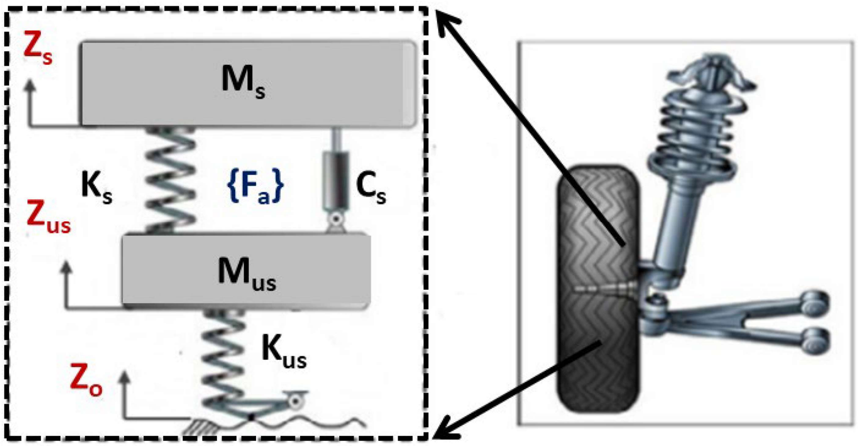
ME 274: Basic Mechanics II

Lecture 35: Vibrations – Intro and Equations of Motion

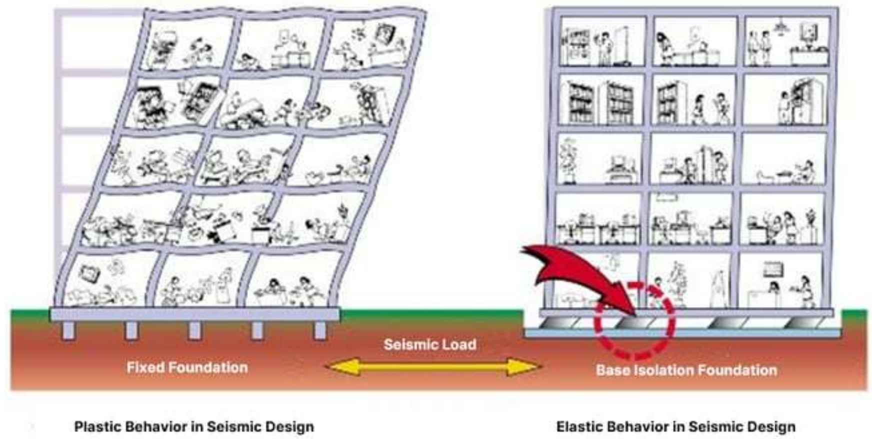
School of Mechanical Engineering

Motivation: Oscillatory motion is everywhere

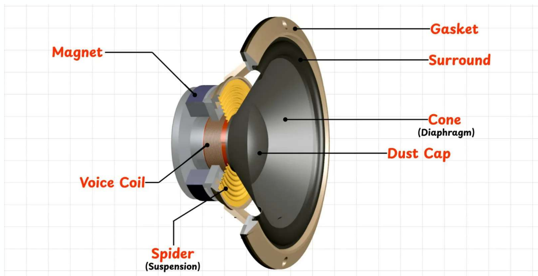
Vehicle Dynamics [1]

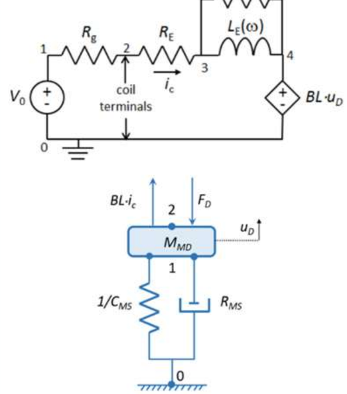


Building Design[2]



Acoustic Systems[3]





Vibrations are periodic oscillatory motion

Goal: solve for acceleration/velocity/position as a general function of time

$$\sum \vec{F} = m\vec{a}_G = m\ddot{x}$$
$$\sum \overrightarrow{M}_A = I_A\vec{\alpha} = I_A\ddot{\theta}$$

How? Developing differential equations of motion governing these oscillatory systems

$$M\ddot{x} + C\dot{x} + Kx = f(t)$$

[1] https://www.mdpi.com/2227-9717/11/9/2715
[2] https://www.motivewith.com/en/insights/seismic-isolation-in-structural-design-concepts-and-applications
[3] https://www.comsol.com/support/learning-center/course/modeling-speaker-drivers-in-comsol-multiphysics-202/modeling-speaker-drivers-lumped-methods-88401
[4] https://englishan.com/parts-of-a-speaker/

Forces in a vibrating system

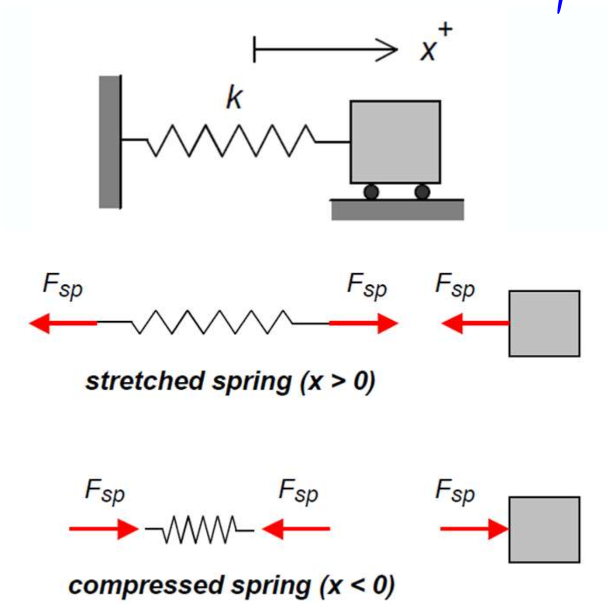
Vibrations EOM: $M\ddot{x} + C\dot{x} + Kx = f(t)$

Vibrational systems generally contain:

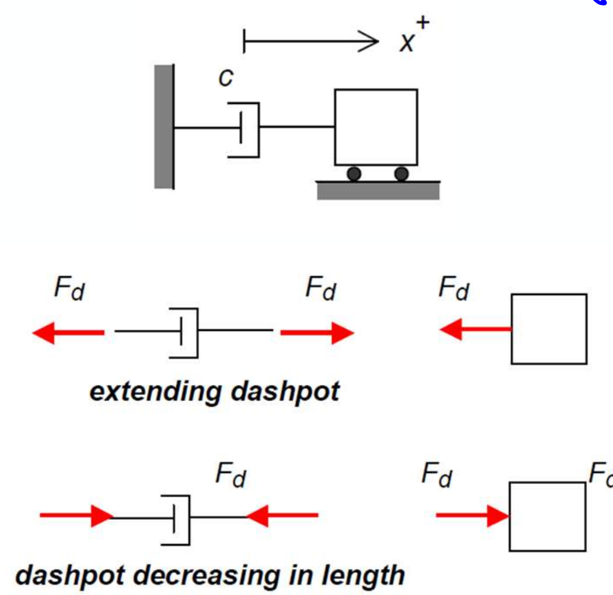
Mass (Inertia) | dashpot (damping) | Spring (stiffness) | External forcing

Oscillatory forces are a result of <u>springs</u> and <u>dashpots:</u>

Linear Springs : $F_{sp} = kx$ ← depends on position



Linear dashpots: $F_d = c\dot{x}$ ← depends on speed



Discussion: Solving vibrations problems

1. Define a set of coordinates to describe the motion of the system. Keep this consistent! Draw a FBD for each body
   - Springs: assume all coords (i.e. x) are in the positive direction and draw forces according to whether springs are in tension or compression. The signs of the forces at other points in time will be accounted for in the math!
   - Dashpots: assume all velocities (i.e. $\dot{x}$) are in the positive direction and draw forces according to whether dashpot is increasing or decreasing in length. The signs of the forces at other points in time will be accounted for in the math!

2. Based on your FBDs, use Newton-Euler equation to determine the dynamical equations for the bodies.

3. Use kinematic equations to relate your coordinates.

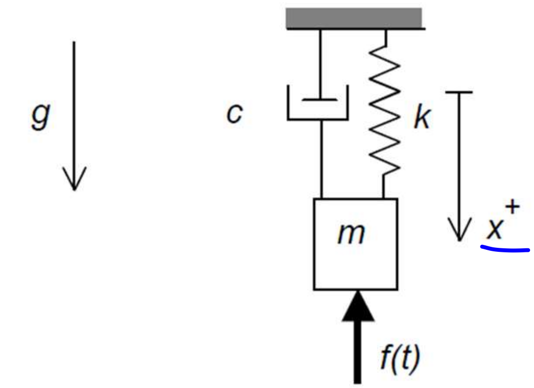
4. From (2) and (3), reduce your equations to create a single differential EOM. It should have the form:

$$M\ddot{x} + C\dot{x} + Kx = f(x)$$

Check: All your coefficients in this EOM should have the same sign. If they don't match, there is an error in the derivation of your EOM!

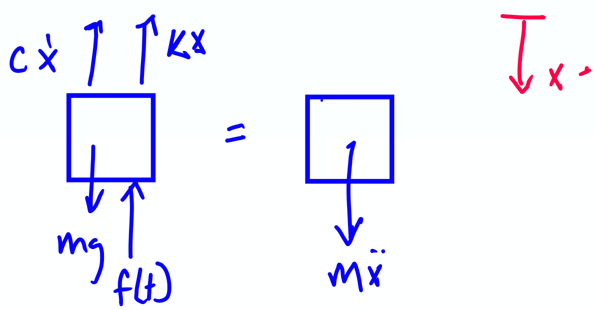
Example 6.A.1

Given: A particle of mass m is supported by a spring of stiffness k and a dashpot with damping constant c. A vertical force $f(t)$ acts on the particle as shown. Let x describe the position of the particle, where x is measured from the position of the particle when the spring is unstretched.



Find: Determine the EOM of this system in terms of the coordinate x.

1) FBD



2) $\overset{+}{\downarrow}\sum F_x = m\ddot{x} \implies mg - c\dot{x} - kx - f(t) = m\ddot{x}$

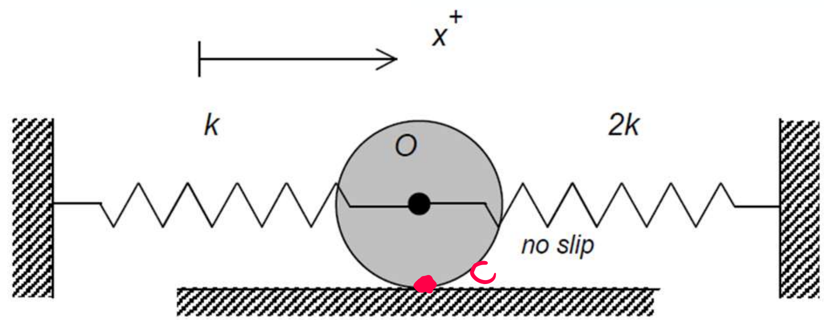
3) Kinematics

4) EOM: $\boxed{m\ddot{x} + c\dot{x} + kx = mg - f(t)}$

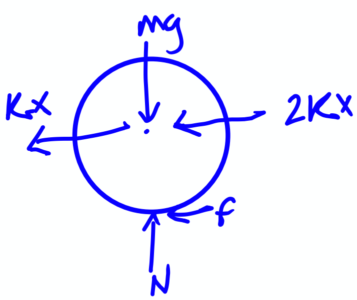
Example 6.A.2

Given: A homogeneous disk of mass m and radius r rolls without slipping on a rough horizontal surface. Two springs, having stiffnesses of k and $2k$, are attached between the disk center O and ground, as shown below. Let x describe the position of O, where the springs are unstretched when $x = 0$.

Find: Determine the EOM for the disk in terms of the coordinate x.



1) FBD



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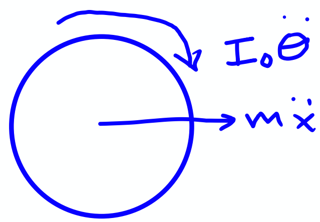


2) Kinetics

$$(\circlearrowleft +)\ \Sigma M_c = I_c\ddot{\theta} = -kxr - 2kxr$$

$$(I_o + mR^2)\ddot{\theta} = \tfrac{3}{2}mr^2\ddot{\theta} = -3kxr \quad \Rightarrow \quad -3kx = \tfrac{3}{2}mr\ddot{\theta} \quad (1)$$

3) Kinematics → roll w/o slip, IC @c

$$x = r\theta, \quad \dot{x} = r\dot{\theta}, \quad \ddot{x} = r\ddot{\theta}$$

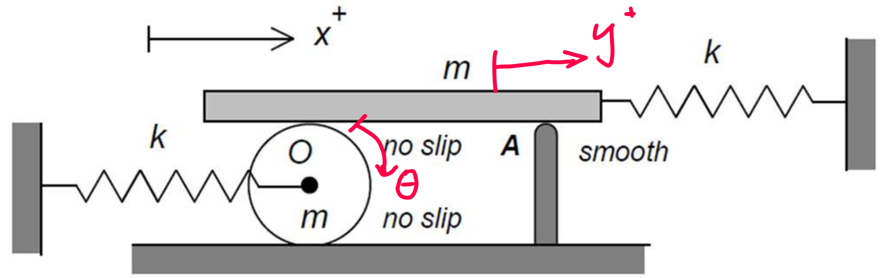
4) EOM ⇒

$$-3kx = \tfrac{3}{2}m\ddot{x} \quad \rightarrow \quad \boxed{\tfrac{3}{2}m\ddot{x} + 3kx = 0}$$

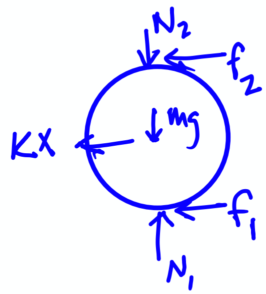
Example 6.A.3

Given: A homogeneous disk of mass of m and radius r rolls without slipping on a rough horizontal surface. A spring, having a stiffness of k, is attached between the disk center O and ground, as shown below. A block, also of mass m, is in *no-slip* contact with the top surface of the disk and with a smooth vertical support at A. A second spring of stiffness k is connected between the block and ground. Let x describe the position of O, where the springs are unstretched when $x = 0$.

Find: Determine the EOM for the disk in terms of the coordinate x.



Disk FBD



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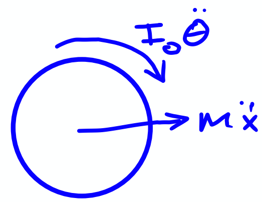


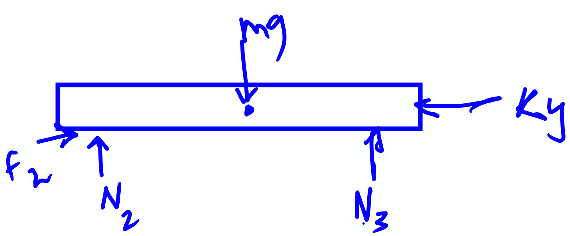
Kinetics → disk

$$\circlearrowleft \sum M_c = I_c \ddot{\theta}$$

$$= -kx\cancel{r} - f_2 2\cancel{r} = \frac{3}{2} m r^{\cancel{2}} \ddot{\theta}$$

$$-kx - 2f_2 = \frac{3}{2} m r \ddot{\theta} \quad (1)$$

Bar FBD



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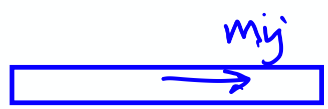
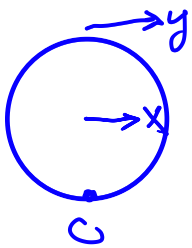


bar

$$\overset{+}{\rightarrow}_y \sum F_y = m\ddot{y}$$

$$f_2 - ky = m\ddot{y} \quad (2)$$

kinematics → roll w/o slip, IC @ c



$y = 2x$ (3)

$x = R\theta$, $\dot{x} = R\dot{\theta}$, $\ddot{x} = R\ddot{\theta}$

<u>Solve for EOM</u>

(3) → (2) $f_2 = 2(kx + mx)$

(2) → (1)

$$-kx - 4(kx + mx) = \frac{3}{2} m r \ddot{\theta}$$
$$= \frac{3}{2} m \ddot{x}$$

write in standard form

$$\boxed{\frac{11}{2} m \ddot{x} + 5kx = 0}$$