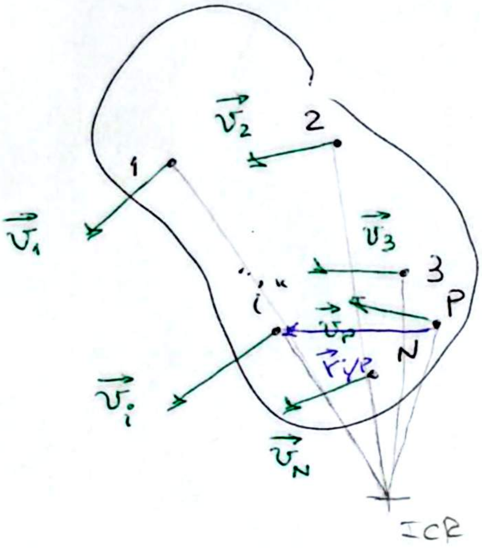
3. After all the examples are shown, you must **choose one example and finish it** (kinematics, solve). Look for the problem in your lecture book:

    Examples: **5.A.12, 5.A.13, 5.A.15, 5.A.16, 5.A.17**

3. Finally, explain how you solved your problem to a peer and hear your peer's explanation on how they solved their problem. After hearing the explanation, **solve the problem**.

4. Upload your work to Gradescope. Make sure to include: all FBDs, all Newton/Euler eqs., your problem, and the problem explained by your peer.

= WORK & ENERGY FOR RIGID BODIES =



For a system of "N" particles:

$$T = \sum_{i=1}^{N} T^{(i)} = \sum_{i=1}^{N} \left(\frac{1}{2} m_i v_i^2 \right)$$

where the speed of the i-th particle:

$$v_i^2 = \left| \vec{v}_i \right| = \vec{v}_i \cdot \vec{v}_i$$

For two points inside the RB:

$$\vec{v}_i = \vec{v}_p + \vec{\omega} \times \vec{r}_{i/p}$$

Thus: $$\left| \vec{v}_i \right| = \vec{v}_i \cdot \vec{v}_i = \left(\vec{v}_p + \vec{\omega} \times \vec{r}_{i/p} \right) \cdot \left(\vec{v}_p + \vec{\omega} \times \vec{r}_{i/p} \right)$$

$v_i^2 = \ldots$ see lecture book... $=$

$$v_i^2 = v_p^2 + 2 \vec{v}_p \cdot \left(\vec{\omega} \times \vec{r}_{i/p} \right) + \left| \vec{\omega} \times \vec{r}_{i/p} \right|^2$$

Back to the kinetic energy equation:

$$T = \frac{1}{2} \left[\sum_{i=1}^{N} m_i v_p^2 + 2 \sum_{i=1}^{N} m_i \vec{v}_p \cdot \left(\vec{\omega} \times \vec{r}_{i/p} \right) + \sum_{i=1}^{N} m_i \left| \vec{\omega} \times \vec{r}_{i/p} \right|^2 \right]$$

$$T = \frac{1}{2} \left[\left(\sum_i m_i \right) v_p^2 + 2 \vec{v}_p \cdot \left(\sum_{i=1}^{N} m_i \vec{\omega} \times \vec{r}_{i/p} \right) + \underbrace{\sum_{i=1}^{N} m_i \left| \vec{r}_{i/p} \right|^2}_{I_p} \omega^2 \right]$$

$$\boxed{T = \frac{1}{2} M v_p^2 + M \vec{v}_p \cdot \left(\vec{\omega} \times \vec{r}_{G/p} \right) + \frac{1}{2} I_p \omega^2}$$

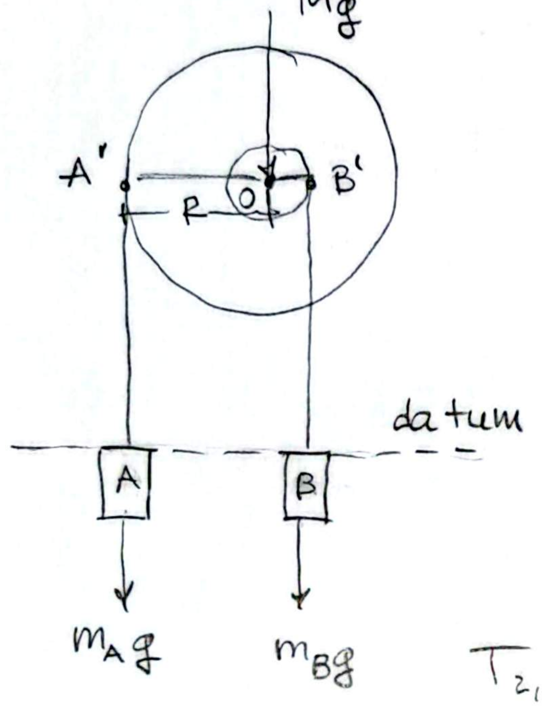
KINETIC ENERGY FOR RIGID BODIES

$M = \sum_i m_i$: total mass

$I_P = \sum_i m_i \left| \vec{r}_{i/p} \right|^2$: mass moment of inertia about P

Example 5.B.#1

1. FBD



2. Kinetics

$$T_2 + V_2 = T_1 + V_1 + \mathcal{U}^{(nc)}_{1\to 2}$$

$T_1 = 0$: released from rest

$\mathcal{U}^{(nc)}_{1\to 2} = 0$: no "nc" forces

$V_1 = 0$: all starts at datum

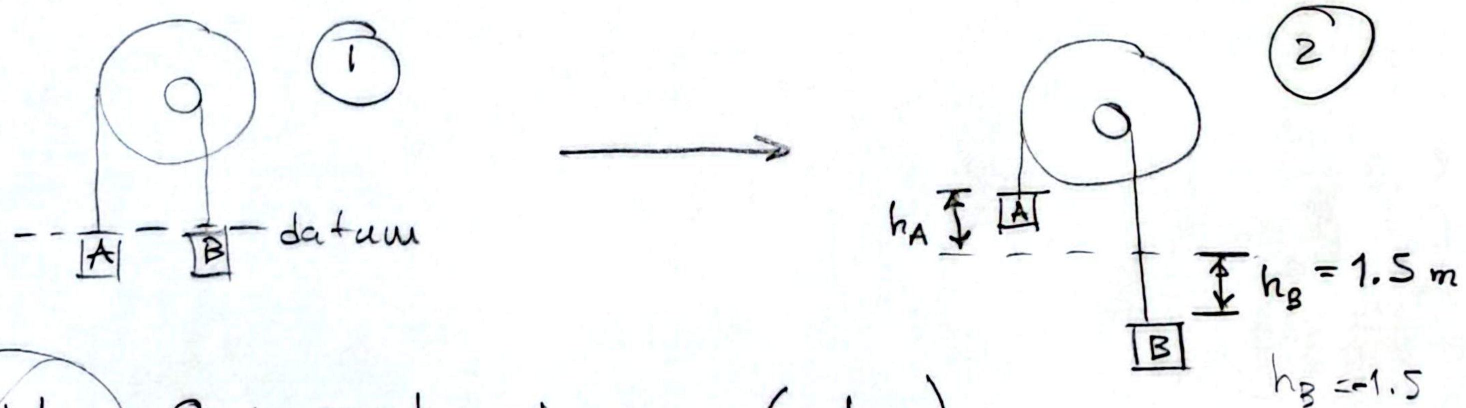
$$T_2 = T_{2,drum} + T_{2,A} + T_{2,B}$$

$$T_{2,drum} = \frac{1}{2}\cancel{M v_0^2}^{\,0} + \cancel{M \vec{v}_0}^{\,\vec{0}} \cdot (\vec{\omega} \times \vec{r}_{G/P}) + \frac{1}{2} I_0 \omega^2$$

$$= \frac{1}{2} I_0 \omega_2^2$$

$$T_{2,A} = \frac{1}{2} m_A v_{A_2}^2 \quad ; \quad T_{2,B} = \frac{1}{2} m_B v_{B_2}^2$$

$$V_2 = V_{2,drum} + V_{2,A} + V_{2,B}$$



$$V_2 = 0 + m_A g h_A + m_B g (-h_B)$$

1 eq, 4 unknowns

3. Kinematics

No slip: $v_{B'} = v_B$, $v_{A'} = v_A$

Recall ICR: $\omega = \dfrac{|\vec{v}_A|}{|\vec{r}_{A/ICR}|} = \dfrac{|\vec{v}_B|}{|\vec{r}_{B/ICR}|} = \ldots$

$$\omega = \frac{v_{A,z}}{R} = \frac{v_{B,z}}{r} \quad +2 \text{ eq.}$$

Positions: $s_B = r\theta \Rightarrow \theta = \dfrac{s_B}{r}$

$$s_A = R\theta = R\left(\frac{h_B}{r}\right) = \frac{R}{r} h_B$$

$$h_A = s_A = \frac{R}{r} h_B$$