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ME 274 Lecture 39

Vibrations: Forced Response – Part 1

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04/24/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. HW 38 (6.G and 6.H) due Today!!

2. HW 39 (6.G and 6.H) due Sunday, 4/26 at 11:59PM!!

3. Office hours are on ME2008. Located on second floor of renovated side of ME.

3. Quizzes, there will be a total of 9 quizzes this semester:

- Remember Quiz 07 is due Sunday. I will be making the Gradescope box after lecture today.
- Quiz 08 – Find a video in the following link below. In your own handwriting, talk about why you enjoyed the video and its relevance to the ME274 coursework. 3-5 sentences.
  - https://www.purdue.edu/freeform/me274/course-material/visualizing-mechanics/
  - Due Friday, May 1st
  - Remote/online quiz. Can be done on white sheet of paper. Just make sure you complete it on your own.
- Quiz 09 - Thank you for filling out QR codes at end of lecture. I will be making a simple multiple-choice quiz asking:
  - “Did you fill out at least one post-lecture feedback survey? Y/N”.
  - Due Friday, May 1st
  - Remote/online quiz. No handout, just make sure you fill it out.

4. Final Exam details:

Tuesday, May 5, 7:00-9:00 PM

Coverage:

- For Problems 1-3: Lectures 27-42 on rigid body kinetics and vibrations
- For Problem 4: throughout entire course

WL locations

- WTHR 200 (260): CA (109), EF (109), LK (35, A-H) and 2 TAs

Review sessions

- ME 274 Instructor, CK: Sunday, May 3, 7:00 PM, live on Zoom for both WL and Indy: https://purdue-edu.zoom.us/j/99152873491?pwd=cl8taKJJeuxJUIl9GmWMILbGYILWIm.1 Video to be posted on website.

Chapter 6: Vibrations

$$\boxed{\begin{aligned} \sum \vec{F} &= m\vec{a}_G \\ \sum \vec{M}_A &= I_A\vec{\alpha} \end{aligned}}$$

$$\boxed{M\ddot{x} + C\dot{x} + Kx = f(t)}$$

$$\boxed{x(t) = x_c(t) + x_p(t)}$$

Goals for Chapter 6:

- *We are used* to solving acceleration/velocity/position for a given **instant in time** (using Newton-Euler Equations, upper left).

- In this chapter, we will study the response of systems whose motion is *oscillatory in nature* (~vibrations).

- We will focus on understanding the change in position as a **general function of time** (ie. **Deriving the EOMs of a system**, upper right).

Chapter 6: Vibrations – Forced Response

Particular solution for harmonic excitation $F(t) = F_0 \sin \omega t$:

$$\boxed{x_p(t) = \left[\frac{F_0/K}{1-(\omega/\omega_n)^2}\right] \sin \omega t = X \ \sin \omega t \qquad \text{(undamped response)}}$$

- Throughout the last three lectures (Lec 36-38) we have looked at "Free response", given by the **complementary/homogeneous solution.**
- Now, we will be evaluating the "Forced Response" of a system (Lec 39-41). Given by the particular solution.

$$x(t) = x_c(t) + x_p(t)$$

- Lec 42 is a review for the final.

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In the next 3 lectures, we will evaluate **the particular solution** to get the Forced Response of the system

$$x(t) = x_c(t) + \boxed{x_p(t)}$$

- In particular for this course, we will consider systems with harmonic excitation:

$$F(t) = F_0 \sin \omega t \quad \text{OR} \quad F(t) = F_0 \cos \omega t$$

- Let's consider problems with sine excitation. The response of the system for the latter would be:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{1}{M} F_0 \sin \omega t$$

- It can be shown that the general form of the particular solution for this EOM is given by:

$$x_p(t) = A \sin \omega t + B \cos \omega t$$

- With this we can equate both equations above. Then, solve for A and B by balancing coefficients of sine/cosine.

$$cosine: \quad -(2\zeta\omega_n\omega) A + \left(-\omega^2 + \omega_n^2\right) B = 0$$

$$sine: \quad \left(-\omega^2 + \omega_n^2\right) A + (2\zeta\omega_n\omega) B = \frac{1}{M} F_0$$

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Undamped Forced Response (zeta ie. damping ratio = 0)

1. Let's consider the undamped case. Picking up from the end of last slide. We can plug in zeta = 0 and get:

$$cosine: \quad \left(-\omega^2 + \omega_n^2\right) B = 0$$

$$sine: \quad \left(-\omega^2 + \omega_n^2\right) A = \frac{1}{M} F_0$$

2. With above, we can solve for A and B. Then plug in back to our original x_p(t) equation in previous slide:

$$x_p(t) = A \sin \omega t = \boxed{\left[\frac{F_0/K}{1-(\omega/\omega_n)^2}\right] \sin \omega t}$$

3. In this course we will not be covering Damped Forced Response, but if you are interested feel free to check out your lecture books page 413.

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Aside: Terms to know about harmonic motion

Remember:

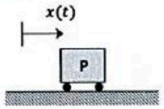
1. Amplitude (X)

2. Angular frequency (omega)

3. Phase (phi)
    - Notice the case when we have 180 degrees of phase shift (lower half of the handout).
        - The response is the exact opposite of when the phase was = 0.
        - We label this response as "out-phase"

Parameters for describing harmonic motion

ME 274 ; 12:30pm - efm



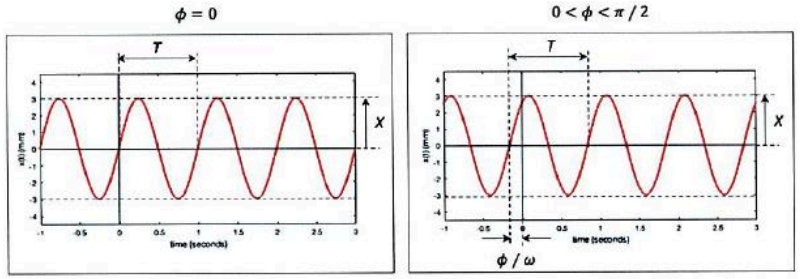
Suppose that the position "x" (in mm) of some point P is written in terms of time "t" (in seconds). If $x(t)$ represents "harmonic motion" of P, $x(t)$ can be written as:

$$x(t) = X sin(\omega t + \phi)$$

As seen in this equation, there are three parameters in this equation that are required to quantify this harmonic motion:

- X = the "amplitude" of the motion (in mm)
- ω = the "angular frequency" (in rad/s)
- ϕ = the "phase" (in rad)

Consider the two plots below of $x(t)$, one with $\phi = 0$, and one with $\phi > 0$.



Period and angular frequency

The time required for x(t) to complete one cycle of motion is called the "period", T. Considering the $\phi = 0$ plot on the left above, the period is the time T over which the argument of the sine function goes from zero to 2π; that is, $\omega T = 2\pi$, or:

$$\omega = \frac{2\pi}{T} = \text{angular frequency (rad/s)}$$

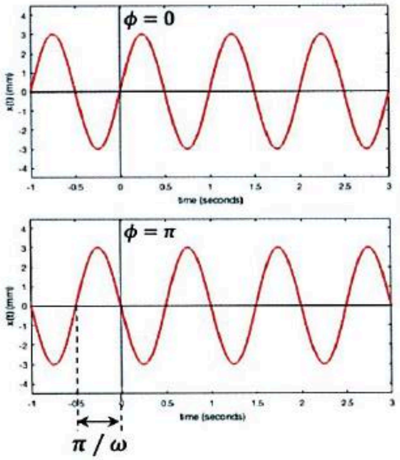
Frequency in hertz

The frequency of the harmonic function is often described in terms of "cycles/second" (or, Hz) using the symbol of "f". Recall that there are 2π radians in one cycle of motion. Suppose that we know that $\omega = 10$ rad/s. From this, the frequency in Hz is given by: (10 rad/s)(cycle/2π rad) = $\frac{10}{2\pi}$ cycles/s = $\frac{5}{\pi}$ Hz. In general, we can write:

$$f = \frac{\omega}{2\pi} = \frac{1}{T} \text{ (Hz)}$$

Phase shift

In this course, we will often encounter a harmonic response that has a "180° phase shift". How does a harmonic function with $\phi = \pi$ look? See the plot to the right. You see that the $\phi = \pi$ phase-shifted harmonic function has its sign "flipped" over the entire motion with unchanged amplitude and frequency.



Undamped Forced Response (zeta ie. damping ratio = 0). (Cont.)

$$x_p(t) = A \sin \omega t = \boxed{\left[\frac{F_0/K}{1-(\omega/\omega_n)^2}\right] \sin \omega t}$$

4. If we plot the equation above, we see the following:

a. For low frequencies (w/w_n < 1), the amplitude is quasistatic. That is, its roughly F/K.

- *In-class demo: Coffee mug moves at same frequency as hand.*

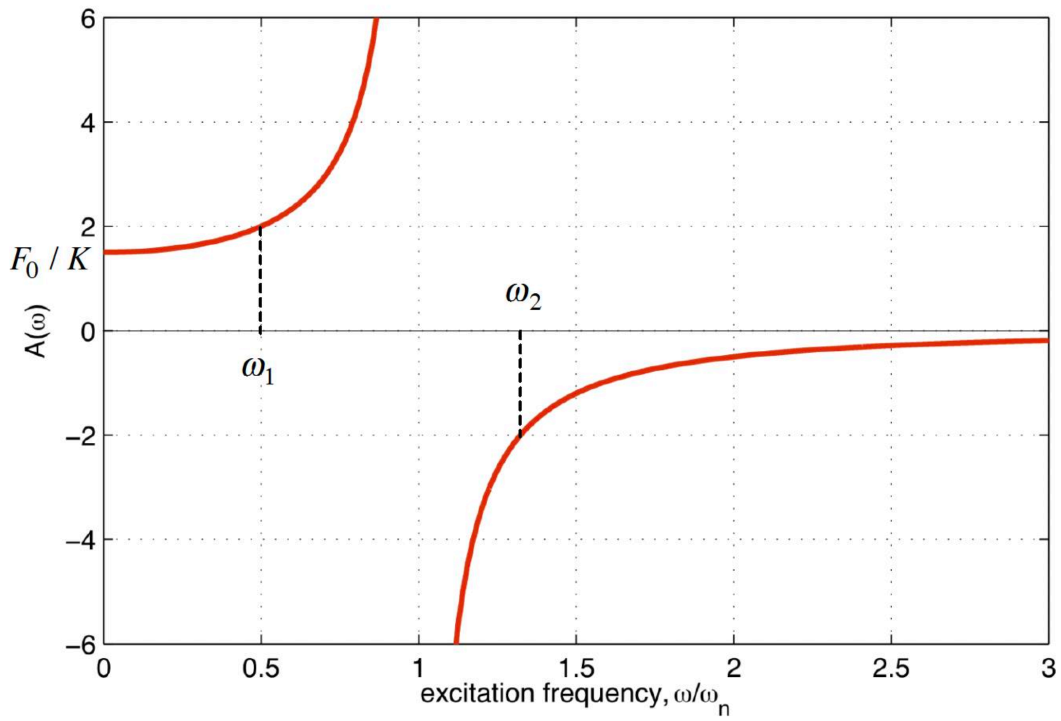
b. For high frequencies (w/w_n >1), the amplitude approaches zero.

- *In-class demo: Coffee mug barely moves when hand moves at high frequency.*

c. The excitation of w/w_n = 1, the amplitude of response is unbounded (ie. Resonance occurs).

d. Note: The sign shifts from positive to negative at w/w_n = 1. We can use this to tell if response is:

- In phase or 180 degrees out-of-phase.



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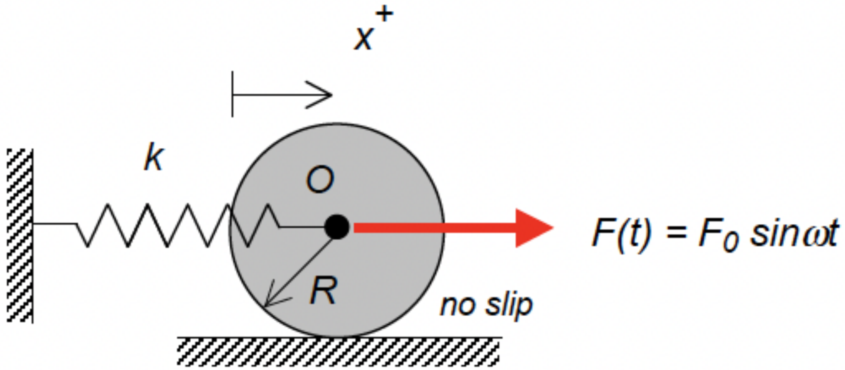
Example 6.C.1

Given: A homogeneous disk having a mass of m and radius R rolls without slipping on a rough horizontal surface. A spring, having a stiffness of k, is attached between the disk center O and ground, as shown below. Let x describe the position of O, where the spring is unstretched when $x = 0$.

Find:
(a) Derive the EOM for the disk in terms of the coordinate x;
(b) Find the particular solution of the EOM; and
(c) Determine the amplitude of response corresponding to $\omega = 15$ rad/s. Is the response in phase or out of phase with the forcing?

Use the following parameters in your analysis: $m = 2$ kg, $k = 300$ N/m and $F_0 = 20$ N.



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Example 6.C.1 p.421

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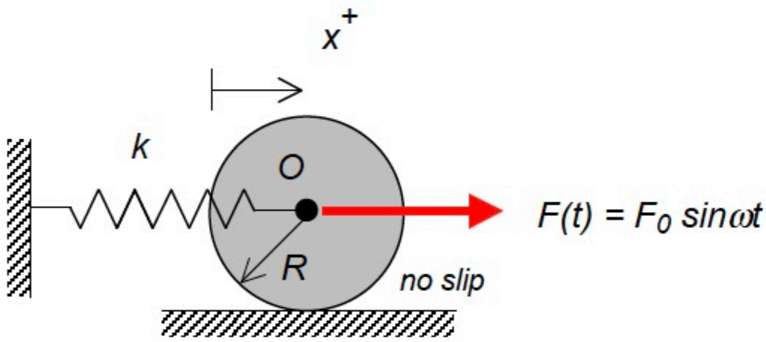


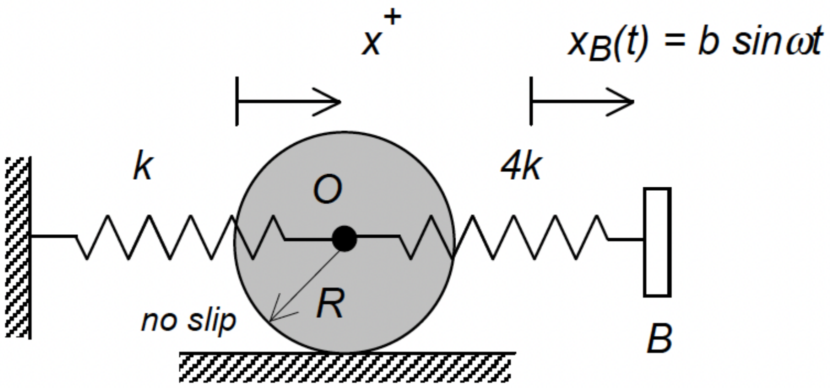
Example 6.C.4

Given: The system shown below consists of a homogeneous disk, of mass m and radius R, which is connected to ground by a spring of stiffness k and a massless moving base B by a spring of stiffness $4k$.

Find:

(a) Derive the EOM for the disk in terms of the coordinate x;
(b) Find the particular solution of the EOM; and
(c) Determine the amplitude of response corresponding to $\omega = 15$ rad/s. Is the response in phase or out of phase with the forcing?

Use the following parameters in your analysis: $m = 4$ kg, $k = 800$ N/m and $b = 20$ mm.



[pg. 424]

Example 6.C.4

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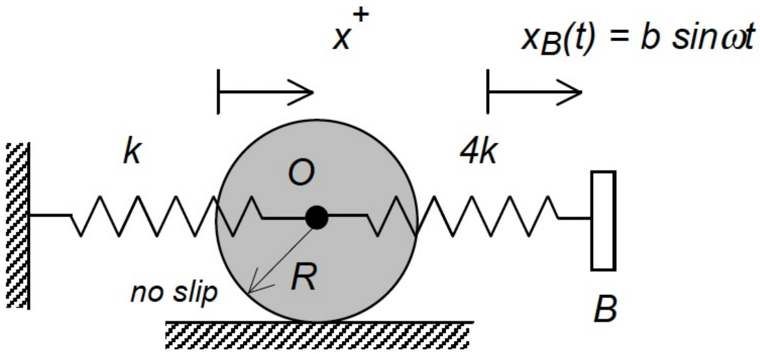
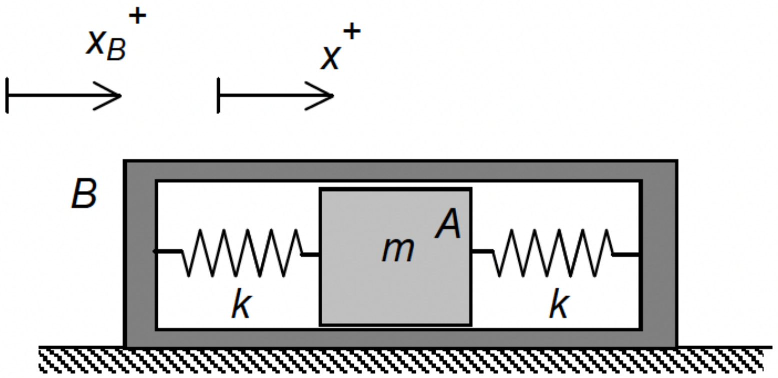


Example 6.C.5

Given: Box B is given a prescribed displacement of $x_B(t) = b \sin \omega t$.

Find: Determine the two values of frequency ω for which the amplitude of forced response of the block A is twice that of the box B.



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Example 6.C.5

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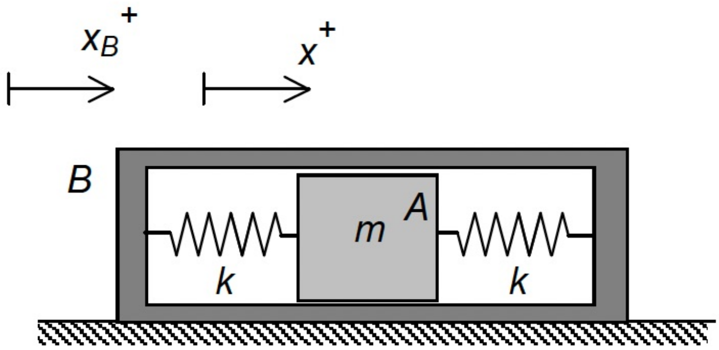

Summary: *Vibrations - Forced Response 1*

EOM: For forced response:

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$$M\ddot{x} + C\dot{x} + Kx = F_0 sin\Omega t \quad \Rightarrow \quad \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{M} sin\Omega t$$

TOTAL RESPONSE: Since this is a linear EOM, we can write:

$$x(t) = x_C(t) + x_P(t)$$

where $x_C(t) = e^{-\zeta\omega_n t}\left(C\cos\omega_d t + S\sin\omega_d t\right)$

HOW TO FIND PARTICULAR SOLUTION?

$$x_P(t) = A sin\Omega t + B cos\Omega t$$

Substitute into EOM and solve for *A* and *B*. If the system is <u>undamped</u>, then you will find that *B = 0*.

ENFORCING IC's: The initial conditions are enforced on the TOTAL solution *x(t)*, NOT on the complementary solution $x_C(t)$!

Lec 39 Short Feedback Form: