ME 274: Basic Mechanics II

Lecture 31: Rigid Body Kinetics – Impulse-Momentum

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Impulse-Momentum comes from integrating Newton-Euler wrt time

Note: we don't separate into angular & linear approach like for particles,as rigid body has both translation & rotation

Remember our Newton-Euler equations for rigid bodies:

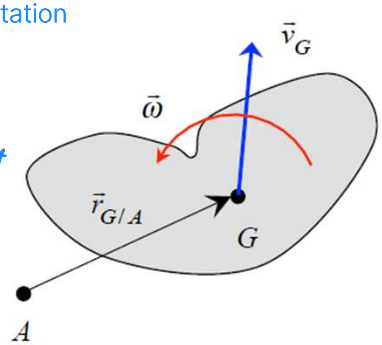
$$\sum \vec{F} = m\vec{a}_G$$

$$\sum \vec{M}_A = \frac{d}{dt}\vec{H}_A, \qquad \vec{H}_A = I_A\vec{\omega}$$

reference pt. A

Angular momentum abt. A

mass moment of inertia abt. A



(fixed point not necessarily on the body)

Integrating both equations with respect to time:

$$\int_1^2 \sum \vec{F}\, dt = m\vec{v}_{G2} - m\vec{v}_{G1}$$

$$\int_1^2 \sum \vec{M}_A dt = \vec{H}_{A2} - \vec{H}_{A1} = I_A\vec{\omega}_2 - I_A\vec{\omega}_1$$

If the point A is **not** on the body, we can solve for the angular momentum $\vec{H}_A$ as:

$$\vec{H}_A = \vec{H}_G + m\vec{r}_{G/A} \times \vec{v}_G = I_G\vec{\omega} + m\vec{r}_{G/A} \times \vec{v}_G$$

about pt. G

motion of G abt. A

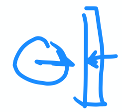
This gives a final angular momentum equation of:

$$\int_1^2 \sum \vec{M}_A dt = \left[I_G\vec{\omega}_2 + m\vec{r}_{G/A} \times \vec{v}_{G2}\right] + \left[I_G\vec{\omega}_1 + m\vec{r}_{G/A} \times \vec{v}_{G1}\right]$$

Note: if A is not on the body, $\vec{r}_{G/A}$ can change over time and the integral can get messy – we will not use this commonly in this course

Important considerations for impulse-momentum

- We generally want to make our system as large as possible to eliminate internal forces
- Only external forces and moments result in a change in momentum
- Internal impulses (for example from impact) cancel



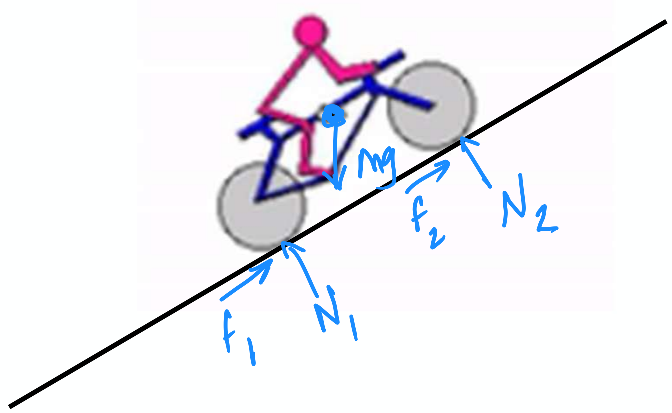
- If there are no net external impulses in a direction, momentum is conserved in that direction
- Generally, this solution method is used to find changes in speed over a change in time; however, in cases where momentum is being conserved, it can relate velocities at different positions without needing time explicitly

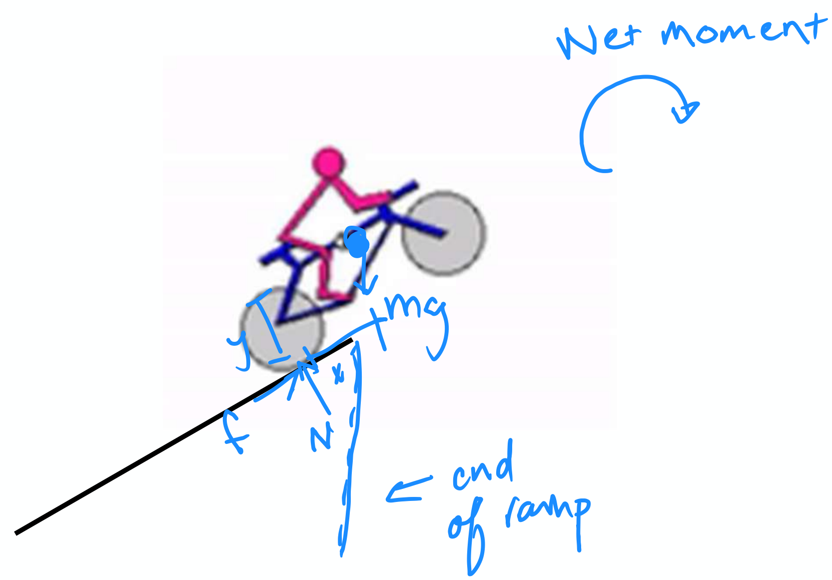
$$\int \sum F$$

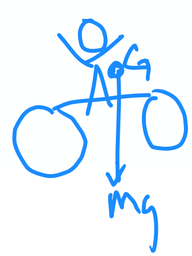
$$\int_{t_1}^{t_2} \sum M$$

Demo: Stunt Cyclist

time 1 → both wheels
contacting ramp



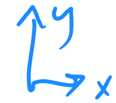


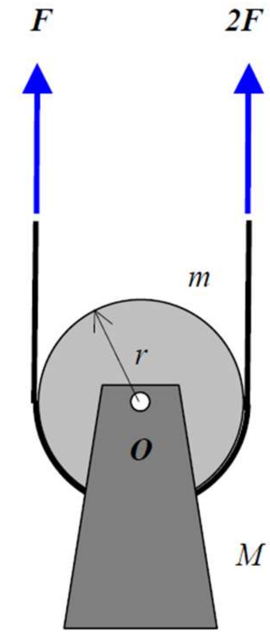


Example 5.C.1

Given: Constant forces of F and $2F$ are applied at the free ends of the hoisting cable as shown. The velocity of O is 4 m/s down and the angular velocity ω_1 of the drum (having a centroidal radius of gyration of $k_O = 0.4$ m) is counterclockwise at time $t = 0$.

Find: Determine, after 10 s:
(a) The velocity of O; and
(b) The angular velocity of the drum.





assume only disk spins

LIM: $\int_1^2 \sum \vec{F}\,dt = m\vec{V}_1 - m\vec{V}_0$

$$\int_{t_1}^{t_2} [3F-(m+M)g]\,dt = (m+M)(V_{02}-V_{01})$$

$$[3F-(m+M)g](t_2-t_1) = (m+M)(V_{02}-V_{01}) \qquad (1)$$

→ solve for V_{02} → $V_{02}\hat{j}$

$\circlearrowleft +$ AIM: $\int_1^2 \sum \vec{M}_0\,dt = I_0\vec{\omega}_2 - I_0\vec{\omega}_1$

$$\int_{t_1}^{t_2} (2Fr - Fr)\,dt = mk_0^2(\vec{\omega}_2-\vec{\omega}_1) \qquad (2)$$

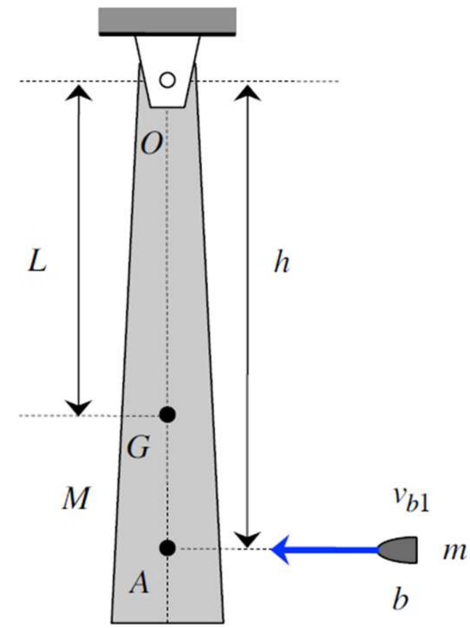
$$Fr\Delta t = mk_0^2(\omega_2-\omega_1)$$

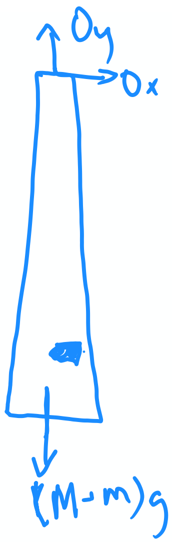
→ solve for ω_2 → $\omega_2\hat{k}$

Example 5.C.5

Given: A bullet having a mass of m strikes a stationary pendulum at a distance of h from the pivot O with a speed of v_{b1}. The pendulum has a mass of M and has a radius of gyration of k_O about the pivot O. The bullet imbeds itself immediately within the pendulum at A on impact.

Find: Determine the angular speed of the pendulum immediately after impact occurs.





$\sum \vec{M}_O = 0$ ← angular momentum conserved

$$(\vec{H}_O)_1 = (\vec{H}_O)_2$$

time 1 → just bullet (particle)

$$(\vec{H}_O)_1 = \vec{r}_1 \times m\vec{v}_1$$
$$= -h\hat{j} \times m(-v_{b1})\hat{i} = -hmv_{b1}\hat{k}$$

time 2 → bullet + pendulum

$$(\vec{H}_O)_2 = I_O \omega_2 \hat{k}$$
$$I_O = I_{O,p} + I_{O,b} = Mk_O^2 + mh^2$$
$$(\vec{H}_O)_2 = (Mk_O^2 + mh^2)\omega_2 \hat{k}$$

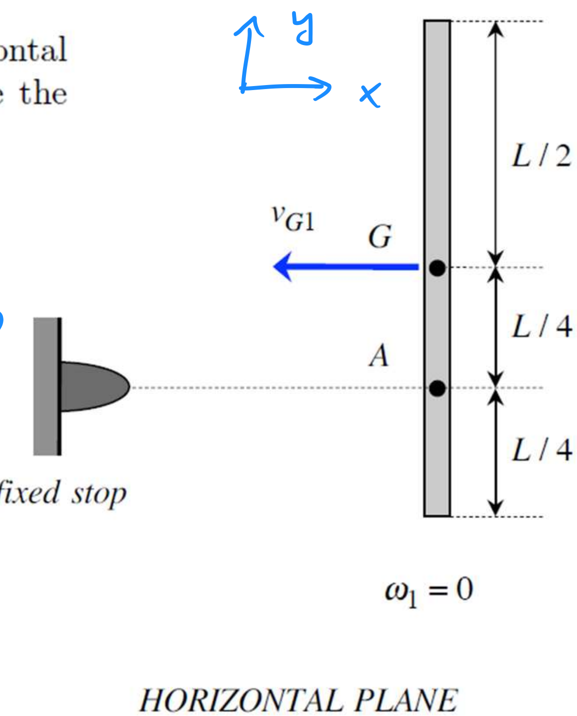
→ solve for $\vec{\omega}_2$

Example 5.C.7

Given: A thin homogeneous bar of mass m and length L translates along a smooth horizontal surface with a speed of v_{G1}. The bar strikes a fixed stop at location A on the bar, where the coefficient of restitution for this impact is e, where $0 < e < 1$.

Find: Determine:

(a) The speed of the center of mass G for the bar after impacting the stop; and
(b) The angular velocity of the bar after impacting the stop.



FBD → why dont we include the stop in our system?
The stop is fixed to ground → unknown moments & reactions

F →

LIM x-dir

$$-mv_{G1} + \int \Sigma F_x \, dt = m v_{G2} \quad (1)$$

AIM

$$0 + \int F \tfrac{L}{4} \, dt = I_G \omega_2$$

$$\tfrac{L}{4} \int F dt = \tfrac{1}{12} m L^2 \omega_2 \quad (2)$$

Impact Rule:

$$e = -\frac{V_{Abx2} - \cancel{V_{AWx2}}}{V_{Abx1} - \cancel{V_{AWx1}}} = -\frac{V_{Abx2}}{V_{Abx1}} \quad (3)$$