ME 274: Basic Mechanics II

Lecture 34: Rigid Body Kinetics – Review

PURDUE UNIVERSITY | School of Mechanical Engineering

When do we use each solution method?

Put effort up front deciding on which method(s) to use!

Use this table to help you decide based on the info you are given and what you are asked to solve for

Ask yourself:

- What should be included in the system?
- What is being conserved?
- Is this acting like a particle, or like a rigid body?

Remember your 4 step problem solving method:

1. FBD
2. Kinetics
3. Kinematics
4. Solve

Kinetics Table (End of 5.D in your lecture book)

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $where\ \ T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $where\ \ T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \bullet \left(\vec{\omega} \times \vec{r}_{G/A}\right)$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\,dt = \vec{H}_{O2} - \vec{H}_{O1}$ $where\ \ \vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\,dt = \vec{H}_{A2} - \vec{H}_{A1}$ $where\ \ \vec{H}_A = I_A\vec{\omega}$

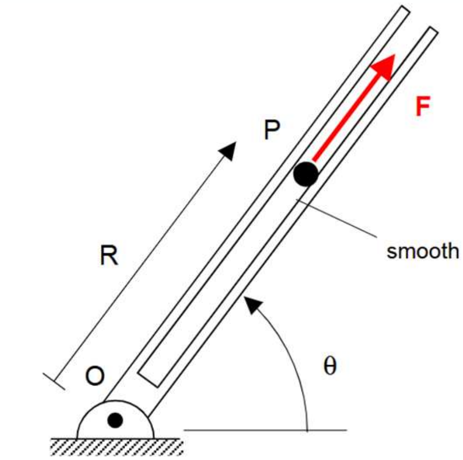
“Rapid Fire” Questions:
Let’s practice deciding how to approach these problems

Given: Pellet P having a mass of m is pulled through a barrel (having negligible mass) by means of radial force $F = 60R$, where F is in Newtons and R is in meters. The barrel is constrained to move in a HORIZONTAL plane by rotating about shaft passing through point O. The system is released with $R = R_1$, $\dot{R} = \dot{R}_1$ and $\dot{\theta} = \dot{\theta}_1$.

Find: For the instant when $R = R_2$:

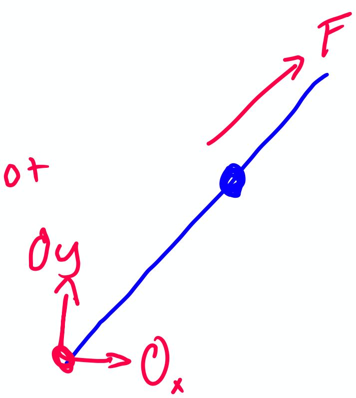
a) determine the rotation rate of the barrel, $\dot{\theta}_2$.

b) determine the value of $\dot{R}_2$.



HORIZONTAL PLANE

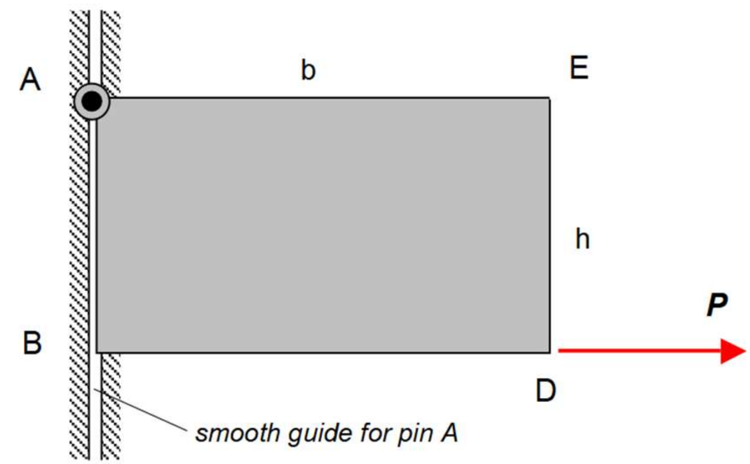
WE & AIM



Ask yourself:

- What should be included in the system? → Rig, particle & slot
- What is being conserved? → Angular Momentum
- Is this acting like a particle, or like a rigid body?

Given: A uniform plate of mass m is able to move on a smooth horizontal plane with corner A of the plate being constrained to move within a smooth guide in the plane of motion. The plate is initially at rest with edge AB aligned with the guide for A. A force P is applied at corner D, with P acting perpendicular to the guide for A.



HORIZONTAL PLANE

Find: Determine the initial angular acceleration of the plate. Write your answer as a vector.

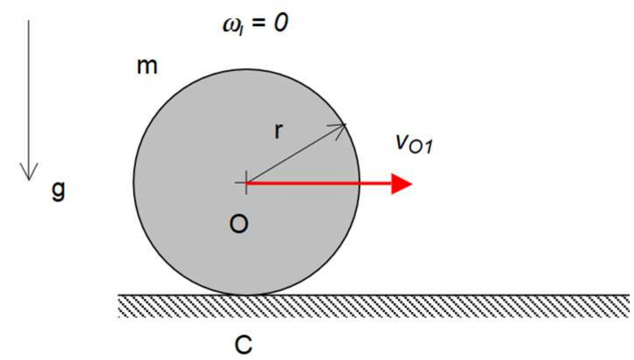
Newton - Euler

Ask yourself:

- What should be included in the system? small → plate
- What is being conserved? nothing
- Is this acting like a particle, or like a rigid body?

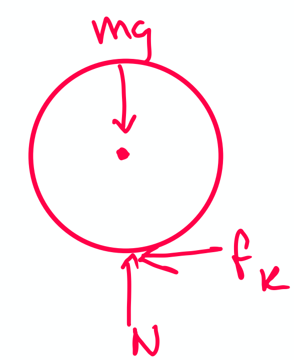
Given: The homogeneous disk shown below has a mass of m and outer radius of r. The disk is placed on a rough horizontal surface (coefficients of static and kinetic friction of μ_s and μ_k, respectively) with a uniform speed (i.e., zero angular velocity) of v_{O1} to the right.

Find: Determine the elapsed time, Δt, during which the disk travels to the right before slipping ceases between the disk and the horizontal surface. Note that slipping ceases when the contact point C has zero velocity.



$\Delta t \rightarrow \Delta v$

Impulse momentum



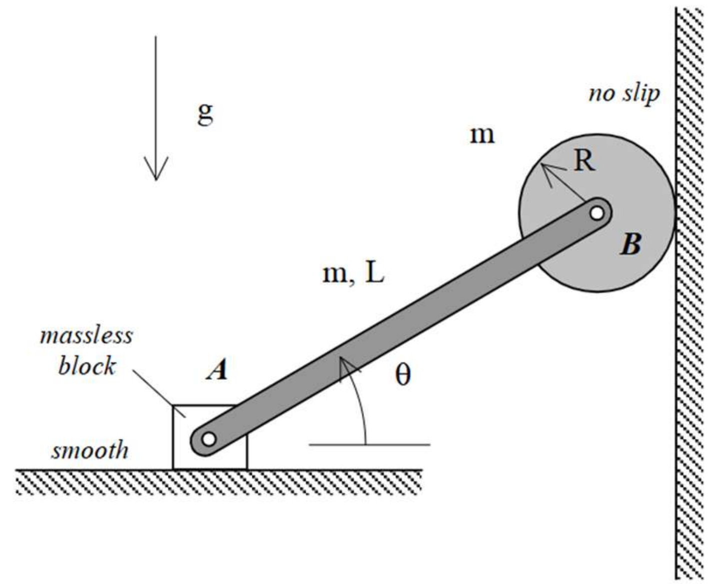
Ask yourself:

- What should be included in the system? disk
- What is being conserved? → nothing, friction changes linear + angular momentum
- Is this acting like a particle, or like a rigid body? → translation + rotation from friction moment

Given: System released from rest with $\theta = 36.87°$. Consider the bar and the disk to be homogeneous.

Find: Determine the *angular velocity* of the bar when $\theta = 0$. Use: $m = 100\ kg$, $L = 2\ meters$ and $R = 0.4\ meters$.

change in height → W-E

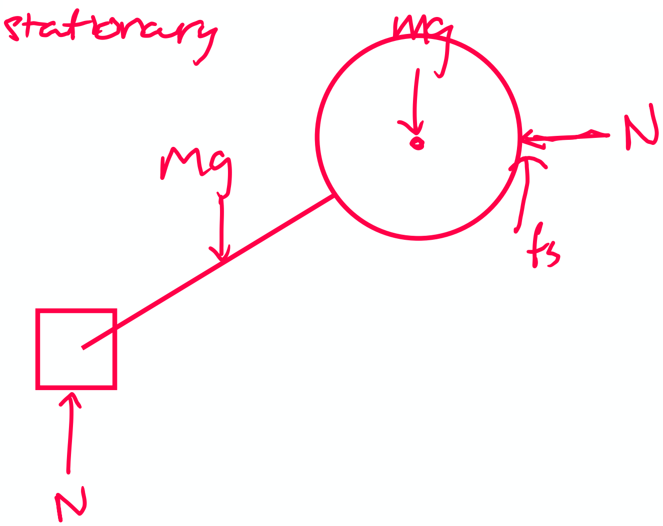


Note: in rolling w/o slip, friction does no work

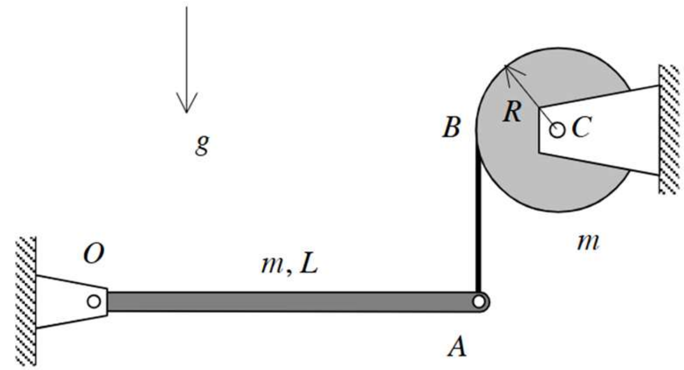
remember the contact point is instantaneously stationary if roling on a stationary surface

Ask yourself:

- What should be included in the system? block, bar, disk
- What is being conserved? ← mechanical energy
- Is this acting like a particle, or like a rigid body?

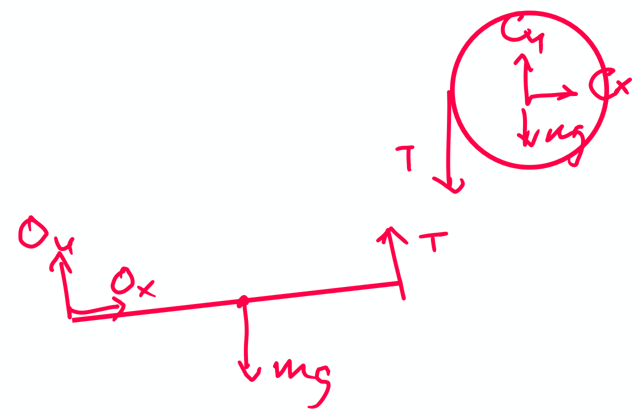


Given: Homogeneous bar OA (of length L and mass m) is pinned to ground at end O. End A of the bar is connected to a cable that is wrapped around a homogeneous disk (of mass m and outer radius R), with the disk being pinned to ground at its center C. Assume that the cable does not slip on the disk. The system is released from rest with OA being horizontal and the cable being vertical.



Find: Determine the *angular acceleration of the disk* on release. Use the following: $m = 10kg$, $L = 4\,meters$ and $R = 2\,meters$.

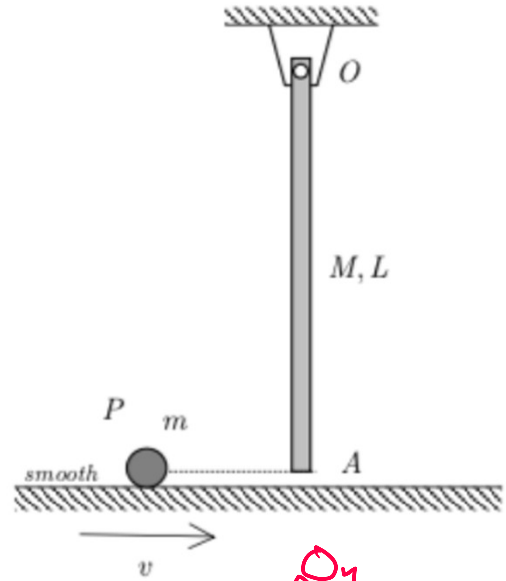
Newton Euler



Ask yourself:

- What should be included in the system? → small, individual FBD
- What is being conserved? → Nothing
- Is this acting like a particle, or like a rigid body?

Given: Particle P (of mass m) slides along a smooth horizontal surface with a speed of v. P then strikes end A of a stationary thin, homogeneous bar (of length L and mass M) that is pinned to ground at O. The coefficient of restitution for the impact of P with end A of the bar is known to be e.



Find: Determine the angular speed of bar OA immediately after the impact. Use the following parameters in your analysis: $m = 10kg$, $M = 30\ kg$, $L = 2\ meters$, $v = 40\ m\ /\ sec$ and $e = 0.5$

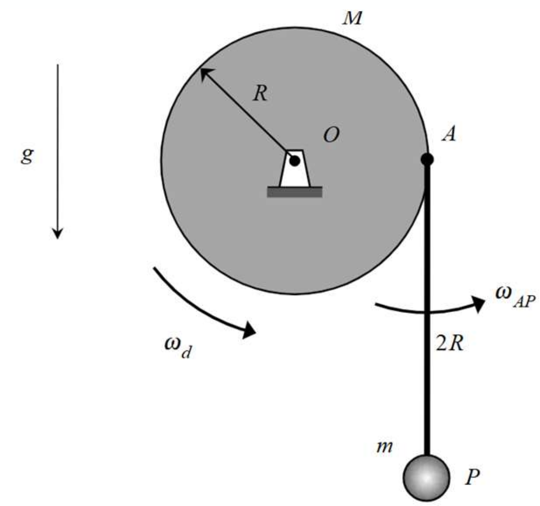
impulse momentum & restitution

Ask yourself:

- What should be included in the system? particle & bar
- What is being conserved? Angular momentum
- Is this acting like a particle, or like a rigid body?

both!

Given: A homogeneous disk with an outer radius of R and of mass M is pinned to ground at its center O. Rigid link AP, of negligible mass and length 2R, is pinned to the outer perimeter of the disk at A. Particle P (of mass m) is attached to the free end of link AP. At the instant shown below: A is directly to the right of O: P is directly below A; link AP is rotating in the counterclockwise sense with an angular speed of ω_{AP}; and, the disk is rotating in the counterclockwise sense with an angular speed of ω_d.

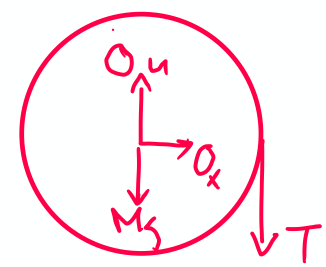


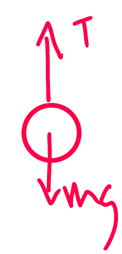
Find: For the position shown, determine the angular acceleration of the disk and the angular acceleration of link AP. Write your answers as vectors. Leave your answers in terms of: m, M, R, g, ω_d and ω_{AP}.

Newton Euler

Small system → individual FBD

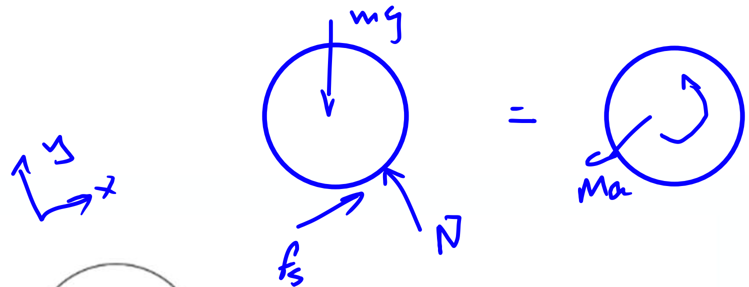
nothing conserved

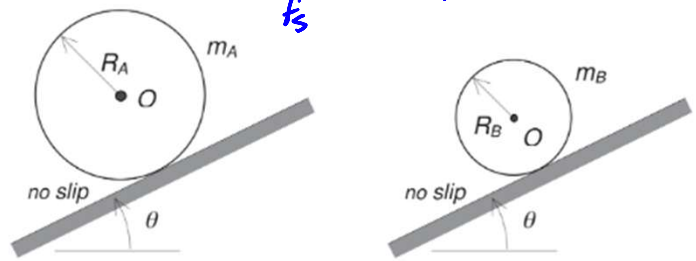




Quiz 5 solutions

Consider the homogeneous disks A and B with $m_A > m_B$ and $R_A > R_B$. The disks are released from rest. Circle the answer below that most accurately describes the relatives sizes of the angular accelerations of the centers of the two disk, α_A and α_B on release.





$$\Sigma F_x = f_s - mg\sin\theta = -ma$$

$$\Sigma M_o = I_o \alpha = f_s R$$

$$I_o = \tfrac{1}{2} m R^2$$

$$a = \alpha R \rightarrow \alpha = \frac{a}{R}$$

$$f_s = \frac{I_o \alpha}{R} = \frac{mR\alpha}{2}$$

$$\frac{\cancel{m}R\alpha}{2} - \cancel{m}g\sin\theta = -\cancel{m}a$$

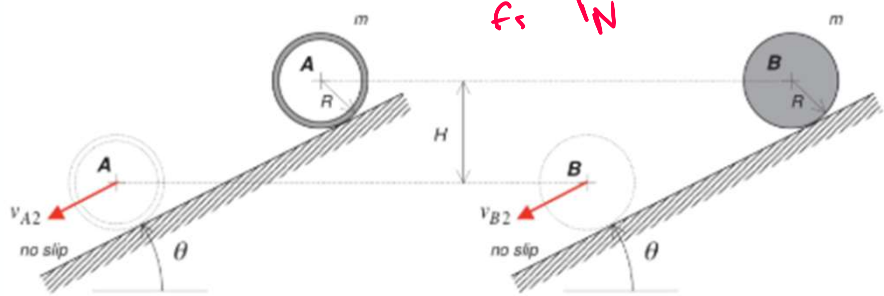
$$\frac{\alpha R}{2} - g\sin\theta = -\alpha R$$

$$\frac{3}{2}\alpha R = g\sin\theta$$

$$\alpha = \frac{2}{3}\frac{g\sin\theta}{R}$$

$$\boxed{\alpha \propto \frac{1}{R} \qquad \alpha_B > \alpha_A}$$

The homogeneous ring and disk are released from rest at the same height. Let v_{A2} and v_{B2} represent the speeds of the centers of the ring and disk after they have dropped through the same vertical distance H. What answer below that most accurately describes the relative sizes of v_{A2} and v_{B2}.



WE: $\cancel{T_1} + V_1 + \cancel{U^{NC}_{1\to2}} = T_2 + \cancel{V_2}$

for rolling: $v = r\omega$

$$mgH = \frac{1}{2}mv^2 + \frac{1}{2}I_o\omega^2$$

$$= \frac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

$$v^2 = \frac{2gH}{1 + \frac{I}{mr^2}}$$

$$v^2 \propto \frac{1}{1 + \frac{I}{mr^2}}$$

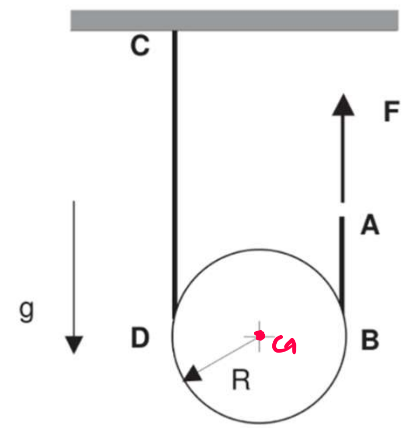
$I\uparrow$ $v\downarrow$

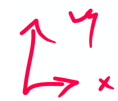
ring: $I_G = mr^2$

disk: $I_G = \frac{1}{2}mr^2$

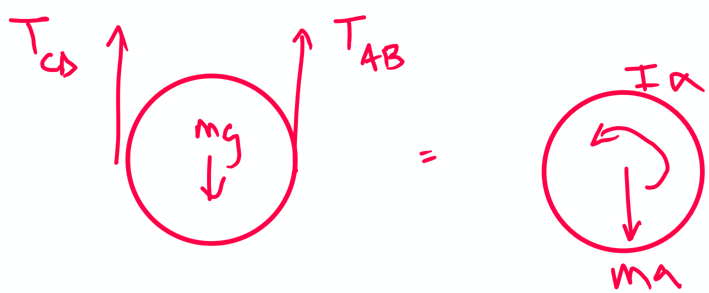
$$\boxed{v_B > v_A}$$

The center of the disk shown below is known to have a downward acceleration. Assume that the cable does no slip on the homogenous disk. Circle the answer below that most accurately describes the tension in sections AB and CD of the cable.





Newton Euler



$$\Sigma F_y = T_{CD} + T_{AB} - mg = -ma$$

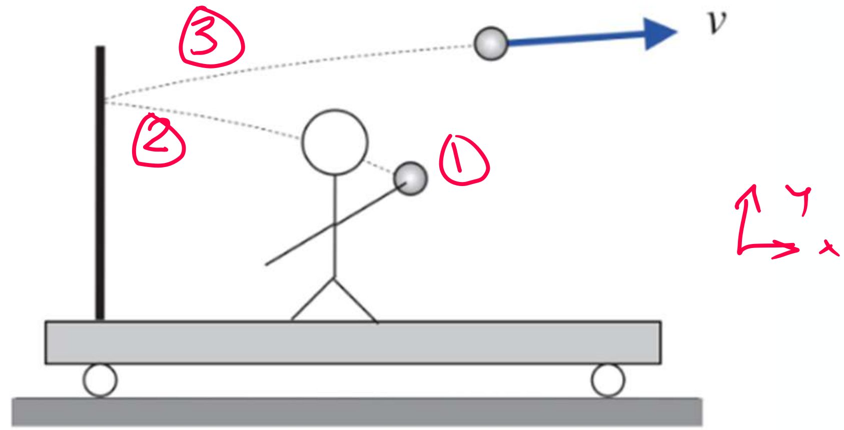
$$+\circlearrowleft \ \Sigma M_{cg} = I\alpha = RT_{AB} - RT_{CD}$$

if the disk is accelerating down, a is CW $\rightarrow a = \alpha r$, $a<0 \rightarrow \alpha < 0$

$$T_{AB} - T_{CD} < 0$$

$$T_{AB} < T_{CD}$$

You are on a cart that is initially at rest on a smooth track. You throw a ball at a partition that is rigidly mounted on the cart. If the ball bounces off the partition as shown in the figure, then at the instant shown in the figure:



1) before throw
2) after throw, before impact
3) after impact

No net impulse in X → momentum conserved in x

① → ②

$$m_B v_{B1} + m_C v_{C2} = m_B v_{B2} + m_C v_{C2}$$

$$0 + 0 = m_B \overset{(-x)}{v_{B2}} + m_C \overset{(+x)}{v_{C2}}$$

If not given, assume perfectly elastic $(e=1)$

$$0 = m_B \overset{(-x)}{v_{B2}} + m_C \overset{(+x)}{v_{C2}} = \underline{m_B \overset{(+x)}{v_{B3}} + m_C \overset{(-x)}{v_{C3}}}$$