Chapter Review

* *Chapter 4.E of the Lecturebook*
* *Chapter 3.D and 3/15 of Meriam Textbook 7th Edition*

Topics

	Newton's Law	Work Energy	L.I.M.	A.I.M.
	1 state	2 states	2 states	2 states
Ask	$\vec{a}$, α, $\vec{F}$	ω, v	$\vec{\omega}$, $\vec{v}$	$\vec{\omega}$, $\vec{v}$
Integral?	—	$\int$ over path	$\int$ over time	$\int$ over time
Coord	Any	Usually Cartesian	Usually Cartesian	Usually path
FBD	YES + KD	YES	YES	YES
Inclusive?	(small) NO	YES	Very Inclusive	Very Inclusive.

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $where\ \ T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $where\ \ T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \bullet \left(\vec{\omega} \times \vec{r}_{G/A}\right)$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_{O2} - \vec{H}_{O1}$ $where\ \ \vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\, dt = \vec{H}_{A2} - \vec{H}_{A1}$ $where\ \ \vec{H}_A = I_A\vec{\omega}$

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Examination No. 2

PROBLEM NO. 1

Given: Pellet P having a mass of m is pulled through a barrel (having negligible mass) by means of radial force $F = 60R$, where F is in Newtons and R is in meters. The barrel is constrained to move in a HORIZONTAL plane by rotating about shaft passing through point O. The system is released with $R = R_1, \dot{R} = \dot{R}_1$ and $\dot{\theta} = \dot{\theta}_1$.

Find: For the instant when $R = R_2$:

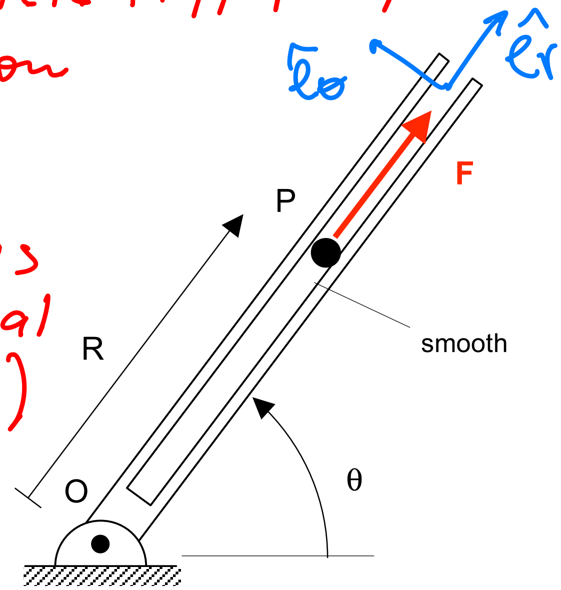
a) determine the rotation rate of the barrel, $\dot{\theta}_2$.

b) determine the value of $\dot{R}_2$.

Use the following parameters in your analysis: $m = 20kg$, $R_1 = 1.5\ meters$, $\dot{R}_1 = 4\ m/sec$, $\dot{\theta}_1 = 8\ rad/sec(CCW)$ and $R_2 = 3\ meters$.

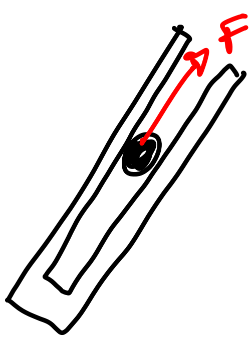
Asking for changes in velocity/speed/rate after changes in position

⇒



HORIZONTAL PLANE

FBD Make system as big as possible ⇒ (Normal Force is internal)



momentum in $\hat{e}_r$ conserved

$$m v_1 + \int \cancel{F}\, dt = m v_2$$

$$\cancel{m}(\dot{r}_1 + r_1\dot{\theta}_1) = \cancel{m}(\dot{r}_2 + r_2\dot{\theta}_2)$$

$$\dot{\theta}_2 =$$

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Examination No. 2

PROBLEM NO. 3

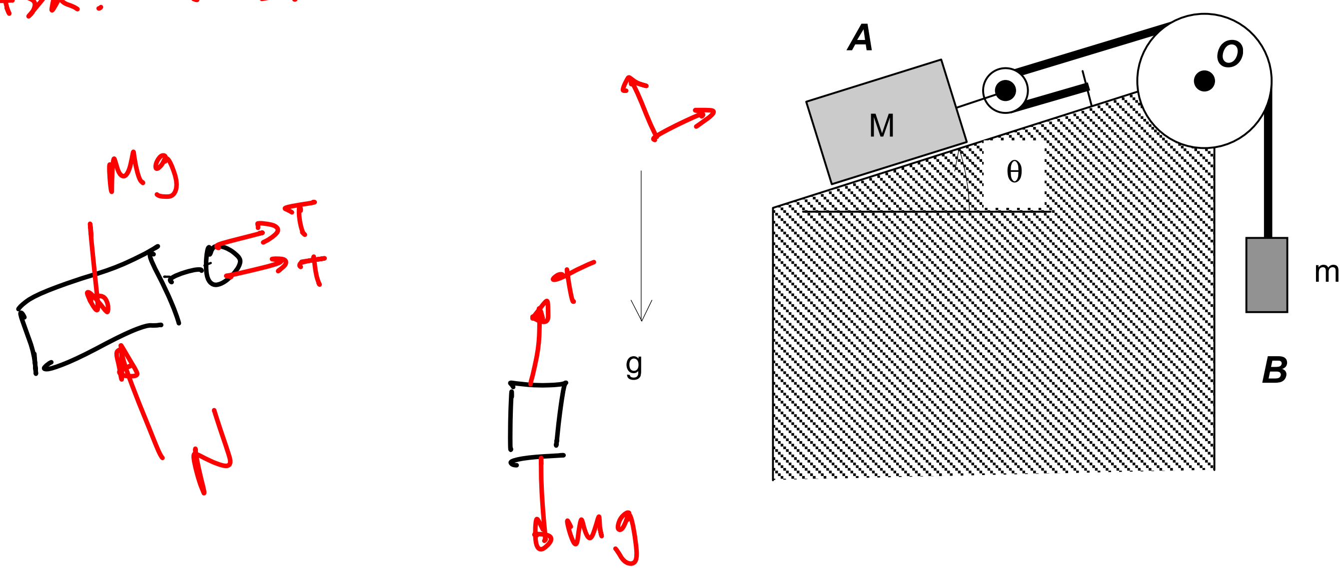
Given: Blocks A and B are connected by the cable-pulley shown. The system is released from rest. Consider all surfaces to be smooth and that the masses of the pulleys are small compared to the masses of A and B.

Find: Upon release,

a) determine the acceleration of block B. Write your answer as a vector.
b) determine the tension in the cable.

Use the following parameters in your analysis: $m = 5 kg$, $M = 20 kg$ and $\theta = 36.87°$.

ASK: accelerations and forces => NEWTON



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Examination No. 2

PROBLEM NO. 4

Given: Block B, having a mass of m, is pressed against a spring (of stiffness k) that is attached to cart A. Cart A (having a mass of M) rests on a horizontal surface. The system is released from rest with the spring compressed by an amount of Δ. After release, block B impacts A, with this impact having a coefficient of restitution of *e*. Assume all surfaces to be smooth. (Note that since B is simply pressed against the spring, the spring *can push but not pull* on B.)

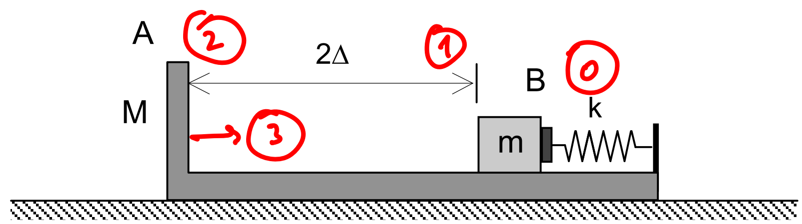
Find: For this problem,

a) determine the velocities of A and B immediately BEFORE impact.

b) determine the velocities of A and B immediately AFTER impact

Write your answers as vectors.

Use the following parameters in your analysis: $m = 20 kg$, $M = 40 kg$, $k = 3000\ N/m$, $\Delta = 0.2\, meters$ and $e = 0.5$.



=> Work - Energy

0-1 $\quad T_0 + V_0 + U_{0-1}^{NC} = T_1 + V_1$

$$\frac{1}{2} m \cancel{V_{B_0}^2}^{\,rfr} + \frac{1}{2} M \cancel{V_{A_0}^2}^{\,rfr} + \frac{1}{2} k\Delta^2 + 0 = \frac{1}{2} m V_{B_1}^2 + \frac{1}{2} M V_{A_1}^2 \quad (1)$$

Conservation of L.M. in x

$$-m V_{B_1} = M V_{A_1}$$

$$\Rightarrow V_{A_1} = -\frac{m}{M} V_{B_1} \quad (2)$$

in (1)

$$\Rightarrow \cancel{\tfrac{1}{2}} k\Delta^2 = \cancel{\tfrac{1}{2}} m V_{B_1}^2 + \cancel{\tfrac{1}{2}} M \left(\frac{m}{M} V_{B_1}\right)^2$$

$$k\Delta^2 = m V_{B_1}^2 + \frac{m^2}{M} V_{B_1}^2 = \frac{mM + m^2}{M} V_{B_1}^2$$

$$\Rightarrow \boxed{\vec{V}_{B1} = -\sqrt{\frac{K\Delta^2 M}{mM+m^2}}\,\hat{i}} \quad (\leftarrow)$$

VEL BEFORE IMPACT

$$V_{A1} = -\frac{m}{M}\left(-\sqrt{\frac{K\Delta^2 M}{mM+m^2}}\right)$$

$$\boxed{\vec{V}_{A1} = \frac{m}{M}\sqrt{\frac{K\Delta^2 M}{mM+m^2}}\,\hat{i}} \quad (\rightarrow)$$

Smooth surfaces: $V_{B2} = V_{B1}$ $\quad V_{A2} = V_{A1}$

2-3

$$e = -\frac{V_{Bn2} - V_{An2}}{V_{Bn1} - V_{An1}} = -\frac{V_{B3} - V_{A3}}{\underbrace{\cancel{V_{B2}}}_{V_{B1}} - \underbrace{\cancel{V_{A2}}}_{V_{A1}}}$$

$$e(V_{A1} - V_{B1}) = V_{B3} - V_{A3}$$

Conservation of L.M

$$mV_{B1} + MV_{A1} = mV_{B3} + MV_{A3}$$

Solve for V_{B3} & V_{A3}

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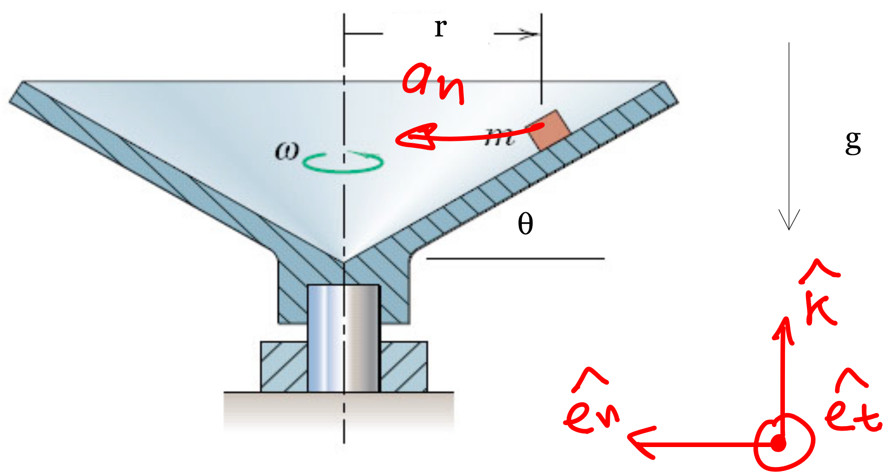
Final Examination

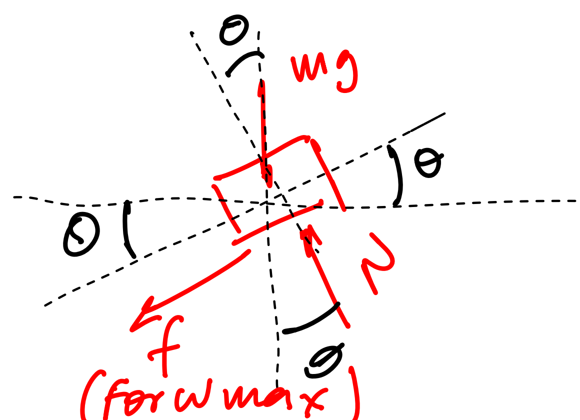
PROBLEM NO. 1

***Given*:** A small object of mass m is placed on the inner surface of a conical dish that is rotating at a constant rate of ω. The coefficients of static and kinetic friction between the object and the dish are known to be μ_s and μ_k, respectively.

***Find*:** Determine the *maximum* rotation rate ω for which the object does not slip on the dish.

Use the following parameters in your analysis: $m = 5kg$, $r = 0.92\ meters$, $\mu_s = 0.4$, $\mu_k = 0.1$ and $\theta = 36.87°$.





$\Sigma F_y = 0$ (not slide)

ωr

$\Sigma F_n = m a_n = m \frac{v^2}{r}$

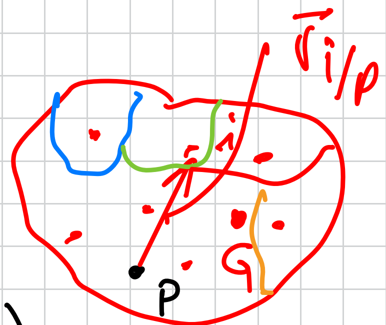
RBK – WORK-ENERGY EQUATION

We derived the W-E eqn for systems of particulas:

$$T_1 + V_1 + U_{1-2}^{(NC)} = T_2 + V_2 ,$$

where 1) $T = \frac{1}{2} \sum m_i v_i^2 = \frac{1}{2} \sum m_i \, \vec{v}_i \cdot \vec{v}_i$

Notice that we are dealing w/ RB now. So, for a RB moving and rotating about an arbitrary point on it, P:



$$\vec{v}_i = \vec{v}_p + \vec{\omega} \times \vec{r}_{i/p}$$

$$\vec{v}_i \cdot \vec{v}_i = (\vec{v}_p + \vec{\omega} \times \vec{r}_{i/p}) \cdot (\vec{v}_p + \vec{\omega} \times \vec{r}_{i/p})$$

$$\vec{v}_i \cdot \vec{v}_i = \vec{v}_p \cdot \vec{v}_p + 2\vec{v}_p \cdot (\vec{\omega} \times \vec{r}_{i/p}) + (\vec{\omega} \times \vec{r}_{i/p}) \cdot (\vec{\omega} \cdot \vec{r}_{i/p})$$

$$\vec{v}_i \cdot \vec{v}_i = |\vec{v}_p|^2 + 2 v_p \cdot (\vec{\omega} \times \vec{r}_{i/p}) + |\vec{\omega} \times \vec{r}_{i/p}|^2$$

Replacing the previous result in T:

$$T = \frac{1}{2}\sum m_i |\vec{v}_p|^2 + \cancel{\frac{1}{2}}\sum m_i \left[\cancel{2}\,\vec{v}_p \cdot (\vec{\omega} \times \vec{r}_{i/p})\right] + \frac{1}{2}\sum m_i |\vec{\omega} \times \vec{r}_{i/p}|^2$$

$$T = \frac{1}{2}\sum m_i |\vec{v}_p|^2 + \vec{v}_p \cdot \Big(\vec{\omega} \times \underbrace{\sum m_i \vec{r}_{i/p}}_{\text{Center of mass}}\Big) + \frac{1}{2}\underbrace{\Big(\sum m_i |r_{i/p}|^2\Big)}_{I_p}\omega^2$$

but, $$\vec{r}_{G/p} = \frac{\sum m_i \vec{r}_{i/p}}{\sum m_i} = \frac{\sum m_i \vec{r}_{i/p}}{m}$$

$$T = \frac{1}{2}\sum m_i |\vec{v}_p|^2 + m\,\vec{v}_p \cdot (\vec{\omega} \times \vec{r}_{G/p}) + \frac{1}{2}\Big(\sum m_i |r_{i/p}|^2\Big)\omega^2$$

$$\boxed{T = \frac{1}{2} m |\vec{v}_p|^2 + m\,\vec{v}_p \cdot (\vec{\omega} \times \vec{r}_{G/p}) + \frac{1}{2} I_p \omega^2}$$

where $m = \sum_i m_i$ ← Total mass of system.

$I_p = \sum_i m_i |\vec{r}_{i/p}|^2 = \int r^2\,dm$ mass m. of I.

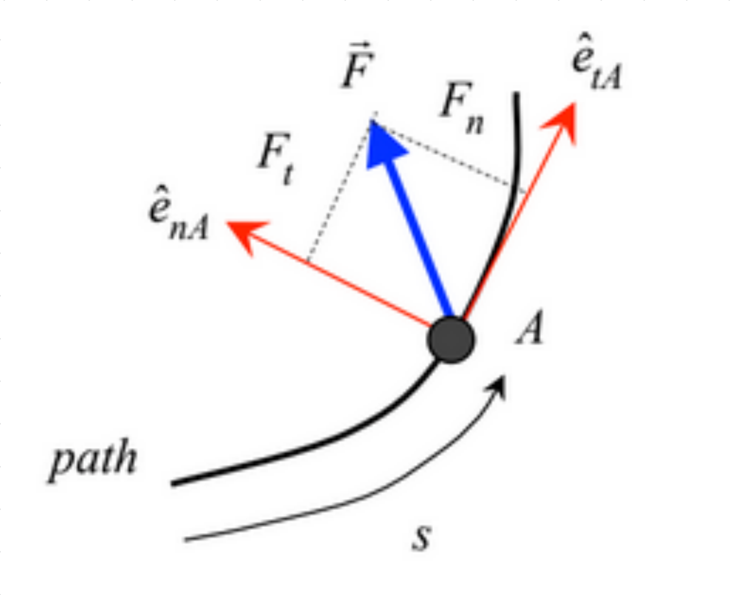
and $\vec{r}_{G/p} = \frac{1}{m}\sum m_i \vec{r}_{i/p}$ Pos. of G wrt P

2) $V = V_e + V_g$: Potential energy of system

3) Work:

$$U_{1-2} = \int_1^2 F \cdot e_{t_A} \, ds = \int_1^2 F_T \, ds$$

If path yields a difficult integral, one can move the force to another point B of less complex path, and translate the force using equivalent force couple system.



Work Energy Equation

** Chapter 5.B of the Lecturebook*
** Chapter 6.B of Meriam Textbook 7th Edition.*

Kinetic Energy of Rigid Body

Special form of Kinetic Energy:

$$T = \frac{1}{2}mv_P^2 + \frac{1}{2}I_P\omega^2 + m\vec{v}_P \cdot (\vec{\omega} \times \vec{r}_{G/P})$$

- P is chosen to be center of mass G:

$\vec{r}_{G/P} = 0 \Rightarrow \quad T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$

* Always safe choice

- P is chosen to be a point with 0 velocity:

P instant center

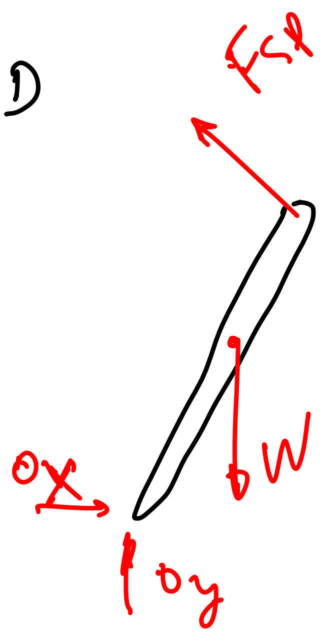
$\vec{v}_P = 0 \Rightarrow \quad T = \frac{1}{2} I_P \omega^2$

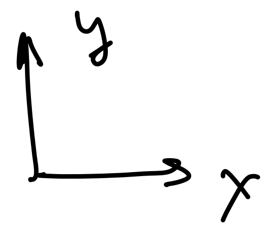
Example 5.B.3

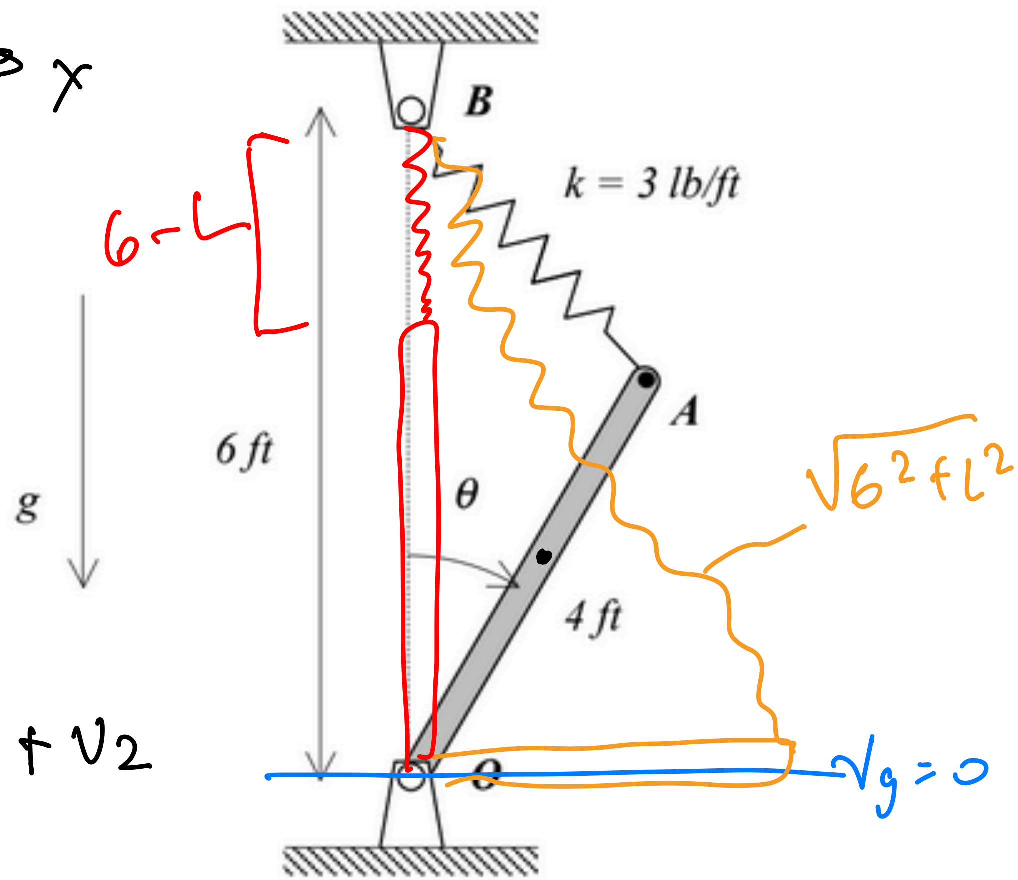
Given: The spring acting on a thin, homogeneous bar (of weight $W = 15$ lb) is unstretched when $\theta = 0$.

Find: Determine the angular speed of the bar at $\theta = 0$, if the bar just reaches $\theta = 90°$ before coming to rest.

1) FBD







2) Kinetics

$$T_1 + V_1 + U^{NC}_{1-2} = T_2 + V_2$$

$$T_1 = \frac{1}{2} I_O \omega_1^2$$

$$T_1 = \frac{1}{2}\left(\frac{1}{12} m L^2 + m \bar{d}_{OG}^{\,2}\right)\omega_1^2$$

$$T_1 = \frac{1}{2}\left(\frac{1}{3} m L^2\right)\omega_1^2 = \frac{1}{6} m L^2 \omega^2$$

$$V_1 = mg\frac{L}{2}$$

$$V_2 = 0 + \frac{1}{2}K\left[\left(\sqrt{6^2+2^2} - (6-L)\right]^2$$

3) Solve For $\underline{\underline{\omega_1}}$