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ME 274 Lecture 41

Vibrations: Forced Response – Part 3

Eugenio “Henny” Frias-Miranda

04/29/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. Office hours are on ME2008. Located on second floor of renovated side of ME.
2. Quizzes, there will be a total of 9 quizzes this semester:
   - Extra credit quizzes 2 and 3
     - Help you make up an unexcused attendance (ie. you forgot to email me before the lecture you missed)
     - **EC Quiz 2: H.6.K. H.6.L (Due Wednesday the 29th of April)**
     - **EC Quiz 3: H.6.M. H.6.N (Due Friday the 1st of May)**
   - Quiz 08 – Find a video in the following link below. In your own handwriting, talk about why you enjoyed the video and its relevance to the ME274 coursework. 3-5 sentences.
     - https://www.purdue.edu/freeform/me274/course-material/visualizing-mechanics/
     - **Due Friday, May 1st**
     - Remote/online quiz. Can be done on white sheet of paper. Just make sure you complete it on your own.
   - Quiz 09 - Thank you for filling out QR codes at end of lecture. I will be making a simple multiple-choice quiz asking:
     - "Did you fill out at least one post-lecture feedback survey? Y/N".
     - **Due Friday, May 1st**
     - Remote/online quiz. No handout, just make sure you fill it out.
3. **Semester Course Evals (make sure to fill out!).** Submit On Gradescope.
4. Final Exam details:

Tuesday, May 5, 7:00-9:00 PM

Coverage:

- For Problems 1-3: Lectures 27-42 on rigid body kinetics and vibrations
- For Problem 4: throughout entire course

WL locations

- WTHR 200 (260): CA (109), EF (109), LK (35, A-H) and 2 TAs

Review sessions

- ME 274 Instructor, CK: Sunday, May 3, 7:00 PM, live on Zoom for both WL and Indy: https://purdue-edu.zoom.us/j/99152873491?pwd=cl8taKJJeuxJUII9GmWMILbGYILWIm.1 Video to be posted on website.

Chapter 6: Vibrations

$$\boxed{\begin{aligned}\sum \vec{F} &= m\vec{a}_G \\ \sum \vec{M}_A &= I_A\vec{\alpha}\end{aligned}}$$

General Form:

$$\boxed{M\ddot{x} + C\dot{x} + Kx = f(t)}$$

Standard Form:

$$\boxed{\ddot{x} + 2\zeta\omega_n\dot{x} + {\omega_n}^2 x = \frac{1}{M}f(t)}$$

Total Response/Solution in terms of: Complementary and Particular Solutions

$$\boxed{x(t) = x_c(t) + x_p(t)}$$

Goals for Chapter 6:

- *We are used* to solving acceleration/velocity/position for a given **instant in time** (using Newton-Euler Equations, upper left).
- In this chapter, we will study the response of systems whose motion is *oscillatory in nature* (~vibrations).
- We will focus on understanding the change in position as a **general function of time** (ie. **Deriving the EOMs of a system**, upper right).

Chapter 6: Vibrations – Forced Response

- Throughout the last three lectures (Lec 36-38) we have looked at "Free response", given by the **complementary/homogeneous solution.**

- Now, we will be evaluating the "Forced Response" of a system (Lec 39-41). Given by the particular solution.

$$x(t) = \boxed{x_c(t)} + \boxed{x_p(t)}$$

- Complementary (a.k.a. Homogeneous) Solution
  - Free Response
- Particular Solution
  - Forced Response

$$\boxed{x_p(t) = A\sin\omega t + B\cos\omega t}$$

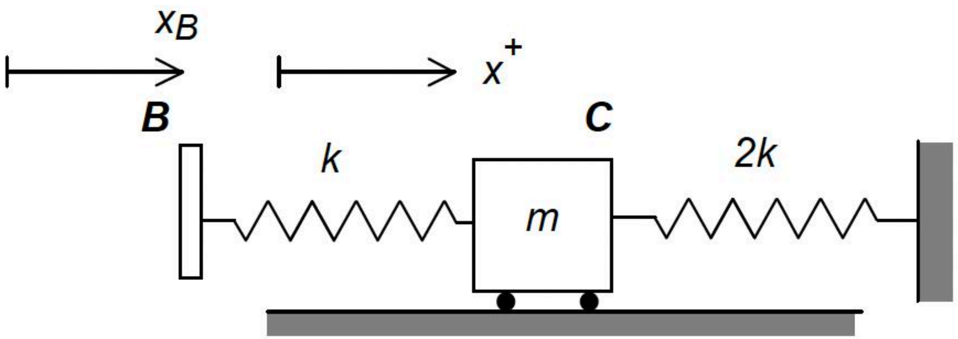
[p. 414]

Systems with Base Excitation

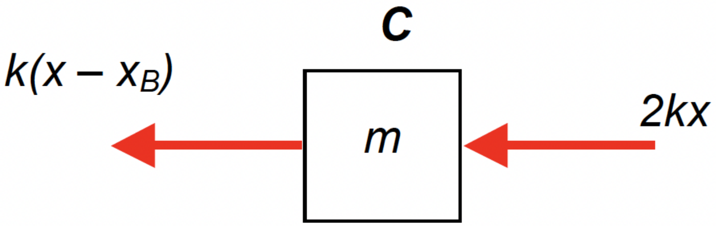
1. Many problems that we will consider for forced vibrations **do not have the force applied on the components of interest** in the system.

2. Therefore, we account for this in our FBDs:

MOTIVATING EXAMPLE: Consider the system shown below where the base B is given a prescribed displacement of $x_B(t) = b \sin \omega t$.



3. FBD:



4. EOM:

$$\sum F_x = -k(x - x_B) - 2kx = m\ddot{x} \quad \Rightarrow$$

$$m\ddot{x} + 3kx = kx_B(t) = kb \sin \omega t$$

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Terms to know about harmonic motion

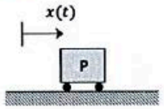
1. Amplitude (A or X)
    - The magnitude of a harmonic function (ie b*sin(wt)) is equal to b.

2. Angular frequency (omega)
    - a.k.a. 'the excitation frequency'
    - Pay attention to units:
        - Angular frequency (omega) = rad /s
        - Frequency (f) = 1 / s
3. Phase (phi)
    - Notice the case when we have 180 degrees of phase shift (lower half of the handout).
        - The response is the exact opposite of when the phase was = 0.
        - We label this response as "out-of-phase"

Note: To determine the kind of phase of a response we compare w with w_n:

- If w < w_n : in-phase
- If w = w_n : resonance or phase is 90 degrees
- If w > w_n : out-of-phase

Parameters for describing harmonic motion
ME 274 ; 12:30pm - efm



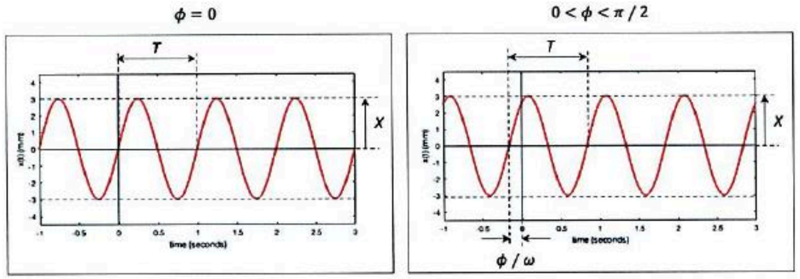
Suppose that the position "x" (in mm) of some point P is written in terms of time "t" (in seconds). If $x(t)$ represents "harmonic motion" of P, $x(t)$ can be written as:

$$x(t) = X\sin(\omega t + \phi)$$

As seen in this equation, there are three parameters in this equation that are required to quantify this harmonic motion:

- X = the "amplitude" of the motion (in mm)
- ω = the "angular frequency" (in rad/s)
- ϕ = the "phase" (in rad)

Consider the two plots below of $x(t)$, one with $\phi = 0$, and one with $\phi > 0$.

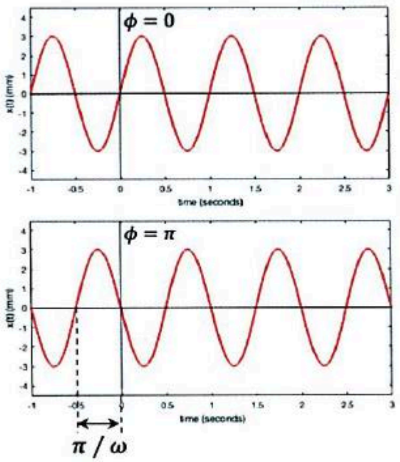


Period and angular frequency
The time required for x(t) to complete one cycle of motion is called the "period", T. Considering the $\phi = 0$ plot on the left above, the period is the time T over which the argument of the sine function goes from zero to 2π; that is, $\omega T = 2\pi$, or:

$$\omega = \frac{2\pi}{T} = \text{angular frequency (rad/s)}$$

Frequency in hertz
The frequency of the harmonic function is often described in terms of "cycles/second" (or, Hz) using the symbol of "f". Recall that there are 2π radians in one cycle of motion. Suppose that we know that $\omega = 10$ rad/s. From this, the frequency in Hz is given by: (10 rad/s)(cycle/2π rad) = $\frac{10}{2\pi}$ cycles/s = $\frac{5}{\pi}$ Hz. In general, we can write:

$$f = \frac{\omega}{2\pi} = \frac{1}{T} \text{ (Hz)}$$



Phase shift
In this course, we will often encounter a harmonic response that has a "180° phase shift". How does a harmonic function with $\phi = \pi$ look? See the plot to the right. You see that the $\phi = \pi$ phase-shifted harmonic function has its sign "flipped" over the entire motion with unchanged amplitude and frequency.

Example 6.C.6

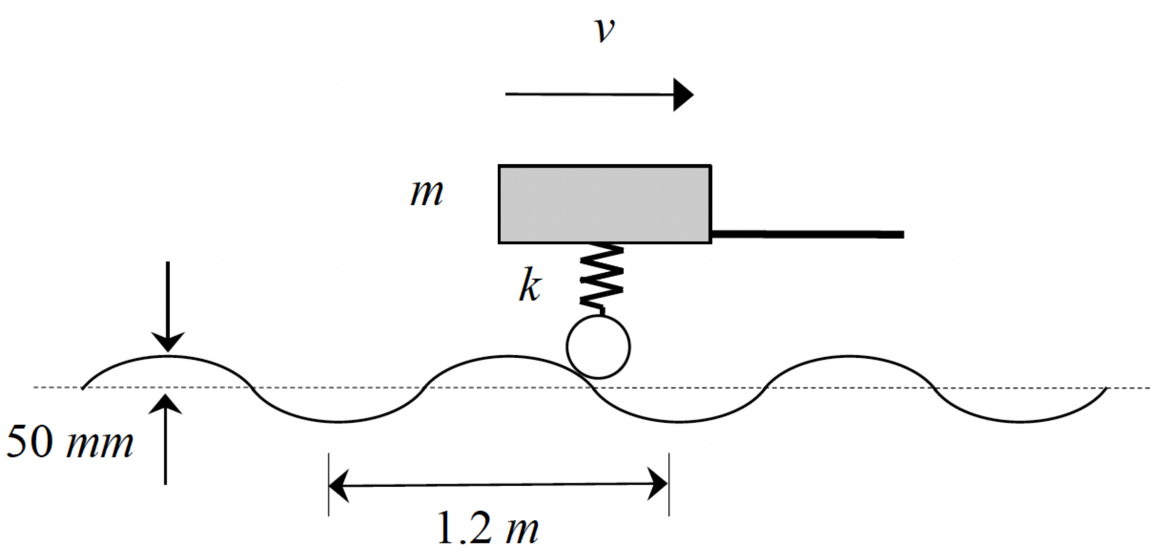
Given: Adding 75 kg of mass to the trailer creates 3 mm of additional sag in the suspension.

Find: Determine:

(a) The vibration amplitude X; and

(b) The critical speed at which the undamped oscillations are the greatest as the trailer is pulled along a roadway with sinusoidal waviness.

Use the following parameters in your analysis: $m = 400$ kg and $v = 25$ km/hr.



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Example 6.C.6 p. 426 "U-haul trailer simulator" problem

Given: Adding 75 kg of mass to the trailer creates 3 mm of additional sag in the suspension.

Find: Determine:

(a) The vibration amplitude X; and

(b) The critical speed at which the undamped oscillations are the greatest as the trailer is pulled along a roadway with sinusoidal waviness.

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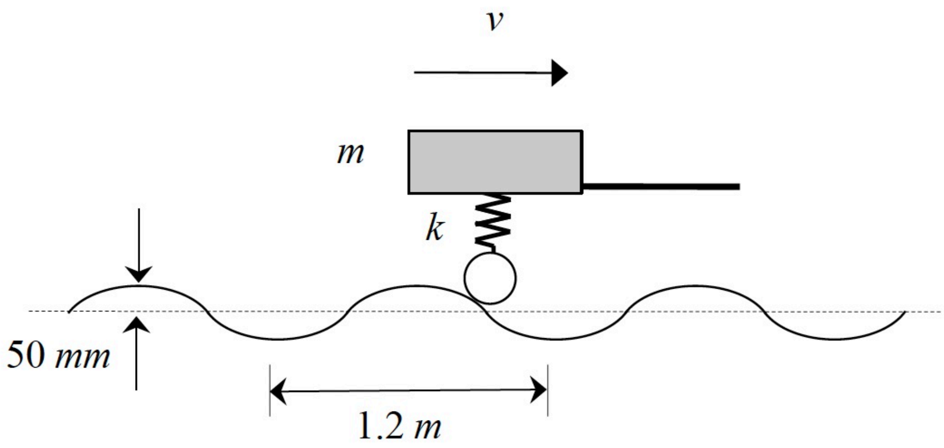
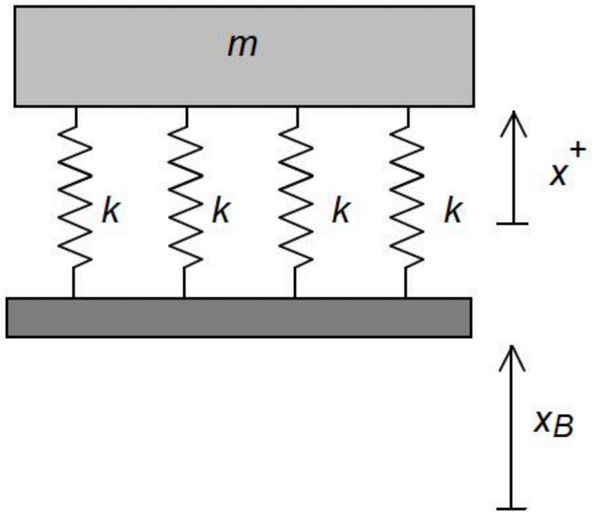


Example 6.C.9

Given: The system shown below.

Find: Determine:
(a) The differential equation of motion for the system shown; and
(b) The amplitude of the steady-state response of the system.

Use the following parameters in your analysis: $m = 50$ kg, $k = 7500$ N/m and $x_B(t) = 0.002 \sin 50t$.



[pg. 429]

Example 6.C.9

p. 429

Given: The system shown below.

Find: Determine:
(a) The differential equation of motion for the system shown; and
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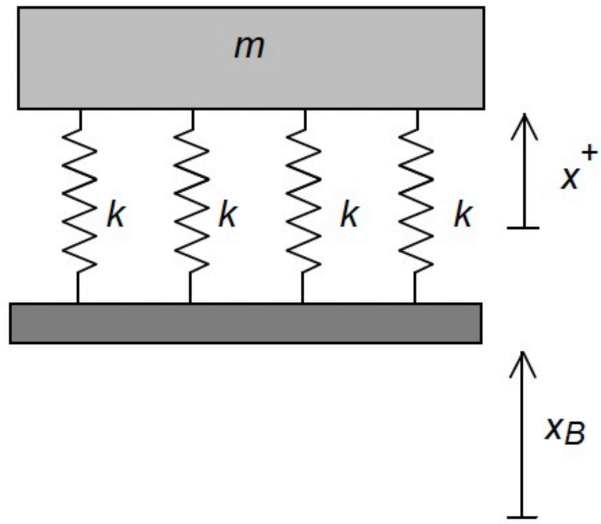
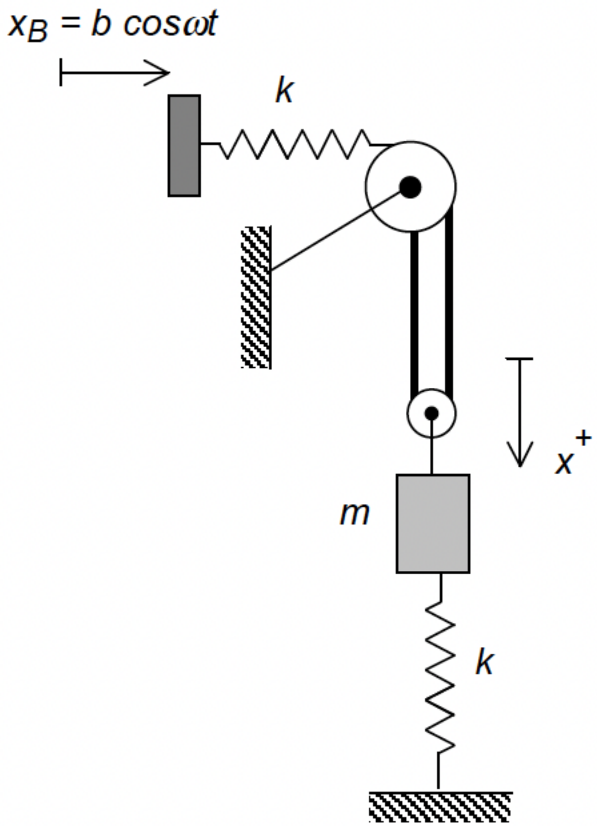


Example 6.C.10

Given: The system shown below.

Find: Determine:
(a) The differential equation of motion for the system shown; and
(b) The resonant frequency of the system.



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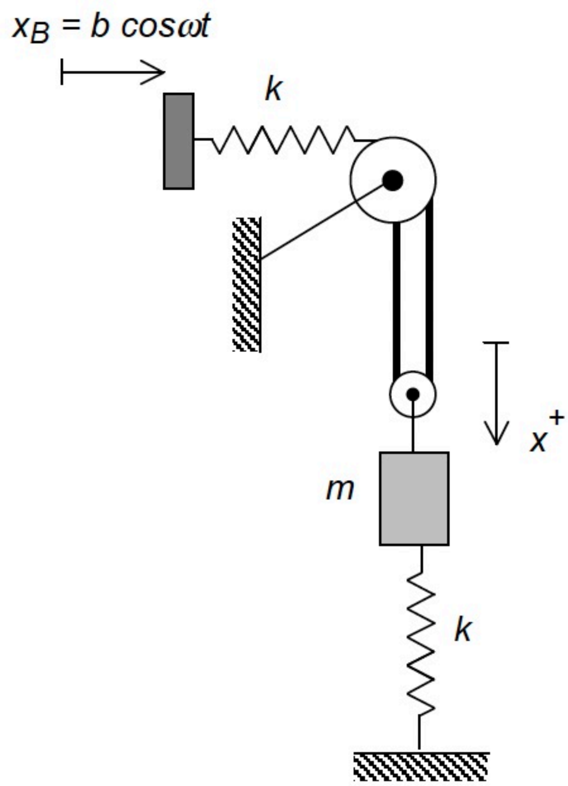
Example 6.C.10 p.430

Given: The system shown below.

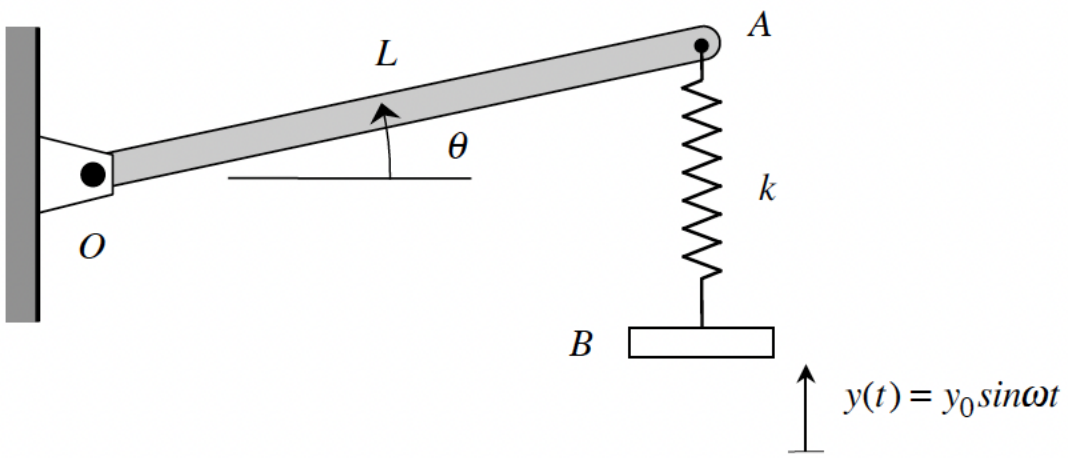
Find: Determine:
(a) The differential equation of motion for the system shown; and
(b) The resonant frequency of the system.



Example 6.C.11

Given: A thin homogeneous bar of length L and mass m is pinned to ground at its left end O. A spring of stiffness k is connected between end A of the bar and a moveable base B, as shown. The base B is given a prescribed motion of $y(t) = y_0 \sin \omega t$.

Find: Find the differential equation of motion for the system in terms of the rotation angle θ. Assume small oscillations.



HORIZONTAL PLANE

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Example 6.C.11 p.431

Given: A thin homogeneous bar of length L and mass m is pinned to ground at its left end O. A spring of stiffness k is connected between end A of the bar and a moveable base B, as shown. The base B is given a prescribed motion of $y(t) = y_0 \sin \omega t$.

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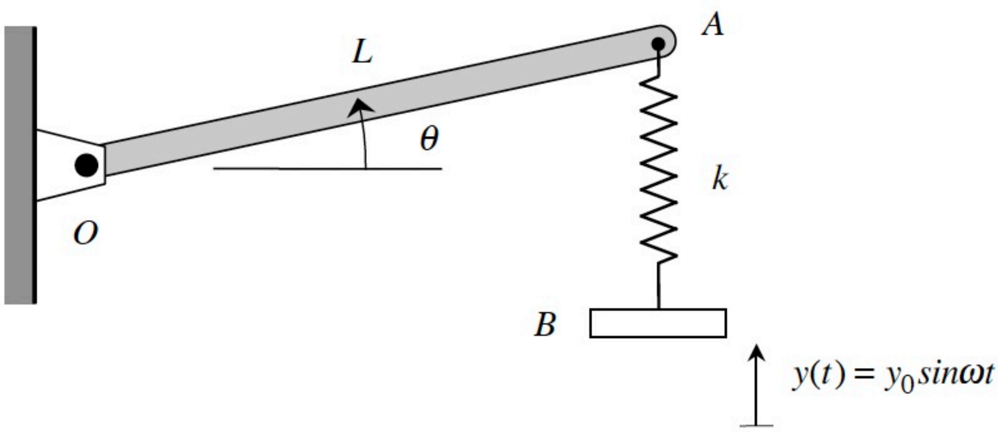

HORIZONTAL PLANE

Summary: Vibrations - Forced Response 2

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EOM: For forced response:

$$M\ddot{x} + C\dot{x} + Kx = F_0 sin\Omega t \quad \Rightarrow \quad \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{M} sin\Omega t$$

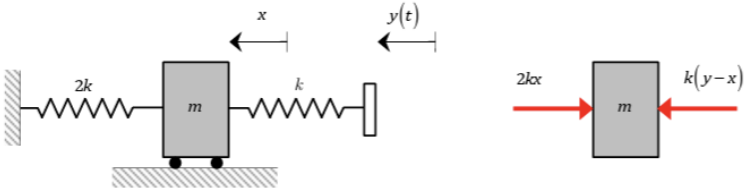
PARTICULAR SOLUTION

$$x_P(t) = Asin\Omega t + Bcos\Omega t$$

Substitute into EOM and solve for A and B. If the system is undamped, then you will find that $B = 0$.

BASE EXCITATION: For many problems, the energy input is through a moving support (prescribed motion, y(t)), rather than an applied force.

Example:



$$\sum F_x = -2kx + k(y - x) = m\ddot{x} \quad \Rightarrow \quad m\ddot{x} + 3kx = \boxed{ky(t)}$$

base excitation

Lec 41 Short Feedback Form: