Practice Problem

Example 5.A.11

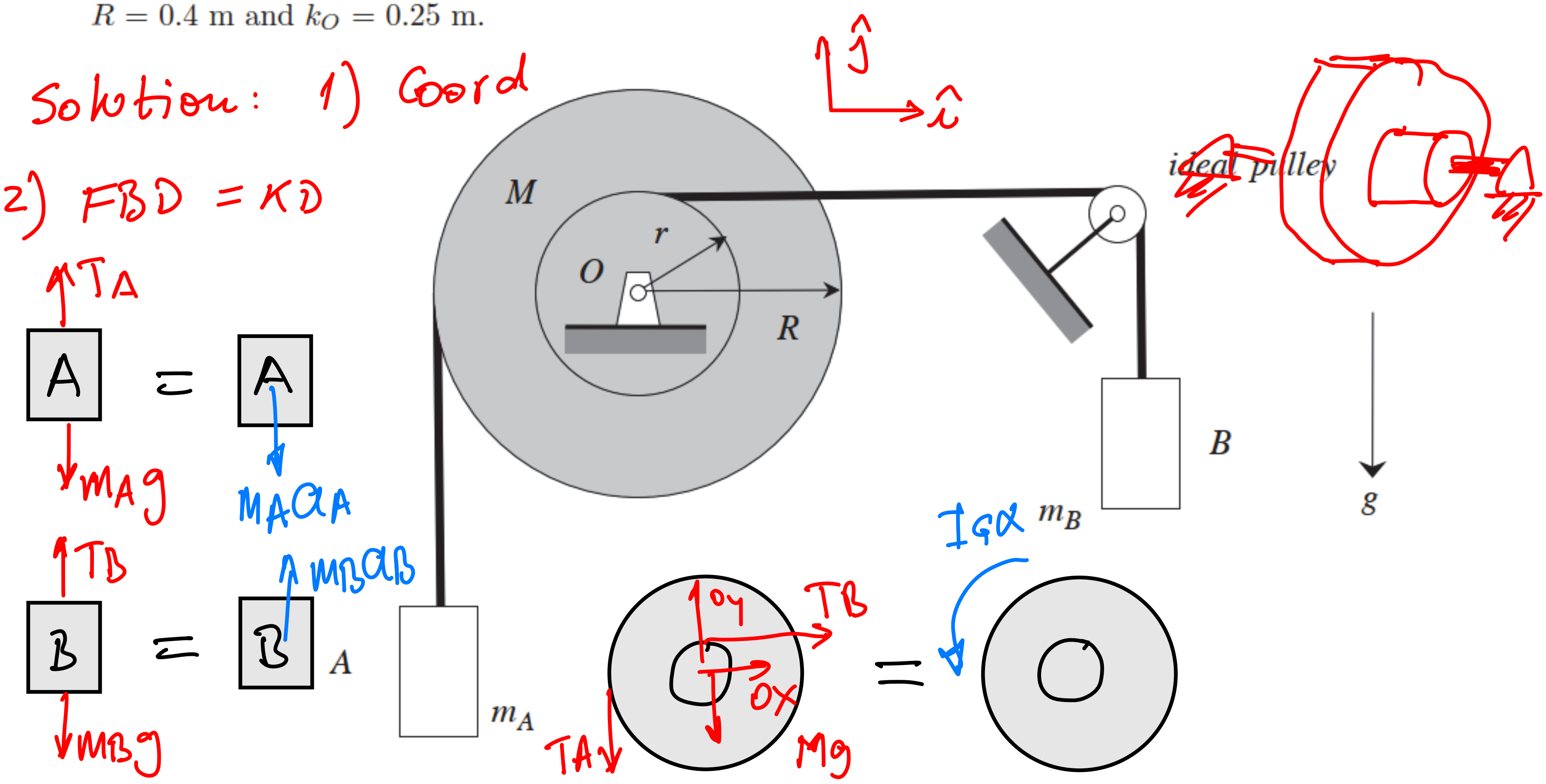
Given: A stepped drum (having a mass of M and radius of gyration about its center O of k_O) is attached to a smooth shaft passing through its center O. A cable wrapped around the outer radius of the drum is attached to block A. A second cable is wrapped around the inner radius of the drum with this cable pulled over an ideal pulley and is attached to block B. Assume that the cables do not slip on the drum. The system is released from rest.

Find: Determine the angular acceleration of the drum on release. Write your answer as a vector.

Use the following parameters in your analysis: $m_A = 10$ kg, $m_B = 30$ kg, $M = 20$ kg, $r = 0.2$ m, $R = 0.4$ m and $k_O = 0.25$ m.

Solution: 1) Coord

2) FBD = KD

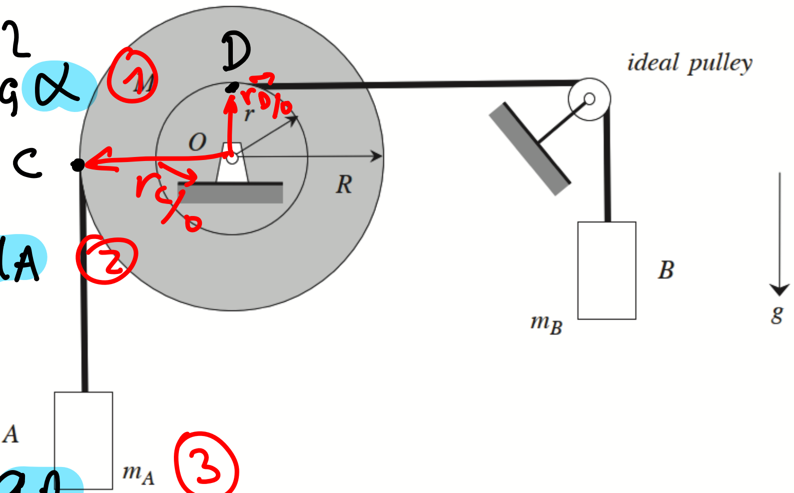


3) Kinetics DRUM

$$+\circlearrowleft \quad \sum M_G = I_G \alpha \Rightarrow T_A R - T_B r = M k_G^2 \alpha \quad (1)$$

Mass A

$$+\uparrow \quad \sum F_y = m_A a_A \Rightarrow T_A - m_A g = -m_A a_A \quad (2)$$



Mass B

$$+\uparrow \quad \sum F_y = m_B a_B \Rightarrow T_B - m_B g = +m_B a_B \quad (3)$$

If we try to do $\sum F_x$ and $\sum F_y$ of the drum, we introduce additional unknowns O_x and O_y. So let's directly try Kinematics.

4) Kinematics

$$\vec{a}_C = \cancel{\vec{a}_O}^{\,0} + \alpha \times \vec{r}_{C/O} - \cancel{\omega^2 \vec{r}_{C/O}}^{\,\text{orFr}}$$

$$a_{Cx}\hat{i} + a_{Cy}\hat{j} = \alpha \hat{k} \times (-R)\hat{i} = -\alpha R \hat{j}$$

$$\hat{i}: \quad a_{Cx} = 0$$

$$\hat{j}: \quad a_{Cy} = -\alpha R \Rightarrow a_A = \alpha R \quad (4)$$

$$\vec{a}_D = \cancel{\vec{a}_O}^{0} + \alpha \times \vec{r}_{D/O} - \cancel{\omega^2} \vec{r}_{D/O}$$

$$a_{Dx}\hat{\imath} + a_{Dy}\hat{\jmath} = \alpha\hat{k} \times r\hat{\jmath} = -\alpha r\hat{\imath}$$

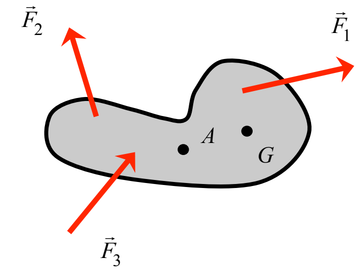
$$\hat{\imath}: \quad a_{Dx} = -\alpha r \Rightarrow a_B = \alpha r \quad 5$$

$$\hat{\jmath}: \quad a_{Dy} = 0$$

$$\alpha = -4.94\,\hat{k}\ \frac{rad}{s^2}$$

3/30/2026

Summary: Newton/Euler Equations 3



FUNDAMENTAL equations:

(1) $\sum \vec{F} = m\vec{a}_G$

(2) $\sum \vec{M}_{\boxed{A}} = I_{\boxed{A}}\vec{\alpha}$; A = c.m. *OR* fixed point *OR* $\vec{r}_{G/A} \parallel \vec{a}_A$

SAME point "A"!

SOLUTION METHOD: the four-step plan

Kinetics: Four-Step Problem Solving Method

The suggested plan of action for solving kinetics problems:

1. ***Free body diagram(s)***. Draw appropriate free body diagrams (FBDs) for the problem. Your choice of FBDs is problem dependent. For some problems, you will draw an FBD for each body; for others, you will draw an FBD for the entire system. An integral part of your FBDs is your choice of coordinate systems. For each FBD, draw the unit vectors corresponding to your coordinate choice.

2. ***Kinetics equations***. At this point, you will need to choose what solution method(s) that you will need to use for the particular problem at hand. In this section of the course we will study four basic methods: Newton/Euler, work/energy, linear impulse/momentum and angular impulse/momentum. Based on your choice of method(s), write down the appropriate equations from your FBD(s) from Step 1.

3. ***Kinematics***. Perform the needed kinematic analysis. A study of the equations in Step 2 above will guide you in deciding what kinematics are needed to find a solution to the problem.

4. ***Solve***. Count the number of unknowns and the number of equations from above. If you do not have enough equations to solve for your unknowns, then you either: (i) need to draw more FBDs, OR (ii) need to do more kinematic analysis. When you have sufficient equations for the number of unknowns, solve for the desired unknowns from the above equations.

Draw INDIVIDUAL free-body diagrams for Newton/Euler.

Be sure to use the correct mass moment of inertia for your choice of point "A". Use PAT if necessary.

Typically the most difficult step. Recall the rigid body kinematics from Chapter 2.

If you are short equations, go back to Step 3 – Kinematics.

Example 5.A.15

Given: The homogeneous sphere (having a mass of m and radius R) is released from rest on a rough incline at an angle of θ. After release, the sphere rolls without slipping on the incline.

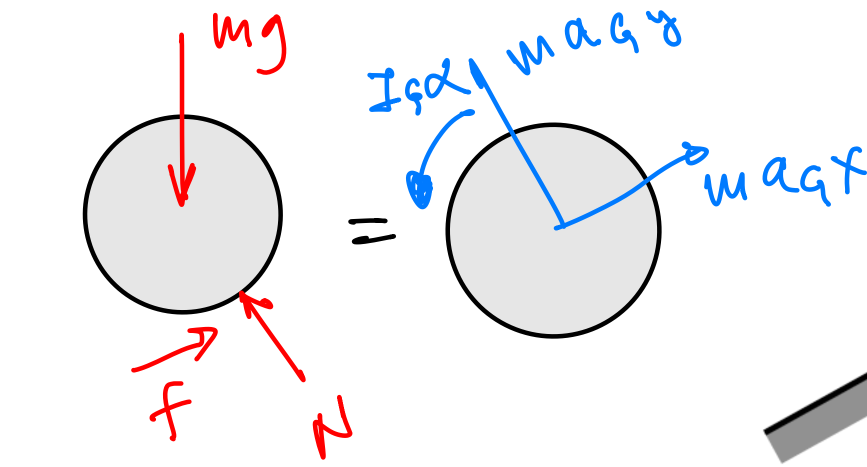
Find: Determine:

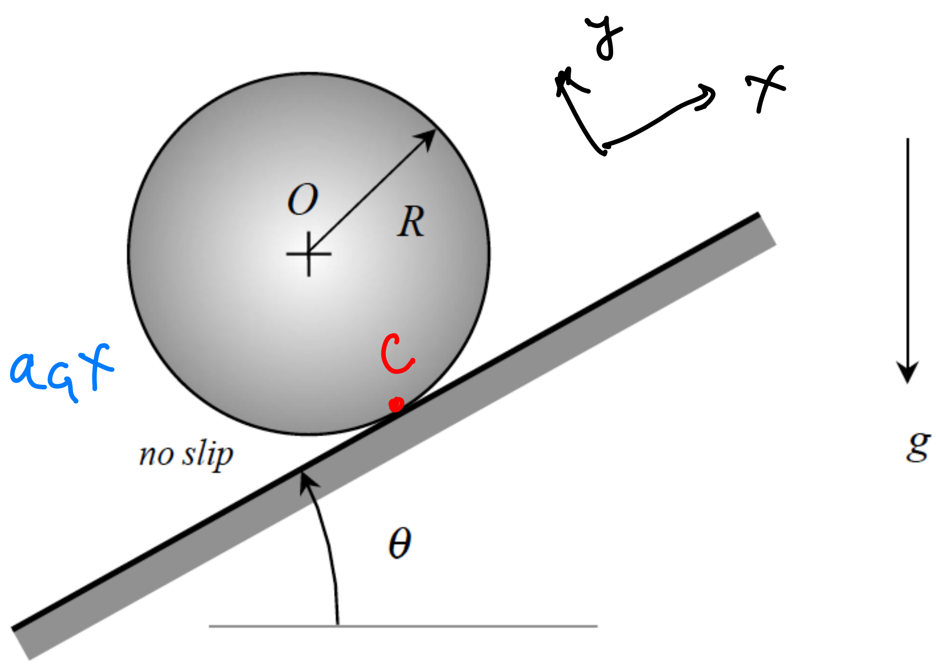
(a) The acceleration of the center of mass G of the sphere; and
(b) The force of friction acting on the sphere on release.

2) Coord

Solution.

1) FBD





3) Kinetics.

$$\Sigma F_x = m a_{Gx} \Rightarrow f - mg\sin\theta = m a_{Gx} \quad (1)$$

$$\Sigma F_y = m a_{Gy} \Rightarrow N - mg\cos\theta = 0 \quad (2)$$

$$N = mg\cos\theta$$

$$\sum M_G = I_G \alpha \implies f\cancel{R} = I_G \alpha = \underbrace{\frac{2}{5} R^{\cancel{2}}}_{\text{check.}} \alpha \quad (3)$$

$$f = \frac{2}{5} m R \alpha$$

4) Kinematics. Relate points C and O (or G)

$$\vec{a}_G = \vec{a}_c + \vec{\alpha} \times \vec{r}_{G/C} - \cancel{\omega^2 \vec{r}_{G/C}}^{\,0} \quad (\text{rfr})$$

$$a_{Gx}\hat{\imath} = a_{cy}\hat{\jmath} + \alpha\hat{k} \times R\hat{\jmath} = a_{cy}\hat{\jmath} - \alpha R \hat{\imath}$$

$$\hat{\imath}: \quad a_{Gx} = -\alpha R \quad (4)$$

$$\hat{\jmath}: \quad 0 = a_{cy}$$

5) Solve

a) From (4):

$$\alpha = -\frac{a_{Gx}}{R}$$

in (3):

$$f = \frac{2}{5} m \cancel{R} \frac{a_{Gx}}{\cancel{R}} \implies f = \frac{2}{5} m a_{Gx}$$

Now, in (1)

$$\frac{2}{5} \cancel{m} a_{Gx} - \cancel{m} g \sin\theta = \cancel{m} a_{Gx}$$

$$g \sin\theta = -\frac{3}{5} a_{Gx}$$

$$\Rightarrow \quad a_G = -\frac{5}{3} g \sin\theta \, \hat{\imath} \qquad m/s^2$$

6)

$$f = \frac{2}{5} m \left(-\frac{5}{3} g \sin\theta\right)$$

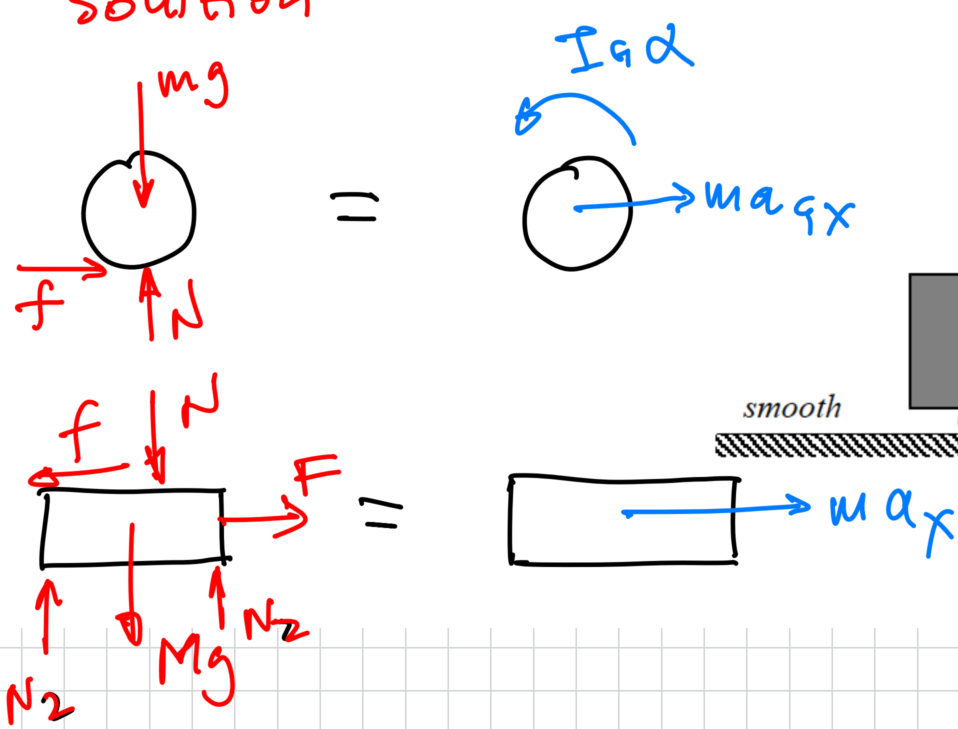
$$\boxed{f = -\frac{2}{3} m g \sin\theta} \quad \hat{\imath} \qquad N$$

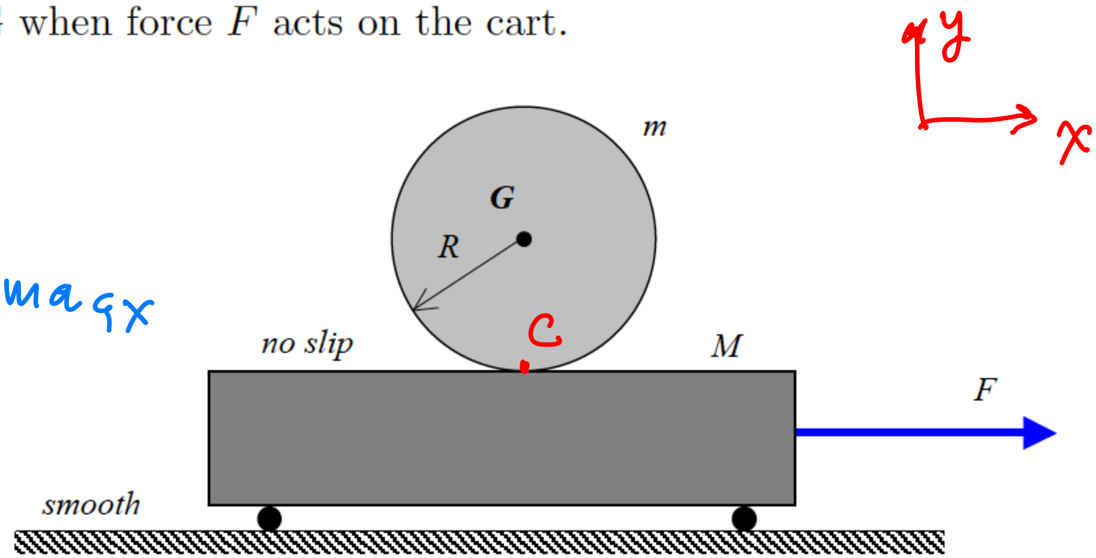
Example 5.A.16

Given: The system is initially at rest.

Find: Determine the acceleration of G when force F acts on the cart.

Solution





Disk

$$\Sigma F_x = m a_{Gx} \Rightarrow f = m a_{Gx} \quad (1)$$

$$\Sigma F_y = 0 \Rightarrow N - mg = 0 \quad (2)$$

$$\Sigma M_G = I_G \alpha \Rightarrow fR = \tfrac{1}{2} m R^2 \alpha \quad (3)$$

Block

$$\Sigma F_x = m a_x \quad \Rightarrow \quad -f + F = M a_x \qquad (4)$$

$$\Sigma F_y = 0 \quad \Rightarrow \quad 2N_2 - N - Mg = 0 \qquad (5)$$

Kinematics

$$\vec{a}_G = \vec{a}_C + \vec{\alpha} \times \vec{r}_{G/C} - \cancel{\omega^2 \vec{r}_{G/C}} \xrightarrow{} 0 \qquad (rfr)$$

$$a_G \hat{i} = a_{Cx} \hat{i} + a_{Cx} \hat{j} \qquad \alpha k \times R \hat{j}$$

$$a_G \hat{i} = a_{Cx} \hat{i} + a_{Cy} \hat{j} - \alpha R \hat{i}$$

$$\hat{i}: \quad a_{Gx} = a_{Cx} - \alpha R \qquad (6)$$

$$\hat{j}: \quad 0 = a_{Cy} \qquad (7)$$

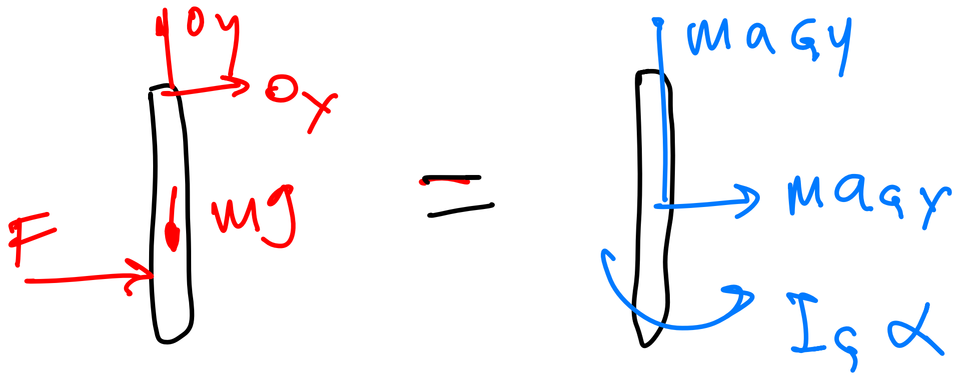
Solve

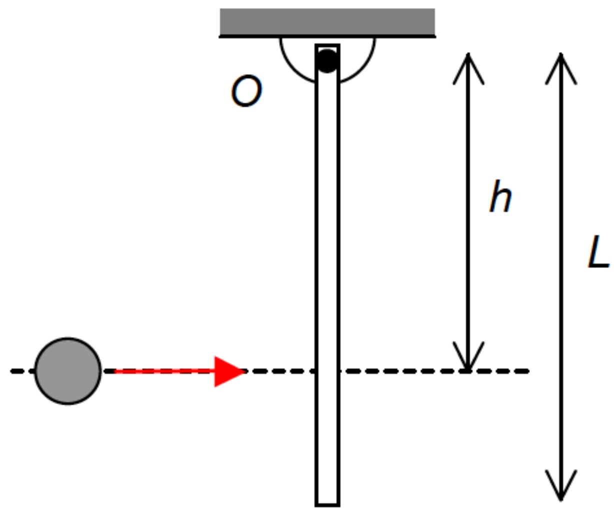
Will include in uploaded material

Example 5.A.17

Given: A ball of mass m strikes a homogeneous bar of length L and mass M at a distance h from the pivot point O.

Find: Determine the distance h such that the horizontal reaction force at O on the bar is zero. (This location on the bar is known as the body's "center of percussion". Can you think of a practical application of knowing the location of the center of percussion of a body?)





$$\Sigma F_x = m a_{Gx} \Rightarrow F + O_x = M a_{Gx}$$

$$\Sigma F_y = m a_{Gy} \Rightarrow O_y - mg = M a_{Gy}$$

$$\Sigma M_O = I_O \alpha \Rightarrow F \cdot h = \tfrac{1}{3} M L^2 \alpha$$

Kinematics

$$a_G = \cancelto{0}{a_O} + \alpha \times r_{G/O} - \cancel{\omega^2 r_{G/O}}$$

$$a_{Gx}\hat{i} + a_{Gy}\hat{j} = \frac{\alpha L}{2}\hat{i}$$

$$\Rightarrow h = \frac{2}{3} L$$