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ME 274 Lecture 38

Vibrations: Free Response – Part 3

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04/22/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 37 (6.E and 6.F) due Today!!**

2. Office hours are on ME2008. Located on second floor of renovated side of ME.

3. Take home quiz07 assigned today, due Sunday April 26th

Chapter 6: Vibrations

$$\sum \vec{F} = m\vec{a}_G$$

$$\sum \vec{M}_A = I_A\vec{\alpha}$$

$$M\ddot{x} + C\dot{x} + Kx = f(t)$$

Goals for Chapter 6:

- *We are used* to solving acceleration/velocity/position for a given **instant in time** (using Newton-Euler Equations, upper left).
- In this chapter, we will study the response of systems whose motion is *oscillatory in nature* (~vibrations).
- We will focus on understanding the change in position as a **general function of time** (ie. **Deriving the EOMs of a system**, upper right).

Standard Form of EOM

A) General Form EOM:

1. Particles/rigid bodies
2. Dashpots (a.k.a. Damper)
3. Springs
4. External forcing

$$\boxed{M\ddot{x}} + \boxed{C\dot{x}} + \boxed{Kx} = \boxed{f(t)}$$

B) Standard Form EOM:

- If you divide general form by M you get standard form (bottom left).
- Standard form gives us the EOM in terms of damping ratio, ζ (zeta), and undamped natural frequency, ω_n (bottom right)
- Why? Damping ratio and undamped natural frequency are values with physical interpretations:
  - Damping ratio -> Ratio for which an objects oscillations will decay over time
  - Undamped natural frequency -> a systems preferred state of motion based on its weight and stiffness
- *Make sure all coefficients on your EOM are equal to the same sign.* In this course, we are only looking at scenarios where all coefficients have the same sign [p. 394].

$$\ddot{x} + \frac{C}{M}\dot{x} + \frac{K}{M}x = \frac{1}{M}f(t)$$

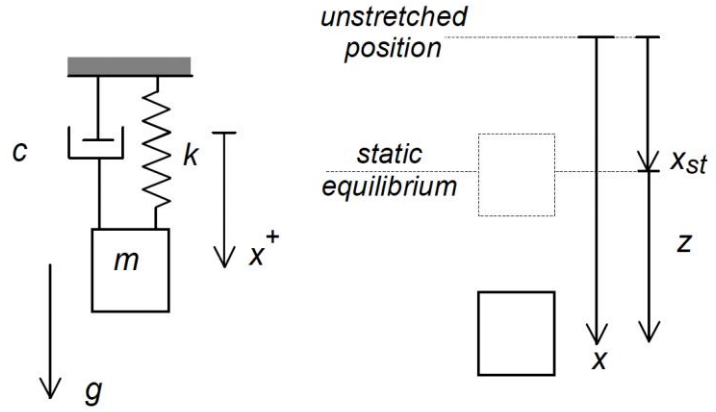
$$\ddot{x} + 2\zeta\omega_n\dot{x} + {\omega_n}^2 x = \frac{1}{M}f(t)$$

[p. 392]

Static Deformation

1. The EOM for the system shown is given as:

$$m\ddot{x} + c\dot{x} + kx = mg$$



2. The **static displacement** can be found by setting $\boxed{\dot{x} = \ddot{x} = 0}$, which gives:

$$kx_{st} = mg \longrightarrow x_{st} = \frac{mg}{k}$$

3. Now instead of measuring the position form the unstretched position, we measure its position from the static deformation state. **We define this coordinate as z:**

$$\boxed{z = x - x_{st}}$$

4. This gives a **homogenous equation (eqn equal to 0):**

$$m\ddot{z} + c\dot{z} + kz = 0$$

[p. 385-386]

Types of Responses

$$\left(\lambda^2 M + \lambda C + K\right) A e^{\lambda t} = 0$$

Solving for the characteristic equation gives us two roots:

$$\lambda_1, \lambda_2 = \frac{-C \pm \sqrt{C^2 - 4MK}}{2M} = \left[-\zeta \pm \sqrt{\zeta^2 - 1}\right] \sqrt{\frac{K}{M}} \quad \text{where } \zeta = C/2\sqrt{KM}$$

The value of the parameter zeta (damping ratio) tells us the type of response of the system:

1. Undamped $\zeta = 0$
- We have two imaginary roots

2. Underdamped $0 < \zeta < 1$
- We have two complex roots

3. Critically damped $\zeta = 1$
- We have two repeated, real roots

4. Overdamped $\zeta > 1$
- We have two distinct, real roots

[p. 388]

The Underdamped Response

1. In this course we will focus **primarily on the Underdamped response**, with undamped being a special case.
2. Substituting the underdamped roots of the characteristic equation into the total complementary solution (first equation below) gives the second equation:

$$x(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$$

$$\boxed{\begin{gathered} = e^{-\zeta \omega_n t}\left[C \cos \omega_d t + S \sin \omega_d t\right] \\ \text{where } \omega_d = \omega_n \sqrt{1 - \zeta^2} \end{gathered}}$$

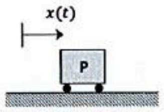
[p. 390]

DUE: Apr 26^{th}

Parameters for describing harmonic motion

ME 274 ; 12:30pm - efm

Suppose that the position "x" (in mm) of some point P is written in terms of time "t" (in seconds). If $x(t)$ represents "harmonic motion" of P, $x(t)$ can be written as:



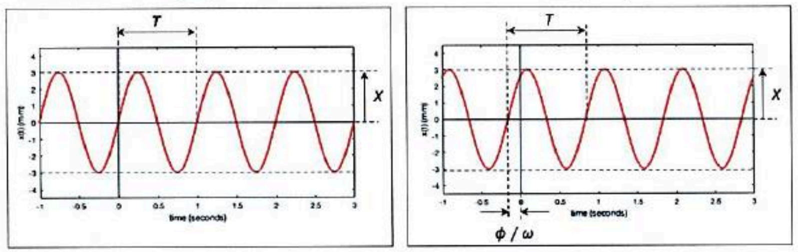
$$x(t) = X\sin(\omega t + \phi)$$

As seen in this equation, there are three parameters in this equation that are required to quantify this harmonic motion:

- X = the "amplitude" of the motion (in mm)
- ω = the "angular frequency" (in rad/s)
- ϕ = the "phase" (in rad)

Consider the two plots below of $x(t)$, one with $\phi = 0$, and one with $\phi > 0$.

$\phi = 0$ $\quad$ $0 < \phi < \pi/2$



Period and angular frequency

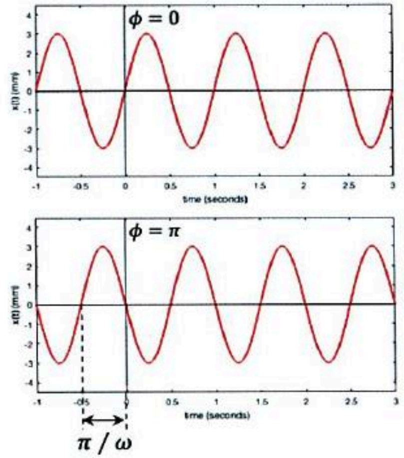
The time required for x(t) to complete one cycle of motion is called the "period", T. Considering the $\phi = 0$ plot on the left above, the period is the time T over which the argument of the sine function goes from zero to 2π; that is, $\omega T = 2\pi$, or:

$$\omega = \frac{2\pi}{T} = \text{angular frequency (rad/s)}$$

Frequency in hertz

The frequency of the harmonic function is often described in terms of "cycles/second" (or, Hz) using the symbol of "f". Recall that there are 2π radians in one cycle of motion. Suppose that we know that $\omega = 10$ rad/s. From this, the frequency in Hz is given by: (10 rad/s)(cycle/2π rad) = $\frac{10}{2\pi}$ cycles/s = $\frac{5}{\pi}$ Hz. In general, we can write:

$$f = \frac{\omega}{2\pi} = \frac{1}{T} \text{ (Hz)}$$



Phase shift

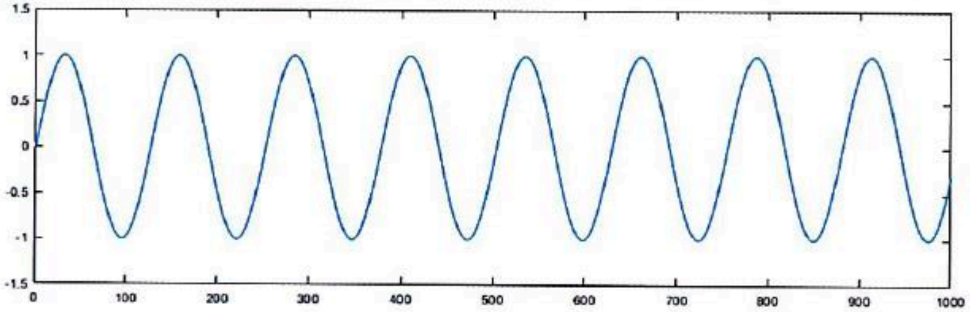
In this course, we will often encounter a harmonic response that has a "180° phase shift". How does a harmonic function with $\phi = \pi$ look? See the plot to the right. You see that the $\phi = \pi$ phase-shifted harmonic function has its sign "flipped" over the entire motion with unchanged amplitude and frequency.

Examples of different frequency levels

https://youtube.com/playlist?list=PLFMXG7jiiXIaKkrOPDbwVeSqXEIxNmzAe&si=RNWZbxnWL4Wm3K8o

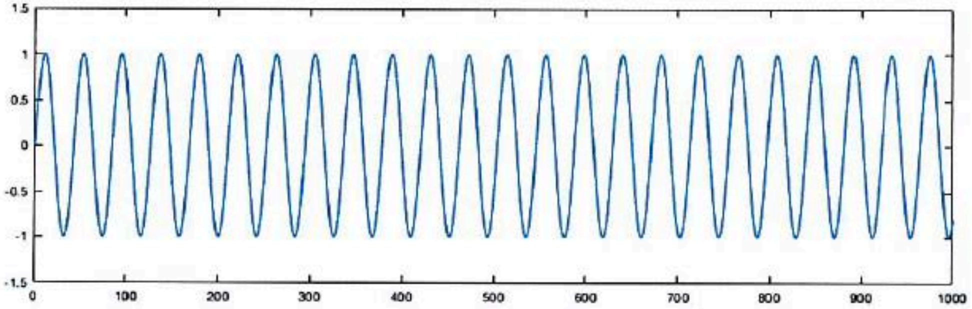
150 hz

f_0



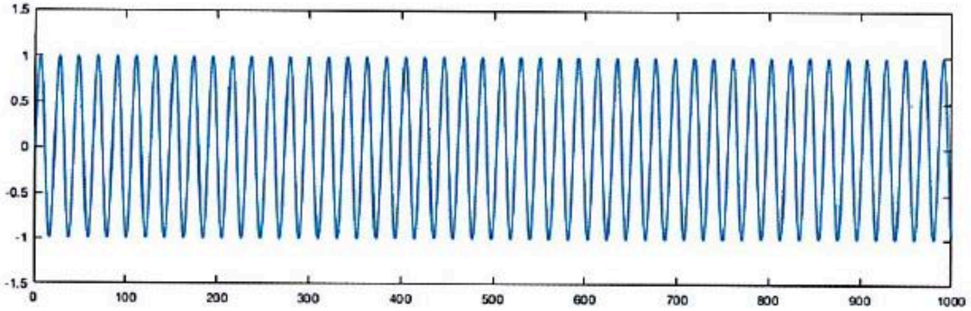
450 hz

$3f_0$



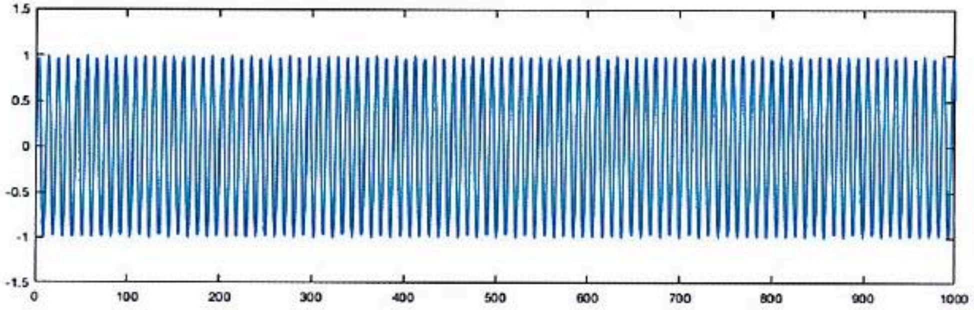
900 hz

$6f_0$



1800hz

$12f_0$



Example 6.B.10

Given: The addition of damping to an undamped system causes the period to increase by 25 percent.

Find: Find the value of the damping ratio after the addition of damping.

[pg. 404]

Example 6.B.10

p. 404

Given: The addition of damping to an undamped system causes the period to increase by 25 percent.

Find: Find the value of the damping ratio after the addition of damping.

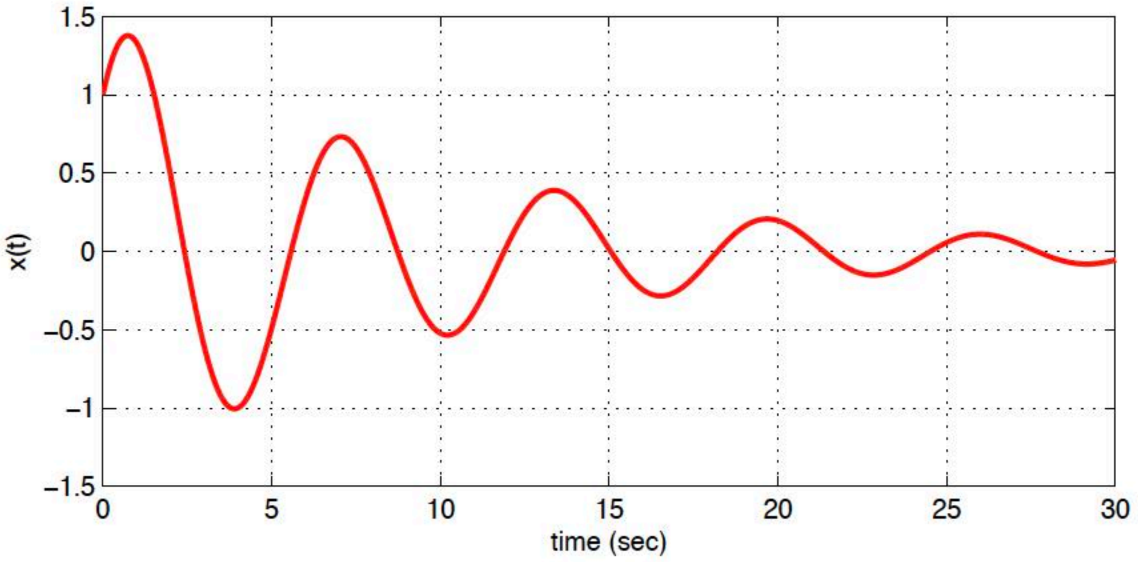
Example 6.B.11

Given: The free response of the single degree of freedom system:

$$M\ddot{x} + C\dot{x} + Kx = f(t)$$

shown below. It is known that $M = 2$ kg.

Find: Find the damping coefficient C from this free response plot.



[pg. 405]

Example 6.B.11 p. 405

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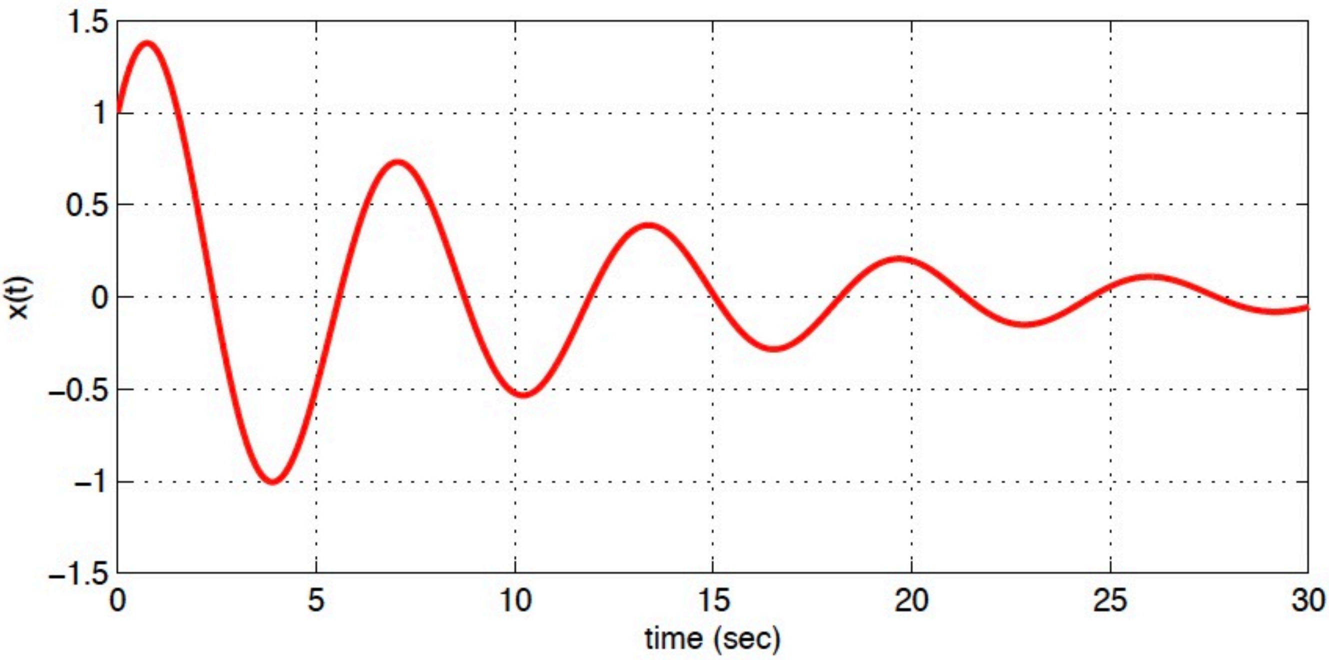
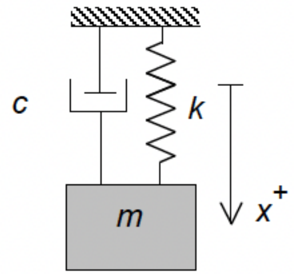


Example 6.B.12

Given: The system shown below is released from rest under the action of gravity.

Find: Determine the initial overshoot past the static equilibrium state of the system.

Use the following parameters in your analysis: $m = 5$ kg, $c = 6$ N-s/m and $k = 180$ N/m.



[pg. 406]

Example 6.B.12

p.406

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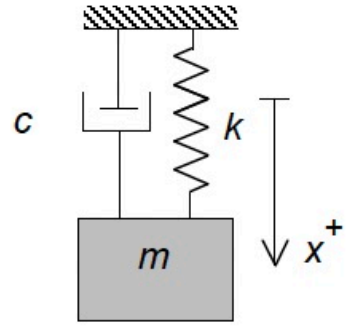
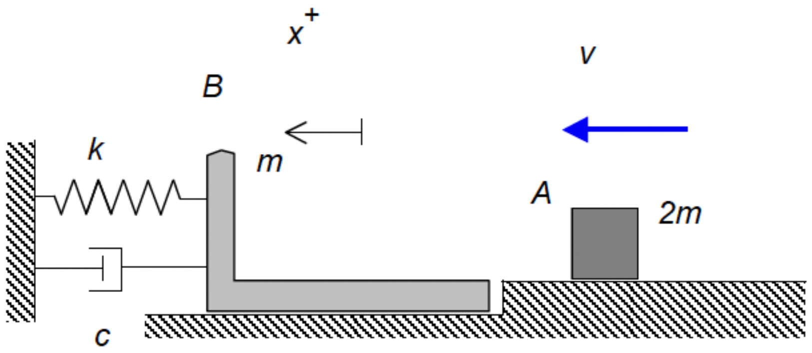


Example 6.B.13

Given: Block A strikes a stationary block, block B, with a speed of v. Upon impact, A sticks to B. Assume all surfaces to be smooth.

Find: Find the time history of motion $x(t)$ of the system after A sticks to B.

Use the following parameters in your analysis: $v = 30$ m/s, $k = 3000$ N/m, $c = 30$ N-s/m and $m = 10$ kg.



[pg. 407]

Example 6.B.13 p.407 Similar to hw due Fri, 4/24

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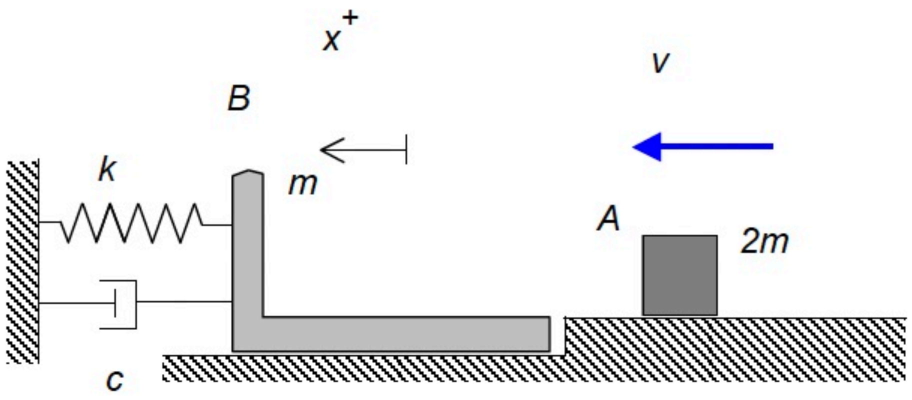
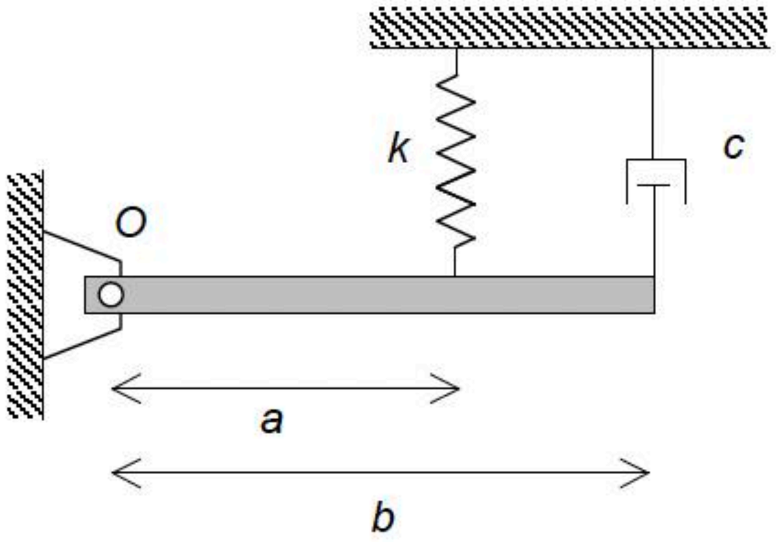


Example 6.B.14

Given: The system shown below, consisting of a homogenous bar of mass m which is pinned to ground at point O, moves in a horizontal plane. Let θ represent the angular orientation of the bar, measured in a counterclockwise positive sense.

Find: Find the value of c for critical damping. Assume small oscillations, that is, assume $\sin\theta \approx \theta$.



[pg. 408]

Example 6.B.14 p. 408

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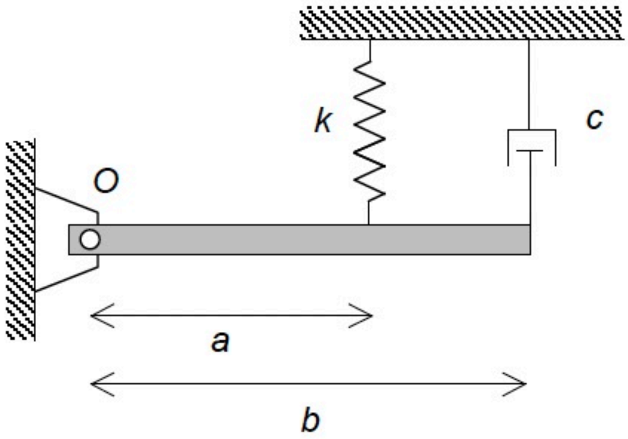

Summary: Vibrations - damped free response

[pg. 392]

EOM: For free response of damped system:

$$m\ddot{x} + c\dot{x} + kx = 0$$

STANDARD FORM OF EOM: Divide EOM by "m" to get:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

where

$$2\zeta\omega_n = c/m \quad \Rightarrow \quad \zeta = c/2\sqrt{km} = damping\ ratio$$

$$\omega_n = \sqrt{k/m} = natural\ \ frequency$$

SOLUTION OF EOM: For $0 \le \zeta < 1\left(UNDERdamped\right)$

$$x(t) = e^{-\zeta\omega_n t}\left(C\,cos\,\omega_d t + S\,sin\,\omega_d t\right) \quad ; \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

HOW TO FIND THE RESPONSE COEFFICIENTS, C AND S?

Enforce the initial conditions on the solution:

$$x(0) = x_0 = e^0\left(C\,cos\,0 + S\,sin\,0\right) \quad \Rightarrow \quad C = x_0$$

$$\dot{x}(0) = \dot{x}_0 = -\zeta\omega_n e^0\left(C\,cos\,0 + S\,sin\,0\right) + \omega_d e^0\left(-Csin0 + Scos0\right) \quad \Rightarrow \quad S = \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d}$$

Lec 38 Short Feedback Form: