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ME 274 Lecture 40

Vibrations: Forced Response – Part 2

Eugenio “Henny” Frias-Miranda

04/27/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. Office hours are on ME2008. Located on second floor of renovated side of ME.

2. Quizzes, there will be a total of 9 quizzes this semester:

- Quiz 08 – Find a video in the following link below. In your own handwriting, talk about why you enjoyed the video and its relevance to the ME274 coursework. 3-5 sentences.
 - <https://www.purdue.edu/freeform/me274/course-material/visualizing-mechanics/>
 - **Due Friday, May 1st**
 - Remote/online quiz. Can be done on white sheet of paper. Just make sure you complete it on your own.
- Quiz 09 - Thank you for filling out QR codes at end of lecture. I will be making a simple multiple-choice quiz asking:
 - “Did you fill out at least one post-lecture feedback survey? Y/N”.
 - **Due Friday, May 1st**
 - Remote/online quiz. No handout, just make sure you fill it out.
- Extra credit quizzes 2 and 3
 - Help you make up an unexcused attendance (ie. you forgot to email me before the lecture you missed)
 - **EC Quiz 2: H.6.K. H.6.L (Due Wednesday the 29th of April)**
 - **EC Quiz 3: H.6.M. H.6.N (Due Friday the 1st of May)**

3. **Semester Course Evals (make sure to fill out!).** Submit On Gradescope.

4. Final Exam details:

Tuesday, May 5, 7:00-9:00 PM

Coverage:

- For Problems 1-3: Lectures 27-42 on rigid body kinetics and vibrations
- For Problem 4: throughout entire course

WL locations

- WTHR 200 (260): CA (109), EF (109), LK (35, A-H) and 2 TAs

Review sessions

- ME 274 Instructor, CK: Sunday, May 3, 7:00 PM, live on Zoom for both WL and Indy:

<https://purdue-edu.zoom.us/j/99152873491?pwd=cl8taKJJeuxJUll9GmWMILbGYlLWlm.1>

Video to be posted on website.

Chapter 6: Vibrations

$$\sum \vec{F} = m\vec{a}_G$$

$$\sum \vec{M}_A = I_A \vec{\alpha}$$

General Form:

$$M\ddot{x} + C\dot{x} + Kx = f(t)$$

Standard Form:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = \frac{1}{M}f(t)$$

Total Response/Solution in terms of: Complementary and Particular Solutions

$$x(t) = x_c(t) + x_p(t)$$

Goals for Chapter 6:

- *We are used* to solving acceleration/velocity/position for a given **instant in time** (using Newton-Euler Equations, upper left).
- In this chapter, we will study the response of systems whose motion is *oscillatory in nature* (~vibrations).
- We will focus on understanding the change in position as a **general function of time** (ie. **Deriving the EOMs of a system**, upper right).

Chapter 6: Vibrations – Forced Response

- Throughout the last three lectures (Lec 36-38) we have looked at “Free response”, given by the **complementary/homogeneous solution**.
- Now, we will be evaluating the “Forced Response” of a system (Lec 39-41). Given by the particular solution.

$$x(t) = x_c(t) + x_p(t)$$

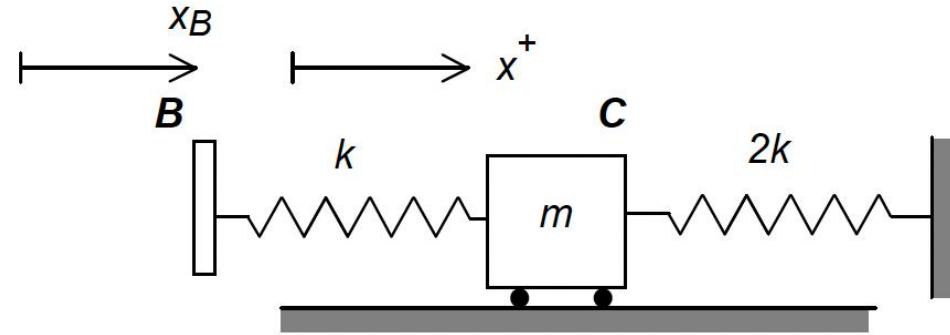
- Complementary (a.k.a. Homogeneous) Solution
 - Free Response
- Particular Solution
 - Forced Response

$$x_p(t) = A \sin \omega t + B \cos \omega t$$

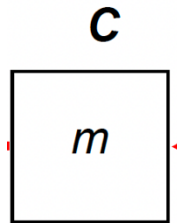
Systems with Base Excitation

1. Many problems that we will consider for forced vibrations **do not have the force applied on the components of interest** in the system.
2. Therefore, we account for this in our FBDs:

MOTIVATING EXAMPLE: Consider the system shown below where the base B is given a prescribed displacement of $x_B(t) = b \sin \omega t$.



3. FBD:



4. EOM:

Terms to know about harmonic motion

1. Amplitude (A or X)

- The magnitude of a harmonic function (ie $b \cdot \sin(\omega t)$) is equal to b .

2. Angular frequency (omega)

- a.k.a. 'the excitation frequency'
- Pay attention to units:
 - Angular frequency (omega) = rad / s
 - Frequency (f) = 1 / s

3. Phase (phi)

- Notice the case when we have 180 degrees of phase shift (lower half of the handout).
 - The response is the exact opposite of when the phase was = 0.
 - We label this response as "out-of-phase"

Note: To determine the kind of phase of a response we compare ω with ω_n :

- If $\omega < \omega_n$: in-phase
- If $\omega = \omega_n$: resonance or phase is 90 degrees
- If $\omega > \omega_n$: out-of-phase

Parameters for describing harmonic motion

ME 274 : 12:30pm - 25m

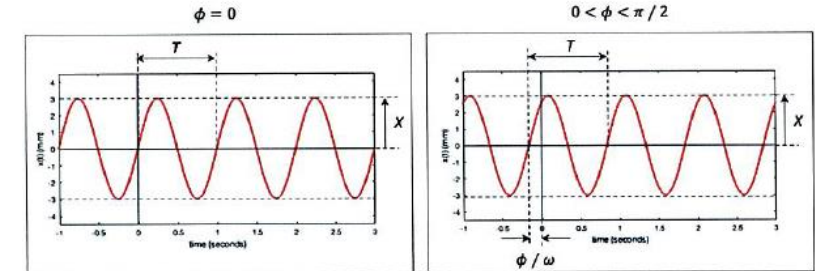
Suppose that the position " x " (in mm) of some point P is written in terms of time " t " (in seconds). If $x(t)$ represents "harmonic motion" of P, $x(t)$ can be written as:

$$x(t) = X \sin(\omega t + \phi)$$

As seen in this equation, there are three parameters in this equation that are required to quantify this harmonic motion:

- X = the "amplitude" of the motion (in mm)
- ω = the "angular frequency" (in rad/s)
- ϕ = the "phase" (in rad)

Consider the two plots below of $x(t)$, one with $\phi = 0$, and one with $\phi > 0$.



Period and angular frequency

The time required for $x(t)$ to complete one cycle of motion is called the "period", T . Considering the $\phi = 0$ plot on the left above, the period is the time T over which the argument of the sine function goes from zero to 2π ; that is, $\omega T = 2\pi$, or:

$$\omega = \frac{2\pi}{T} = \text{angular frequency (rad/s)}$$

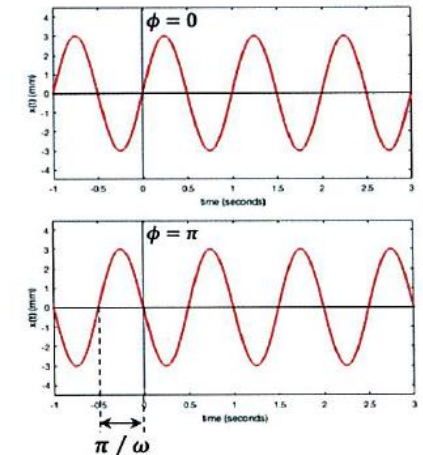
Frequency in hertz

The frequency of the harmonic function is often described in terms of "cycles/second" (or, Hz) using the symbol of " f ". Recall that there are 2π radians in one cycle of motion. Suppose that we know that $\omega = 10$ rad/s. From this, the frequency in Hz is given by: $(10 \text{ rad/s}) / (2\pi \text{ rad}) = \frac{10}{2\pi} \text{ cycles/s} = \frac{5}{\pi} \text{ Hz}$. In general, we can write:

$$f = \frac{\omega}{2\pi} = \frac{1}{T} \text{ (Hz)}$$

Phase shift

In this course, we will often encounter a harmonic response that has a "180° phase shift". How does a harmonic function with $\phi = \pi$ look? See the plot to the right. You see that the $\phi = \pi$ phase-shifted harmonic function has its sign "flipped" over the entire motion with unchanged amplitude and frequency.



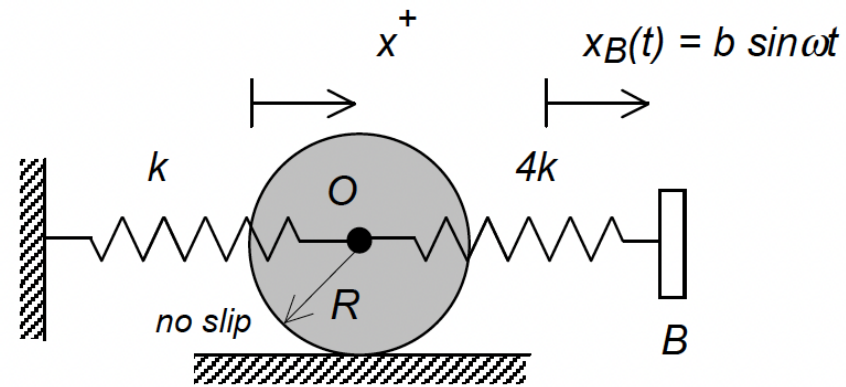
Example 6.C.4

Given: The system shown below consists of a homogeneous disk, of mass m and radius R , which is connected to ground by a spring of stiffness k and a massless moving base B by a spring of stiffness $4k$.

Find:

- (a) Derive the EOM for the disk in terms of the coordinate x ;
- (b) Find the particular solution of the EOM; and
- (c) Determine the amplitude of response corresponding to $\omega = 15$ rad/s. Is the response in phase or out of phase with the forcing?

Use the following parameters in your analysis: $m = 4$ kg, $k = 800$ N/m and $b = 20$ mm.



Example 6.C.4

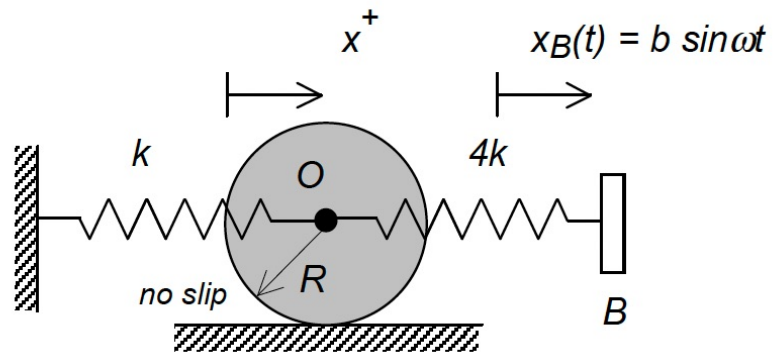
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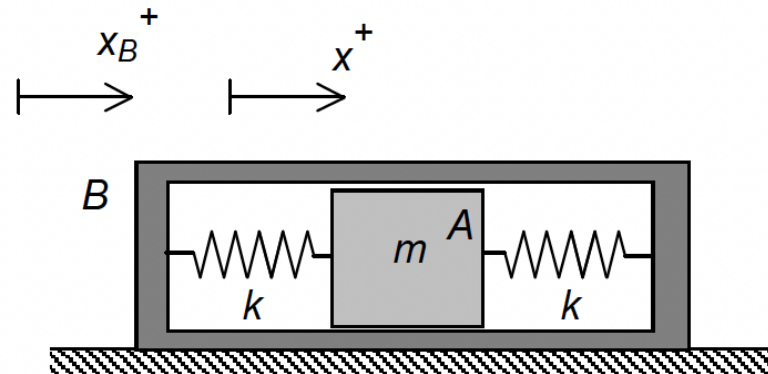
Use the following parameters in your analysis: $m = 4$ kg, $k = 800$ N/m and $b = 20$ mm.



Example 6.C.5

Given: Box B is given a prescribed displacement of $x_B(t) = b \sin \omega t$.

Find: Determine the two values of frequency ω for which the amplitude of forced response of the block A is twice that of the box B.

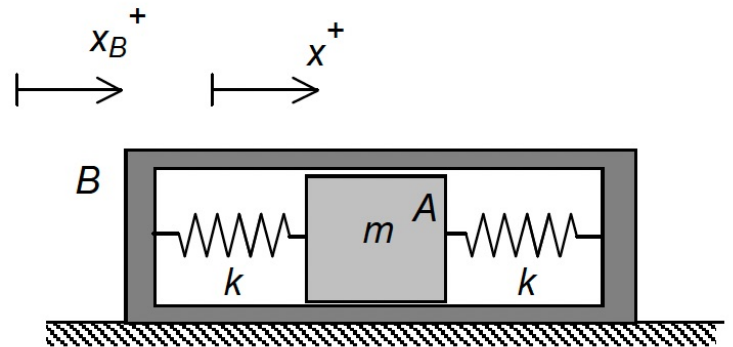


Example 6.C.5

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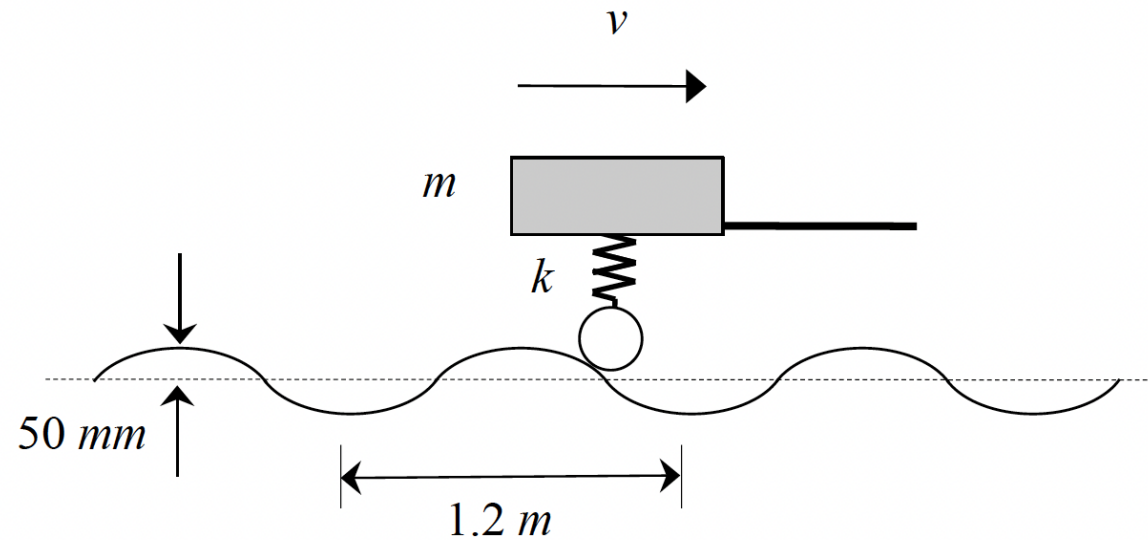
Example 6.C.6

Given: Adding 75 kg of mass to the trailer creates 3 mm of additional sag in the suspension.

Find: Determine:

- (a) The vibration amplitude X ; and
- (b) The critical speed at which the undamped oscillations are the greatest as the trailer is pulled along a roadway with sinusoidal waviness.

Use the following parameters in your analysis: $m = 400$ kg and $v = 25$ km/hr.



Example 6.C.6

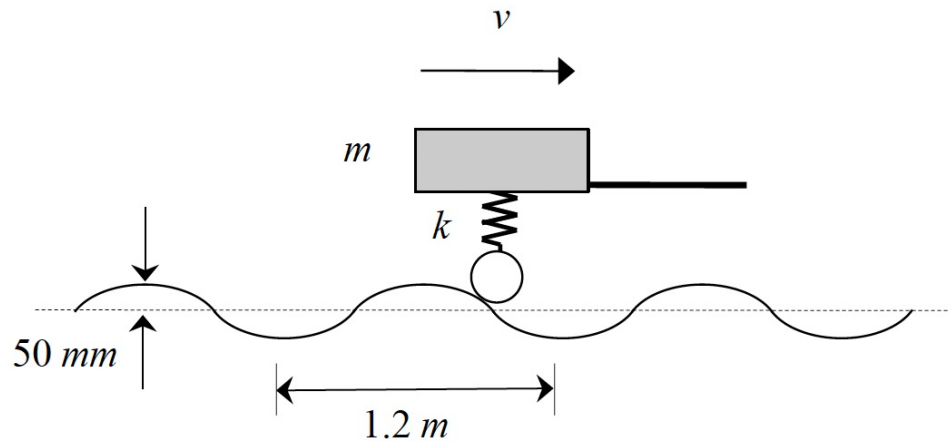
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Given: Adding 75 kg of mass to the trailer creates 3 mm of additional sag in the suspension.

Find: Determine:

- (a) The vibration amplitude X ; and
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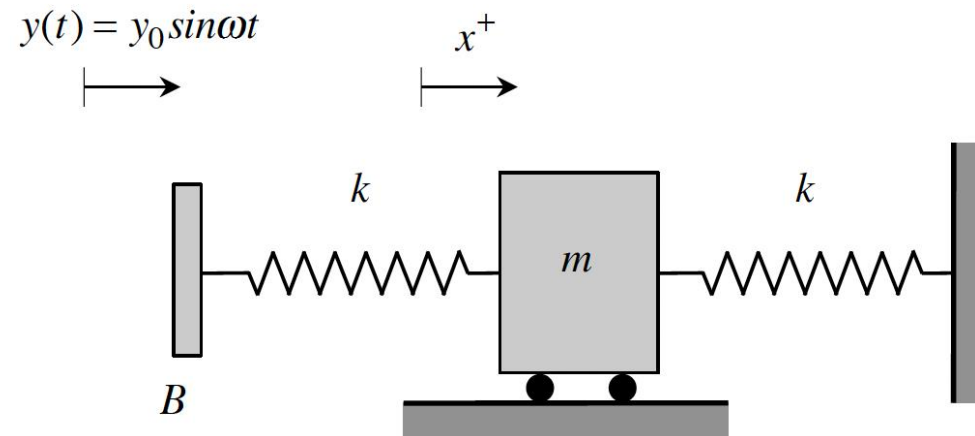


Example 6.C.7

Given: The system shown below is given a prescribed motion for the base B: $y(t) = y_0 \sin \omega t$.

Find: Determine the range of frequencies ω for which the amplitude of the motion for block A does not exceed $1.5y_0$.

Use the following parameters in your analysis: $m = 4 \text{ kg}$ and $k = 800 \text{ N/m}$.



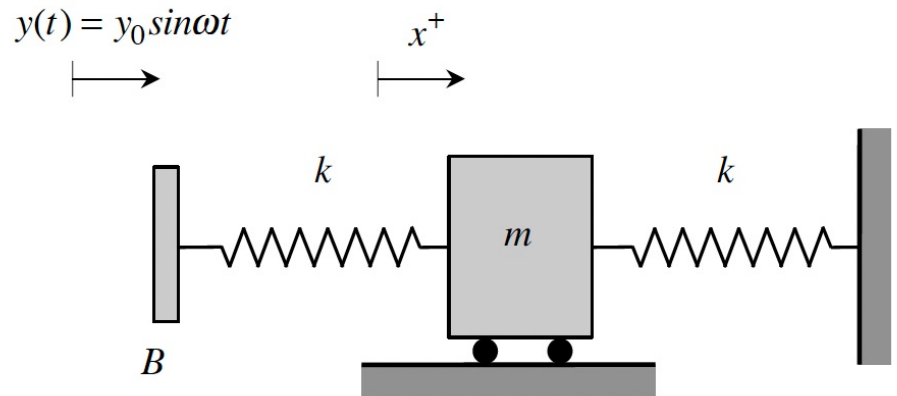
Example 6.C.7

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Given: The system shown below is given a prescribed motion for the base B: $y(t) = y_0 \sin \omega t$.

Find: Determine the range of frequencies ω for which the amplitude of the motion for block A does not exceed $1.5y_0$.

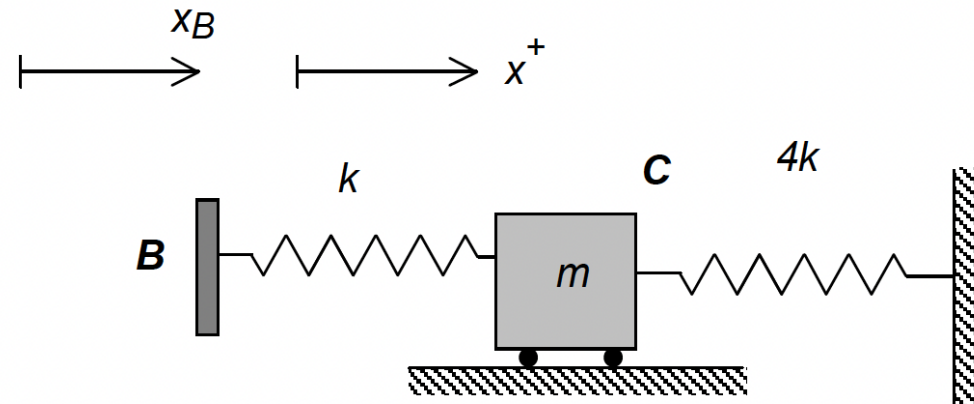
Use the following parameters in your analysis: $m = 4$ kg and $k = 800$ N/m.



Example 6.C.8

Given: Determine the differential equation of motion for the system shown where $x_B(t) = b \sin \omega t$.

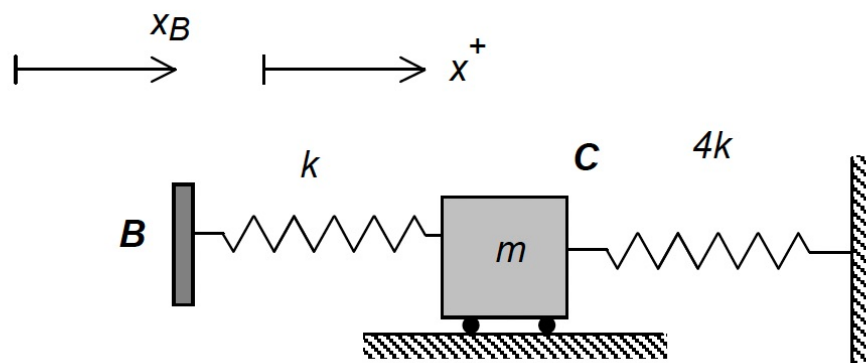
Find: Find the particular solution of this equation of motion.



Example 6.C.8*p.428*

Given: Determine the differential equation of motion for the system shown where $x_B(t) = b \sin \omega t$.

Find: Find the particular solution of this equation of motion.



Summary: Vibrations - Forced Response 2

EOM: For forced response:

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$$M\ddot{x} + C\dot{x} + Kx = F_0 \sin \Omega t \Rightarrow \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{M} \sin \Omega t$$

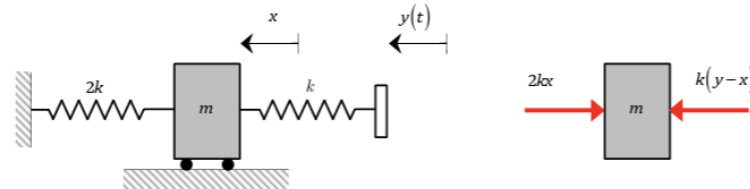
PARTICULAR SOLUTION

$$x_p(t) = A \sin \Omega t + B \cos \Omega t$$

Substitute into EOM and solve for A and B . If the system is undamped, then you will find that $B = 0$.

BASE EXCITATION: For many problems, the energy input is through a moving support (prescribed motion, $y(t)$), rather than an applied force.

Example:



$$\sum F_x = -2kx + k(y-x) = m\ddot{x} \Rightarrow m\ddot{x} + 3kx = \boxed{ky(t)} \text{ base excitation}$$

Lec 40 Short
Feedback Form:

