ME 274: Basic Mechanics II
Spring 2026

Lecture 20
Rigid Body Kinetics Review

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Office Hours in EL-104: Monday from Noon to 1:30 PM, Tuesday from 10 AM to Noon, Wednesday from 3:30 PM to 4:30 PM, and Friday from 11:30 AM to 1 PM

Information adapted from “DYNAMICS: A Lecturebook”

By Charles M. Krousgrill, Jeffrey F. Rhoads

Reminders

- Keep up with daily homework
- Try your favorite programming language to help solve systems of equations for the homework
- Final Exam on Tuesday (5/5) from 7-9 PM
  - Either in LE-102 or LE-103

NEW 1-CREDIT CLASSES NOW AVAILABLE!

BUILD YOUR OWN CUSTOM ME ELECTIVE OUT OF THESE OPTIONS:

ME 49601 Innovation and Customer Discovery *(CRN 30126)*
For students taking Senior Design in Spring 2027, get a jump on your project by identifying and validating problems worth solving through customer discovery.

ME 49601 AI for Research and Innovation *(CRN 29080)*
Learn how AI tools are being used to enhance research, accelerate innovation, and solve complex engineering problems in modern ME contexts.

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(INDY: CRN 30113, 30114 — WEST LAFAYETTE: CRN 30115, 30116, 30118, 30122)
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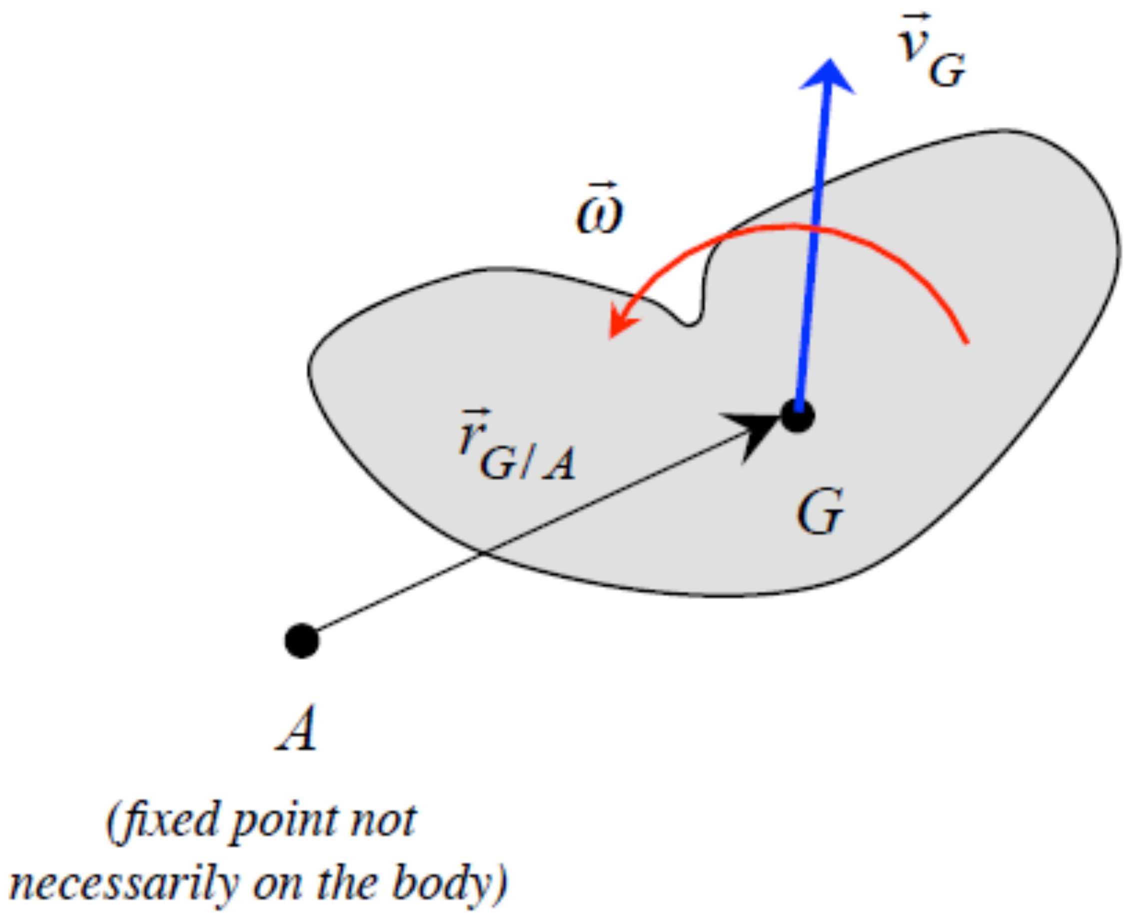
ME 49700 Essentials of Undergraduate Research *(CRN 19636)*
Explore undergraduate research and prepare for graduate school by developing research skills, identifying opportunities, and connecting with faculty.

ME 49601 Succeeding as a Woman Engineer *(CRN 28710)*
Develop the skills, confidence, and network to navigate and succeed through community, mentorship, and professional development.

ENROLL BY WEDNESDAY, APRIL 15!

Last Lecture

- Impulse-momentum for rigid bodies



Newton-Euler Equations

$$\sum \vec{F} = m\vec{a}_G \qquad \sum \vec{M}_A = I_A\vec{\alpha} + \vec{r}_{G/A} \times (m\vec{a}_A)$$

- Used for dynamic analysis of planar motion of rigid bodies
- Must use acceleration of G in Newton's equation
- You can choose any point A, just be consistent and some will make your life easier.

$$\sum \vec{M}_{\boxed{A}} = I_{\boxed{A}}\vec{\alpha} + \vec{r}_{G/\boxed{A}} \times \left(m\vec{a}_{\boxed{A}}\right)$$

SAME point A for <u>ALL</u> terms

- Remember your sign conventions!
- Moments of inertia are given for center of mass. Use parallel axis theorem

$$I_A = I_G + md_{AG}^2$$

Comparison: N-E

Newton-Euler (relates forced to acceleration)	Particle	$\sum \vec{F} = m\vec{a}_G$
	Rigid Body G= c.m. & A= point on Body	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$

Work-Energy Equations

$$T_2 + V_2 = T_1 + V_1 + U_{1\to 2}^{(nc)}$$

LKE RKE

$$T = \frac{1}{2} m v_P^2 + \frac{1}{2} I_P \omega^2 + m \vec{v}_p \cdot (\vec{\omega} \times \vec{r}_{G|P}) \qquad U_{1\to 2} = \int_1^2 \vec{F} \cdot \hat{e}_{tB}\, ds + \int_1^2 \vec{M} \cdot d\vec{\theta}$$

- The point P for the KE can be any point on the rigid body! Usually we choose…
  - The center of mass (G) so $\vec{r}_{G|P} = 0$

$$T = \frac{1}{2} m v_P^2 + \frac{1}{2} I_P \omega^2 + \cancel{m \vec{v}_p \cdot (\vec{\omega} \times \vec{r}_{G|P})}$$

  - Fixed point O (possibly the instant center) so $\vec{v}_p = 0$

$$T = \cancel{\frac{1}{2} m v_P^2} + \frac{1}{2} I_P \omega^2 + \cancel{m \vec{v}_p \cdot (\vec{\omega} \times \vec{r}_{G|P})}$$

Comparison: W-E

Work-Energy (Relates change in speed to position)	Particle	$T_1 + V_1 + U^{nc}_{1\to2} = T_2 + V_2$ Where $T = \frac{1}{2} m v^2$
	Rigid Body G= c.m. & A= point on Body	$T_1 + V_1 + U^{nc}_{1\to2} = T_2 + V_2$ Where $T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$

Impulse-Momentum Equations

ext.

$$\int_1^2 \sum \vec{F}\, dt = m(\vec{v}_{G,2} - \vec{v}_{G,1})$$

LIM

- G is the center of mass and A could be anywhere

- If A is on the body

A must be on the rigid body

$$\int_1^2 \sum \vec{M}_A\, dt = \vec{H}_{A,2} - \vec{H}_{A,1} = I_A(\vec{\omega}_2 - \vec{\omega}_1)$$

AIM

- Coefficient of Restitution

$$e = -\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}$$

Comparison: IM

Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\, dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A \vec{\omega}$

How to choose a method?

- Let the FBD guide you!

- When is Newton-Euler useful?
  - How many moments in time? · 1 moment in time
  - What can you solve for?

    · $\vec{a}$, $\vec{\alpha}$ and forces

- When is W-E useful?
  - How many moments in time? · 2 moments in time
  - What can we solve for with W-E?

    · distance

    · velocity, angular velocity,

- When is AIM and/or LIM useful?
  - How many moments in time?
  - What can you solve for? · impulse, Δt

Summary

Method	Newton-Euler	W-E	LIM	AIM
# of states	1	2	2	2
Impacts?	n.a.	Cannot use for impacts	Can use for impacts	Can use for impacts
Integral term is over…	n.a.	Distance (ds)	Time (dt)	Time (dt)
Coordinate System	Will need to use all three!	Usually, cartesian	Usually, cartesian	Usually, polar
Diagrams	FBD & Kinetics	FBD	FBD	FBD
Systems	Usually only consider one object	Usually want to include all objects in system	Usually want to include all objects in system	Usually want to include all objects in system

What method? (Q1)

a) N-E
b) W-E
c) LIM
d) AIM

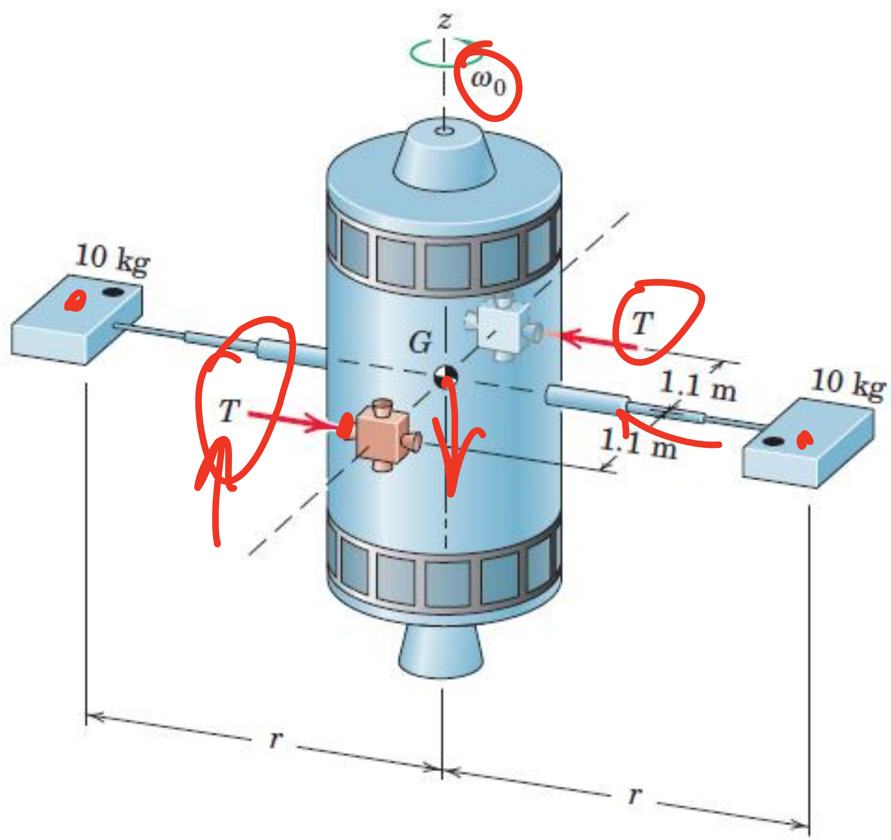
~~N-E: 2 states~~

~~WE: $\int T\, ds$~~

~~LIM:~~

AIM:

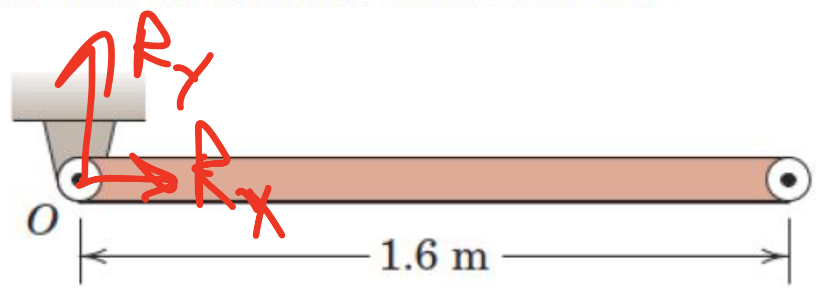
6/182 Two small variable-thrust jets are actuated to keep the spacecraft angular velocity about the z-axis constant at $\omega_0 = 1.25$ rad/s as the two telescoping booms are extended from $r_1 = 1.2$ m to $r_2 = 4.5$ m at a constant rate over a 2-min period. Determine the necessary thrust T for each jet as a function of time where $t = 0$ is the time when the telescoping action is begun. The small 10-kg experiment modules at the ends of the booms may be treated as particles, and the mass of the rigid booms is negligible.



What method? (Q2)

a) N-E
b) W-E
c) LIM
d) AIM

The uniform 20-kg slender bar is pivoted at O and swings freely in the vertical plane. If the bar is released from rest in the horizontal position, calculate the initial value of the force R exerted by the bearing on the bar an instant after release.



Problem 6/33

NE - One moment

Intime, asked to find some force

~~WE~~

LIM

AIM

• no Time 1 or time 2

• cant solve for this force

What method? (Q3)

a) N-E
b) W-E
c) LIM
d) AIM

The 1.2-kg uniform slender bar rotates freely about a horizontal axis through O. The system is released from rest when it is in the horizontal position $\theta = 0$ where the spring is unstretched. If the bar is observed to momentarily stop in the position $\theta = 50°$, determine the spring constant k. For your computed value of k, what is the angular velocity of the bar when $\theta = 25°$?

1: $\theta = 0$

2: $\theta = 50°$

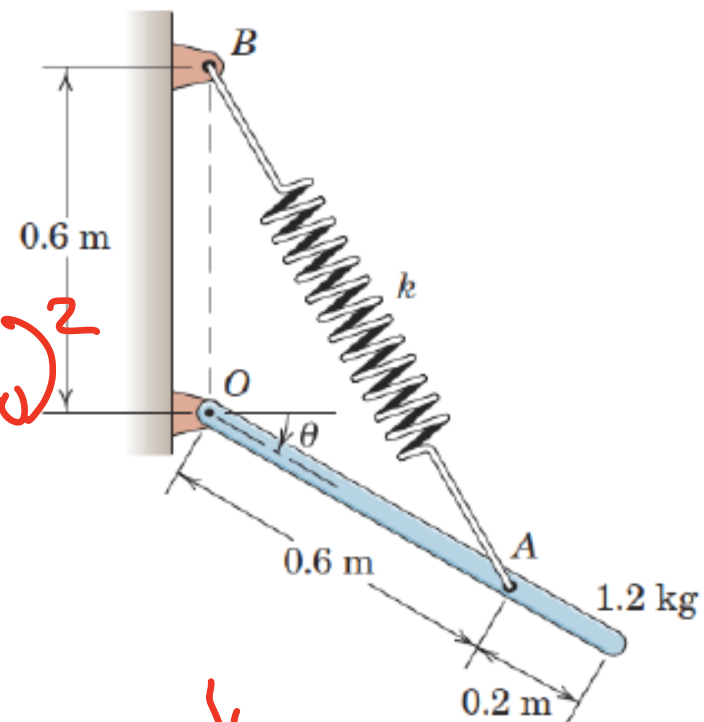
NE: 2 moments in time

WE

AIM $V_{sp} = \frac{k}{2}(L-L_0)^2$

LIM

$\int F_{spring}(t)\,dt \rightarrow F_{spring}$



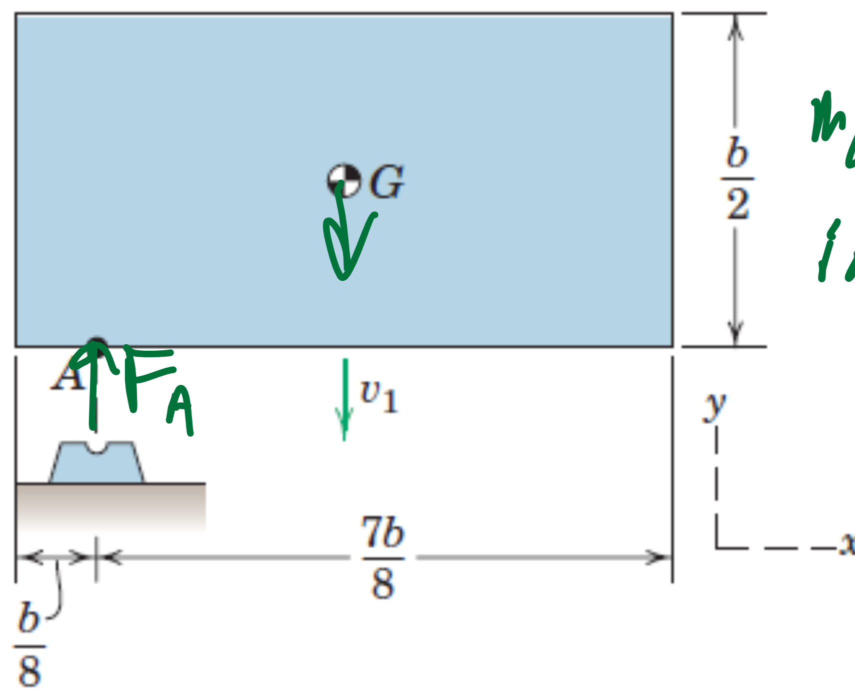
What method? (Q4)

a) ~~N-E~~
b) ~~W-E~~
c) LIM
d) AIM

LIM

AIM

6/178 The uniform rectangular panel is falling vertically with speed v_1 when its small peg A engages in the receptacle. Determine the angular velocity of the body as well as the x- and y-components of its mass-center velocity just after the impact.

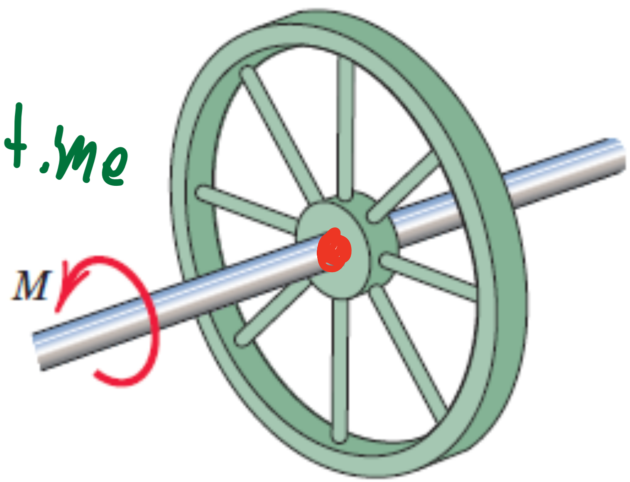


momentum in X

What method? (Q5)

a) ~~N-E~~
b) W-E
c) LIM
d) AIM

6/167 The 75-kg flywheel has a radius of gyration about its shaft axis of $\bar{k} = 0.50$ m and is subjected to the torque $M = 10(1 - e^{-t})$ N·m, where t is in seconds. If the flywheel is at rest at time $t = 0$, determine its angular velocity ω at $t = 3$ s.



WE: 2 moments in time

$U_{12}^{nc} = \int M\, d\theta$

~~LIM~~

AIM

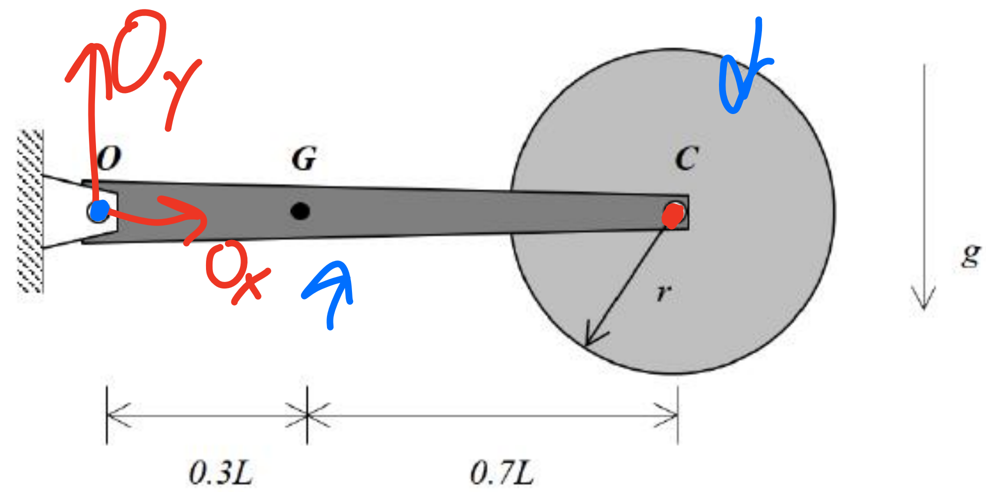
$\int M\, dt$

What method? (Q6)

a) ~~N-E~~
b) W-E
c) ~~LIM~~
d) AIM

Given: Link OC and the circular disk are released from rest when OC is horizontal. Link OC has a length of $L = 1.2$ m, mass of 10 kg and radius of gyration about O of $k_O = 0.5$ m. The disk has a mass of 8 kg and outer radius of $r = 0.4$ m.

Find: Determine the angular speed of OC when OC has moved to a vertical orientation.



~~LIM~~

AIM

W-E

- changes in height for v or ω

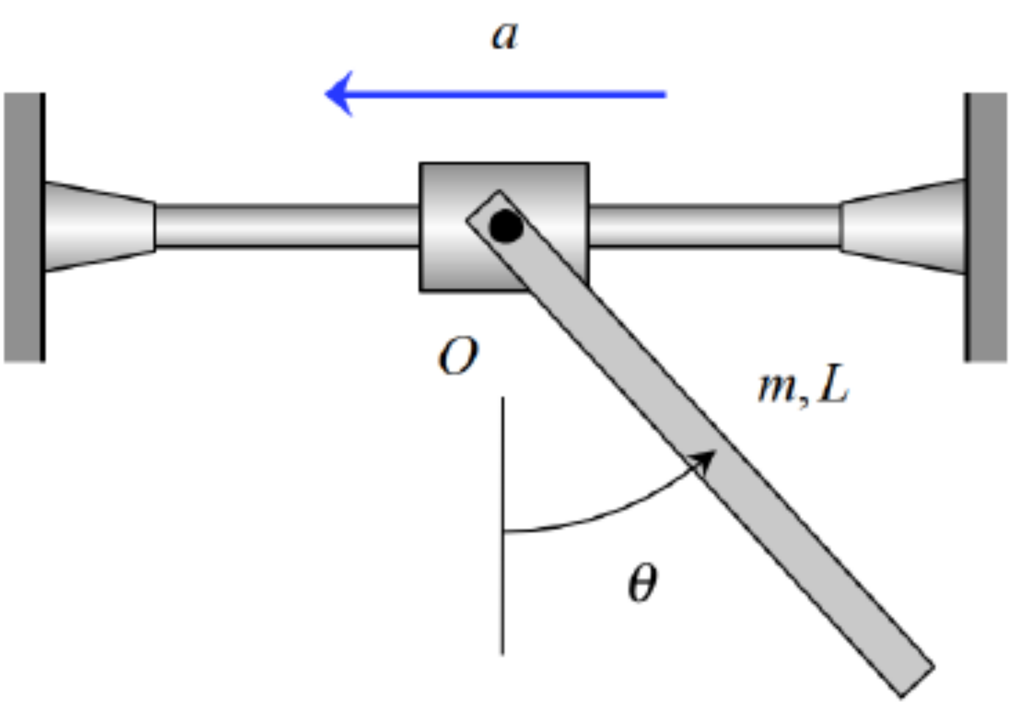
What method? (Q7)

a) N-E
b) ~~W-E~~
c) ~~LIM~~
d) ~~AIM~~

Given: The collar O has a constant acceleration of a to the left. A thin, homogeneous bar of length L and mass m is pinned to the collar.

Find: Find a value of a such that the bar is held at a constant angle of $\theta = 25°$.

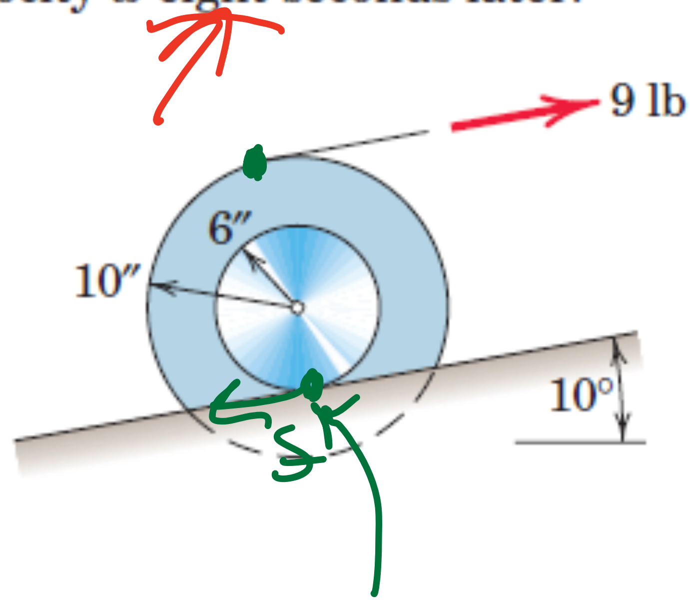
N-E



What method? (Q8)

a) ~~N-E~~
b) ~~W-E~~
c) LIM
d) AIM

6/170 The constant 9-lb force is applied to the 80-lb stepped cylinder as shown. The centroidal radius of gyration of the cylinder is $\bar{k} = 8$ in., and it rolls on the incline without slipping. If the cylinder is at rest when the force is first applied, determine its angular velocity ω eight seconds later.



$U_{AC}^{12} = \int F \, \underline{ds}$

LIM

AIM

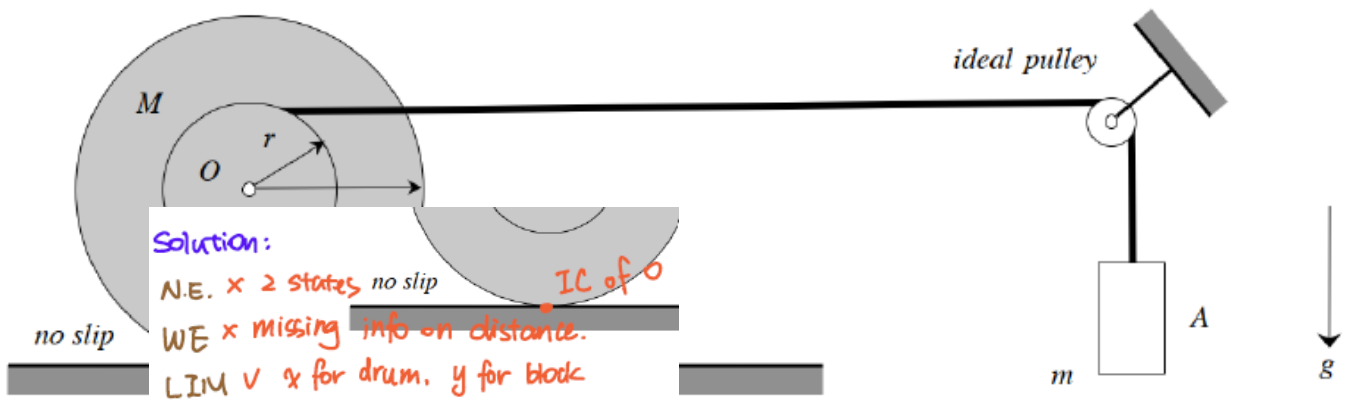
What method? (Q9)

a) N-E
b) W-E
c) LIM
d) AIM

Given: A spool (having a mass of M and radius of gyration about its center O of k_O) rolls without slipping on a rough, horizontal surface. A cable is wrapped around the inner radius of the spool. This cable is pulled over an ideal pulley and is attached to block A. Assume that the cable does not slip on the drum. The system is released from rest.

Find: Find the angular velocity of the spool after 10 s.

Use the following parameters in your analysis: m = 10 kg, M = 20 kg, r = 0.2 m, R = 0.4 m, k_O = 0.25 m.

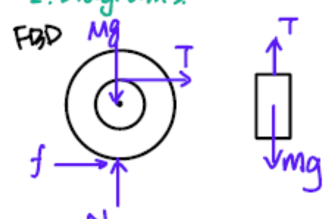


Solution:
N.E. × 2 states
WE × missing info on distance.
LIM ✓ x for drum. y for block
AIM ✓ about O for drum only.
IC of O

1. Coor. y x

2. Diagrams.

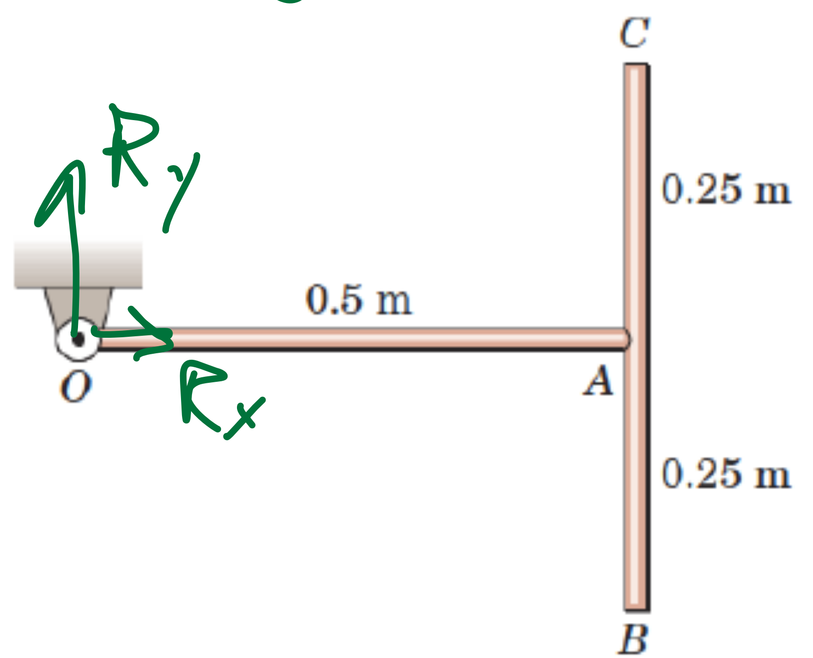
FBD



What method? (Q10)

a) N-E
b) W-E
c) LIM
d) AIM

Each of the two uniform slender bars OA and BC has a mass of 8 kg. The bars are welded at A to form a T-shaped member and are rotating freely about a horizontal axis through O. If the bars have an angular velocity ω of 4 rad/s as OA passes the horizontal position shown, calculate the total force R supported by the bearing at O.

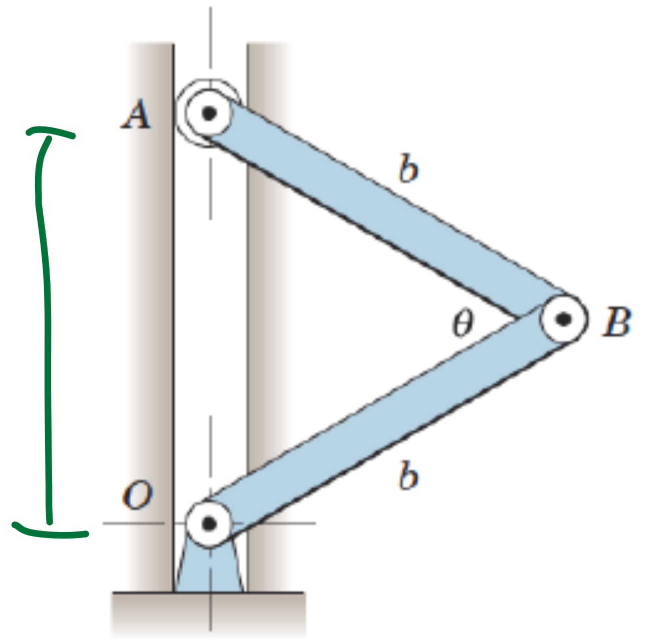


What method? (Q11)

a) N-E
b) W-E
c) LIM
d) AIM

The two identical links, each of length b and mass m, may be treated as uniform slender bars. If they are released from rest in the position shown with end A constrained by the smooth vertical guide, determine the velocity v with which A reaches O with θ essentially zero.

$V_{12}^{nc} = 0$



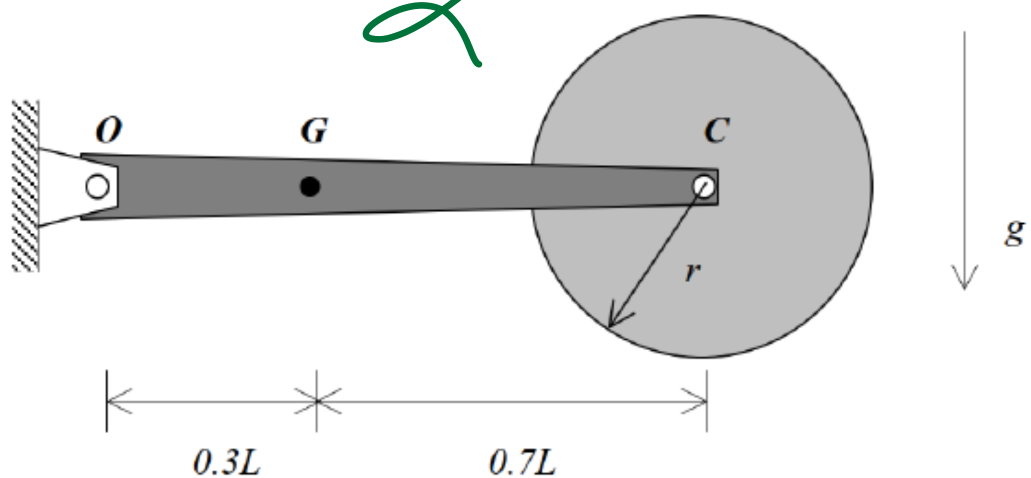
What method? (Q12)

a) N-E
b) W-E
c) LIM
d) AIM

Given: An inhomogeneous bar OC (having a length of $L = 2$ m, mass $m = 10$ kg, center of mass at G and centroidal radius of gyration of $k_G = 0.4$ m) is pinned to ground at end O. A homogeneous disk (having an outer radius of $r = 0.6$ m and mass $M = 15$ kg) is pinned to the right end of the bar at the disk's center C. The system is released from rest with OC being horizontal. Assume that all bearings are smooth.

Find: Determine the angular acceleration of arm OC on release.

α

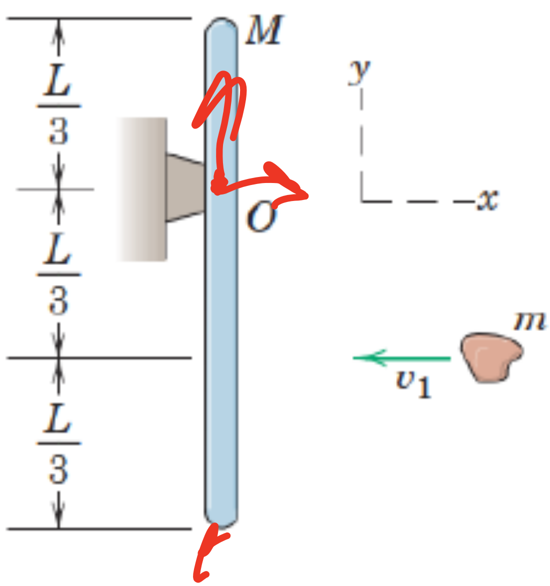


What method? (Q13)

a) ~~N-E~~
b) ~~W-E~~
c) LIM
d) AIM

linear momentum

The wad of clay of mass m is initially moving with a horizontal velocity v_1 when it strikes and sticks to the initially stationary uniform slender bar of mass M and length L. Determine the final angular velocity of the combined body and the x-component of the linear impulse applied to the body by the pivot O during the impact.



Question 1

Consider a homogeneous disc rolling without slipping on the ground with the contact point being A and center of mass being G. Can A and G be used for reference point for Euler Equation, Work-Energy Equation, Linear Impulse Momentum equation and Angular Impulse Momentum Equation?

$$\int_1^2 \sum \vec{F}\, dt = m(\vec{v}_{G,2} - \vec{v}_{G,1})$$

No, only G here for LIM

Yes both A & G

$$\sum \vec{M}_A = I_A\vec{\alpha} + \vec{r}_{G/A} \times (m\vec{a}_A)$$

$$T = \frac{1}{2}mv_P^2 + \frac{1}{2}I_P\omega^2 + m\vec{v}_p \cdot (\vec{\omega} \times \vec{r}_{G|P})$$

$$T_2 + V_2 = T_1 + V_1 + U_{1\to 2}^{(nc)}$$

@ yes, both A & G for W-E eq

$$\int_1^2 \sum \vec{M}_A\, dt = \vec{H}_{A,2} - \vec{H}_{A,1} = I_A(\vec{\omega}_2 - \vec{\omega}_1)$$

Yes, both A & G for AIM

Question 2

Which expressions below are not an acceptable formulation for the Euler Equation for relating moments and angular acceleration for the disk with the center of mass G and the geometric center O?

(a) $\sum M_D = I_D \vec{\alpha}$

(b) $\sum M_E = I_E \vec{\alpha}$

(c) $\sum M_O = I_O \vec{\alpha}$

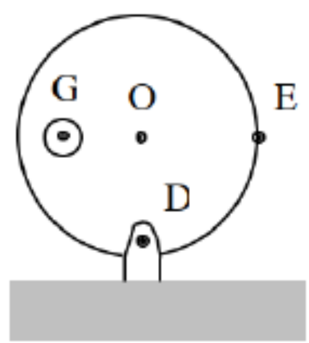
(d) $\sum M_G = I_G \vec{\alpha}$

L is not on body

and for E

$m\vec{r} \times \vec{a}$ sticks

around



Question 3

The center of the disk shown below is known to have a downward acceleration. Assume that the cable does not slip on the homogeneous disk. Circle the answer below that most accurately describes the tension in sections AB and CD of the cable.

(a) The tension in section AB is smaller than the tension in section CD.

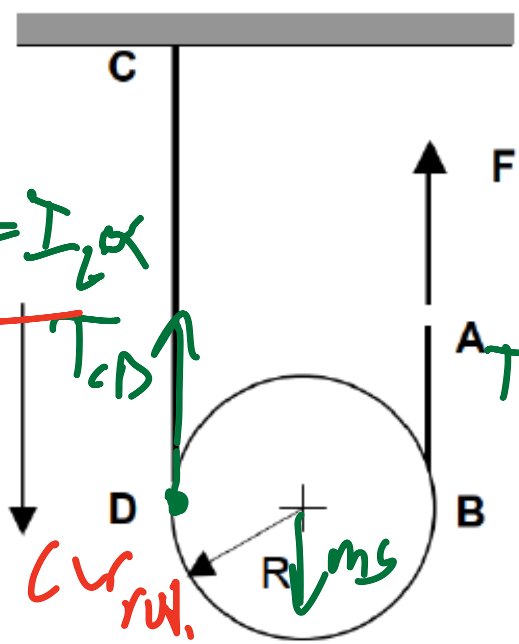
(b) The tension in section AB is equal to the tension in section CD.

(c) The tension in section AB is larger than the tension in section CD.

(d) More information is needed on the problem to answer this question.

a_y = neg, which means α is CW

$$\sum M_G = T_{AB}R - T_{CD}R = I_G\alpha$$



α must be negative b/c CW rot.

$T_{AB} < T_{CD}$

Question 4

The rigid body shown below has its center of mass at G. Circle the answer below that most accurately describes the mass moment of inertia of the body about point B.

(a) $I_B = 0$

(b) $I_B = I_G$

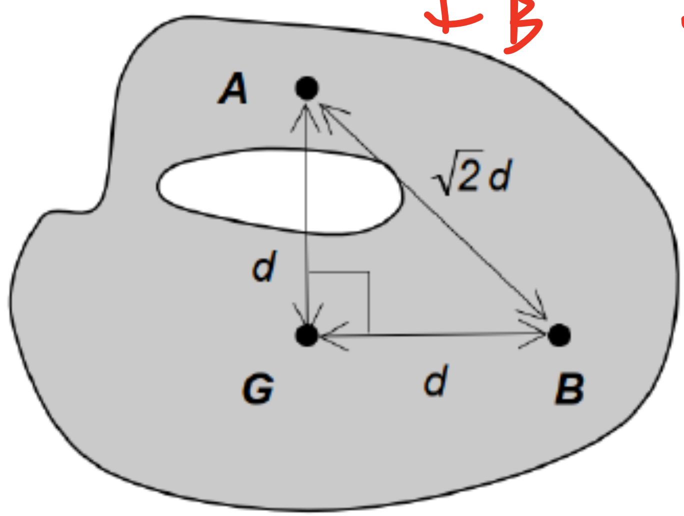
(c) $I_B = I_A$

(d) $I_B = I_A + m(\sqrt{2}d)^2$

(e) None of the above

$I_A = I_G + md^2$

$I_B = I_G + md^2$ ✓



Question 5

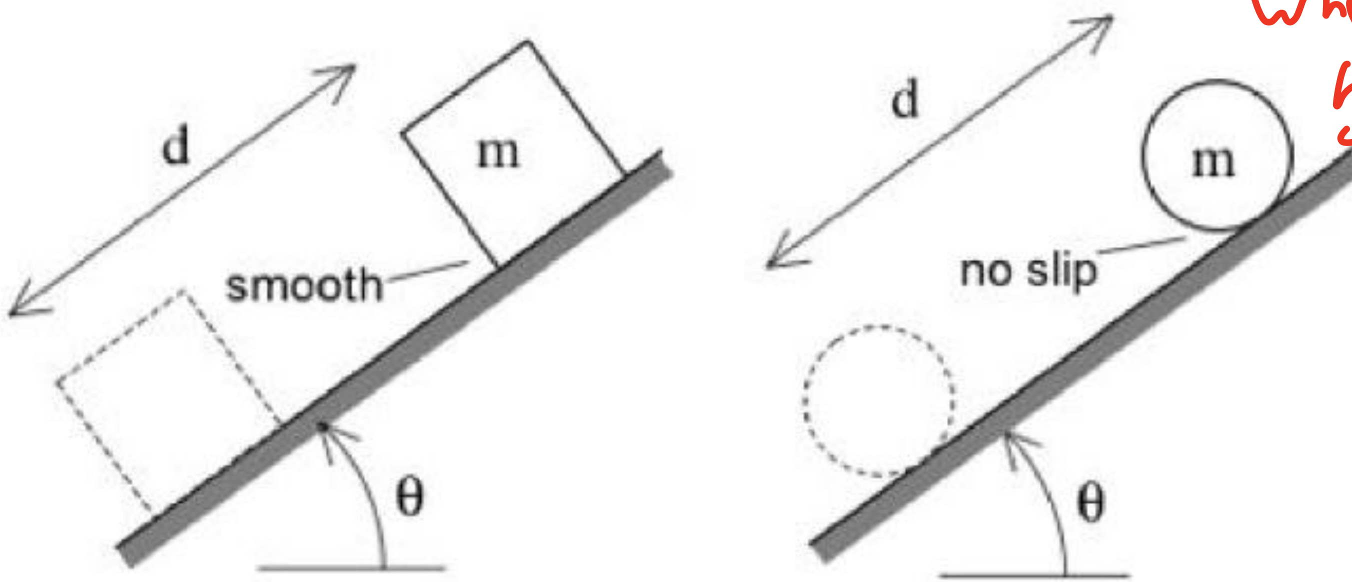
The block and homogeneous sphere are released from the same height. Circle the answer below that most accurately describes the relative speeds of the block and the center of the sphere after each has moved a distance down the inclines.

(a) The block is moving faster than the center of the sphere

(b) The block is moving more slowly than the center of the sphere

(c) The block and the center of the sphere are moving with the same speed

ΔE all converted to linear KE unlike disk where some goes to rot. KE



Lecture Summary

- We…
  - Summarized rigid body kinetics

- Next lecture we will…
  - Introduce vibrations!