

Not Filled

ME 274 Lecture 34

Rigid body kinetics: Summary

Eugenio “Henny” Frias-Miranda

04/13/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. HW 33 (5.M and 5.N) due Today!!

2. Office hours are changing to ME2008B...

- Second floor of renovated side of ME.

Kinetics Summary Handout

Kinetics Table (page 352 of the lecture book)

Method	Body model	Fundamental equations
Newton-Euler (relating forces to accelerations)	particle	$\sum \vec{F} = m\vec{a}$
	rigid body (G = c.m. and A = any point on body)	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy (relating change in speed to change in position)	particle	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body (G = c.m. and A = any point on body)	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$
Linear impulse-momentum (relating change in velocity to change in time)	particle	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body (G = c.m.)	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum (relating change in angular velocity to change in time)	particle (O = fixed point)	$\int_{t_1}^{t_2} \sum \vec{M}_O dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body (A = fixed point or c.m.)	$\int_{t_1}^{t_2} \sum \vec{M}_A dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A \vec{\omega}$

Notes on the *four-step* method:

1. FBD(s)

- Newton/Euler (N/E): typically individual FBDs
- Work/energy (W/E), linear impulse momentum (LIM) and angular impulse/momentum (AIM): typically make it "BIG" (include all moving bodies)

2. Kinetics (see suggestions in the table to the right as to which method(s) to use)

- Choose coordinate system(s) based on the "Given" and "Find", including consideration of any motion constraints in the system.
- N/E: point G in the Newton equation must be the center of mass for the body
- W/E: mechanical energy is conserved if $U_{1 \rightarrow 2}^{(nc)} = 0$ (no non-conservative work being done on your system)
- LIM: linear momentum in the x-direction is conserved if $\sum F_x = 0$ (no net force in the x-direction for your system)
- AIM: angular momentum in the z-direction is conserved if $\sum M_{Oz} = 0$ (no net moment in the z-direction about point O for your system). For particles, O must be a fixed point. For rigid bodies, O can be either a fixed point or the center of mass.

3. Kinematics

- N/E: typically need *acceleration* kinematics
- W/E, LIM and AIM: typically need *velocity* kinematics

4. Solve

Chapter 5: Planar Rigid Body Kinetics

Newton–Euler (N/E) Equations

- Newton–Euler is a vector-based approach.
- For planar motion, each free body provides up to three scalar equations (two force + one moment).
- Force equations always use the center of mass acceleration.
- Moment equations can be taken about any point and involve mass moment of inertia.
- **These equations relate forces/reactions to accelerations at a specific instant.**

Work–Energy (W/E) Equation

- The work–energy equation is a scalar equation derived from Newton–Euler via path integrals.
- It is expressed using kinetic energy, potential energy, and work done.
- **It is useful for relating changes in speed to changes in position.**
- A system approach is recommended.
- Including multiple bodies helps treat internal forces as workless.

Impulse–Momentum (L.I.M & A.I.M) Equations

- Impulse–momentum equations are vector equations derived from time integration of Newton–Euler.
- For planar motion, they provide up to three components.
- **They are useful for relating changes in velocity to time.**
- A system approach is recommended, especially when momentum is conserved.
- They are commonly used with coefficient of restitution to solve impact problems.

Discussion - Newton-Euler Equation - 3 Special Forms of Euler's Equation where $\vec{r}_{G/A} \times (m\vec{a}_A) = 0$

$$\left(\sum \vec{M}_A \right)_{ext} = I_A \vec{\alpha} + \vec{r}_{G/A} \times (m\vec{a}_A)$$

1. If you choose A to be the **center of mass, G**. $\vec{r}_{G/A} = \vec{0}$.

$$\sum \vec{M}_G = I_G \vec{\alpha}$$

2. If you choose A to be a **fixed point on the body, O**. $\vec{a}_O = \vec{0}$.

$$\sum \vec{M}_O = I_O \vec{\alpha}$$

3. If you choose a point A which has an acceleration vector that is **parallel to** $\vec{r}_{G/A}$. $\vec{r}_{G/A} \times \vec{a}_A = \vec{0}$

$$\sum \vec{M}_A = I_A \vec{\alpha}$$

Picking one of these 3 locations will simplify our problem, **it is recommended to do this as often as possible.**

Discussion – Work/Energy Equation for Rigid Bodies

$$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$$

$$\text{where } T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$$

The point P in the above equation for K.E. can be any point on the rigid body:

- **However, typically we pick one of the two**, since it makes the equation above simpler:

1. Center of Mass, G, for the body. Equation reduces to:

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$$

2. A fixed point, O. Equation above reduces to:

$$T = \frac{1}{2}I_O\omega^2$$

- **Can be the instant center of the body**, we will typically do this
- **Need to use Parallel Axis Theorem**, since its not center of mass, G

Impulse-Momentum Equations for Planar Rigid Bodies

- LIM and AIM for Planar Rigid Bodies (lectures 32-33)
 - Where A is a **fixed point on the body or center of mass**

$$\int_1^2 \sum \vec{F} dt = m\vec{v}_{G2} - m\vec{v}_{G1}$$

$$\int_1^2 \sum \vec{M}_A dt = \vec{H}_{A2} - \vec{H}_{A1} = I_A \vec{\omega}_2 - I_A \vec{\omega}_1$$

Kinetics Table

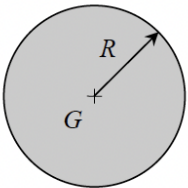
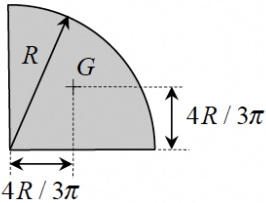
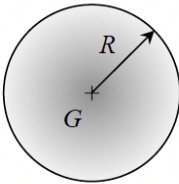
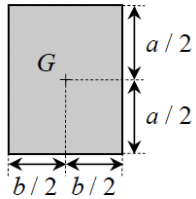
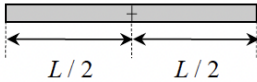
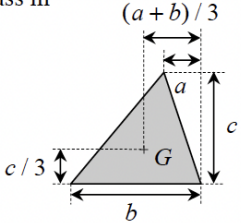
Method	Body model	Fundamental equations
Newton-Euler (relating forces to accelerations)	particle	$\sum \vec{F} = m\vec{a}$
	rigid body (G = c.m. and A = any point on body)	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy (relating change in speed to change in position)	particle	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body (G = c.m. and A = any point on body)	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$
Linear impulse-momentum (relating change in velocity to change in time)	particle	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body (G = c.m.)	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum (relating change in angular velocity to change in time)	particle (O = fixed point)	$\int_{t_1}^{t_2} \sum \vec{M}_O dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body (A = fixed point or c.m.)	$\int_{t_1}^{t_2} \sum \vec{M}_A dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A \vec{\omega}$

Mass Moment of Inertia of Commonly Used Objects and **when to use Parallel Axis Theorem (P.A.T.)**

1. Often times you will reference a table such as below and get your I_G
2. **Note:** These values below are presented in reference to G.
 - If you are not using your Euler's Equation in terms of G, you will have to use **Parallel Axis Theorem (P.A.T.)**

$$I_A = I_G + md_{AG}^2$$

3. When do I use P.A.T.? -> What point are you evaluating?

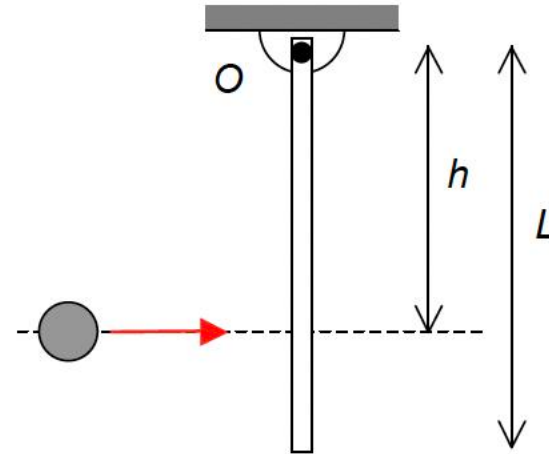
Uniform disk of mass m  $I_G = \frac{1}{2}mR^2$	Quarter circular plate of mass m  $I_G = \left(\frac{9\pi^2 - 64}{18\pi^2} \right) mR^2$	Uniform sphere of mass m  $I_G = \frac{2}{5}mR^2$	Uniform rectangular plate of mass m  $I_G = \frac{1}{12}m(a^2 + b^2)$	Uniform thin bar of mass m  $I_G = \frac{1}{12}mL^2$	Uniform triangular plate of mass m  $I_G = \frac{1}{18}m(a^2 + b^2 + c^2 - ab)$
---	---	---	---	--	--

[pg. 303-304 content]

Example 5.A.17

Given: A ball of mass m strikes a homogeneous bar of length L and mass M at a distance h from the pivot point O.

Find: Determine the distance h such that the horizontal reaction force at O on the bar is zero. (This location on the bar is known as the body's “center of percussion”. Can you think of a practical application of knowing the location of the center of percussion of a body?)

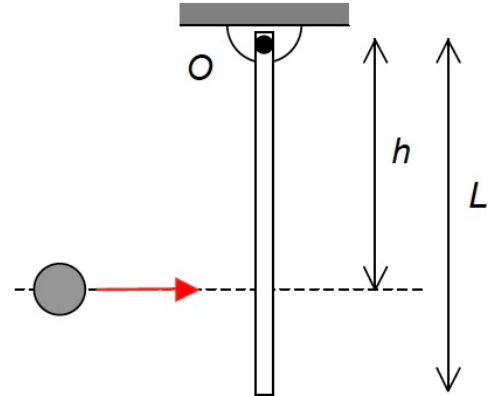


Example 5.A.17

p.324

Given: A ball of mass m strikes a homogeneous bar of length L and mass M at a distance h from the pivot point O.

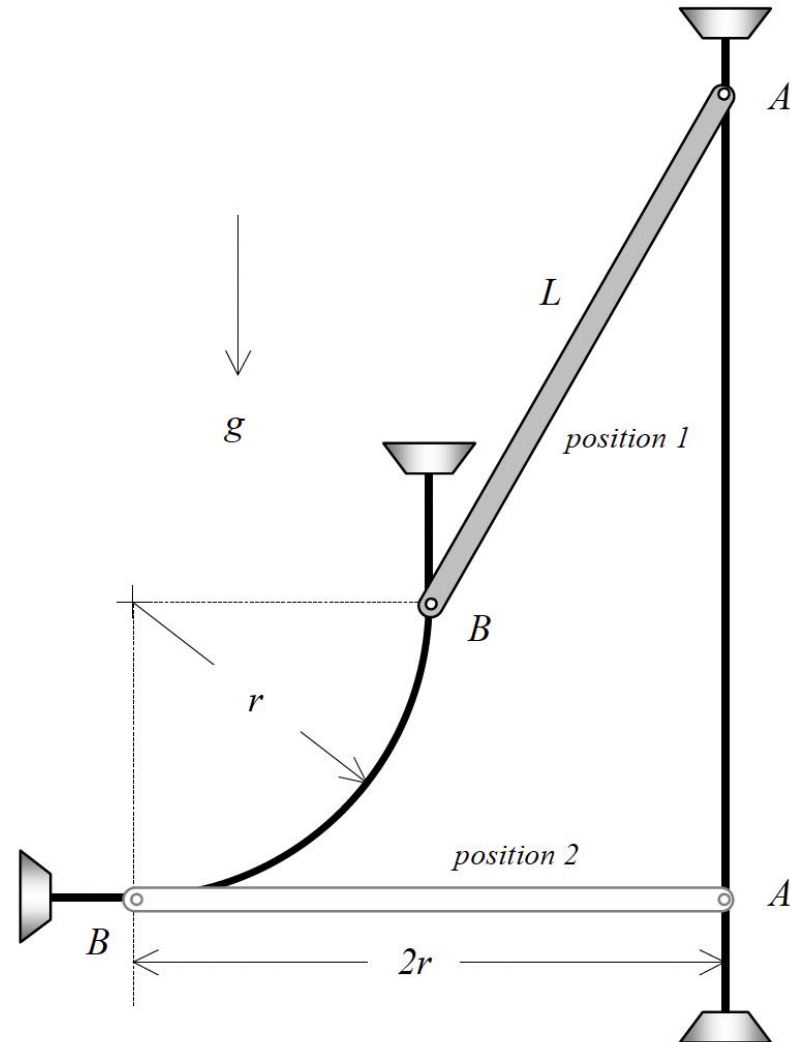
Find: Determine the distance h such that the horizontal reaction force at O on the bar is zero. (This location on the bar is known as the body's "center of percussion". Can you think of a practical application of knowing the location of the center of percussion of a body?)



Example 5.B.9

Given: Ends A and B of the thin, homogeneous bar AB (having a mass of m and a length of $L = 2r$) are constrained as follows: A moves along a smooth vertical guide and B moves along a smooth semi-circular guide of radius r . The bar is released from rest at position 1.

Find: Find the speed of end A when the bar is horizontal (i.e. is at position 2).

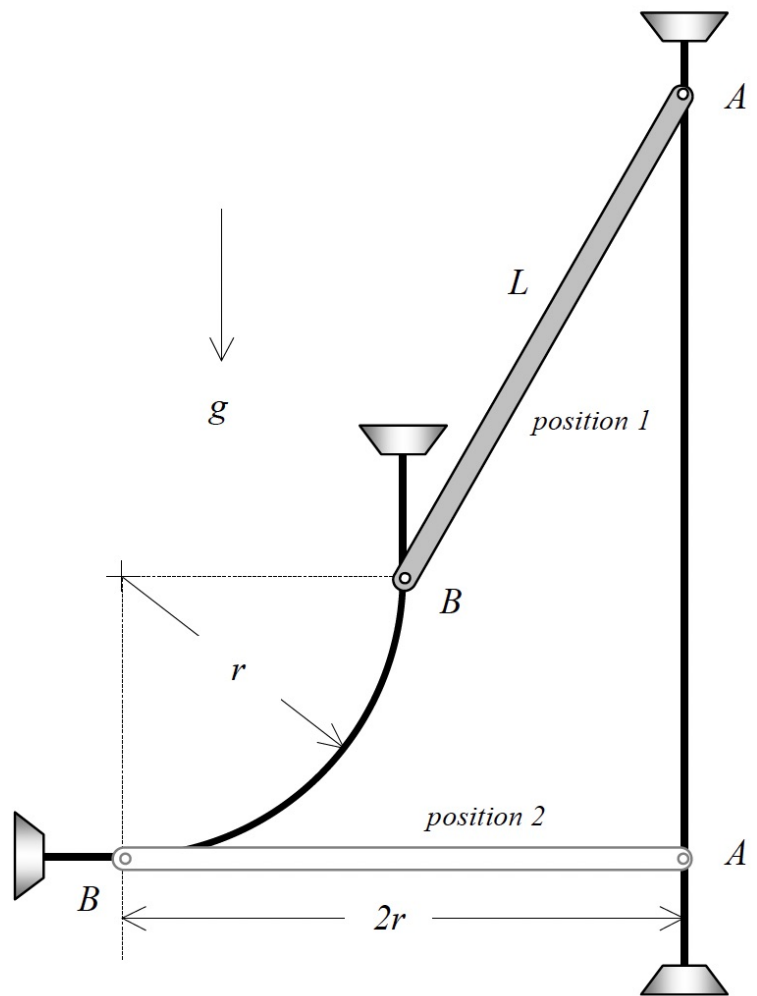


Example 5.B.9

p.338

Given: Ends A and B of the thin, homogeneous bar AB (having a mass of m and a length of $L = 2r$) are constrained as follows: A moves along a smooth vertical guide and B moves along a smooth semi-circular guide of radius r . The bar is released from rest at position 1.

Find: Find the speed of end A when the bar is horizontal (i.e. is at position 2).



Summary: Particle and Planar Rigid Body Kinetics

WHICH TOOL(s) TO USE?

Put effort up front deciding on which method(s) to use: *Newton-Newton/Euler*, *work/energy*, *linear impulse momentum* or *angular impulse momentum*. Use the Kinetics Table in Section 5.D of the lecture book as a guide.

PARTICLE or RIGID BODY?

How is a particle distinguished from a rigid body? For a particle, we have:

$$\sum \bar{M}_G = I_G \bar{\alpha} = \bar{0} \quad (\text{EITHER } I_G = 0 \text{ OR } \bar{\alpha} = \bar{0})$$

THE FOUR-STEP PLAN: Follow it...it is your friend!

Kinetics Table		
Method	Body model	Fundamental equations
Newton-Euler (relating forces to accelerations)	particle	$\sum \bar{F} = m\bar{a}$
	rigid body (G = c.m. and A = any point on body)	$\sum \bar{F} = m\bar{a}_G$ $\sum \bar{M}_A = I_A \bar{\alpha} + m\bar{r}_{G/A} \times \bar{a}_A$
Work-energy (relating change in speed to change in position)	particle	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body (G = c.m. and A = any point on body)	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\bar{r}_{A/G} \cdot (\bar{\omega} \times \bar{r}_{G/A})$
Linear impulse-momentum (relating change in velocity to change in time)	particle	$\int_{t_1}^{t_2} \sum \bar{F} dt = m\bar{v}_2 - m\bar{v}_1$
	rigid body (G = c.m.)	$\int_{t_1}^{t_2} \sum \bar{F} dt = m\bar{v}_{G2} - m\bar{v}_{G1}$
Angular impulse-momentum (relating change in angular velocity to change in time)	particle (O = fixed point)	$\int_{t_1}^{t_2} \sum \bar{M}_O dt = \bar{H}_{O2} - \bar{H}_{O1}$ where $\bar{H}_O = m\bar{r}_{P/O} \times \bar{v}_P$
	rigid body (A = fixed point or c.m.)	$\int_{t_1}^{t_2} \sum \bar{M}_A dt = \bar{H}_{A2} - \bar{H}_{A1}$ where $\bar{H}_A = I_A \bar{\omega}$

Lec 34 Short Feedback Form:

