ME 274: Basic Mechanics II

Lecture 31: Rigid Body Kinetics – Work Energy

PURDUE UNIVERSITY | School of Mechanical Engineering

Work-Energy for rigid bodies

The Work-Energy equation for rigid bodies has the same general form as we saw for systems of particles:

$$T_1 + V_1 + U_{1\to2}^{NC} = T_2 + V_2$$

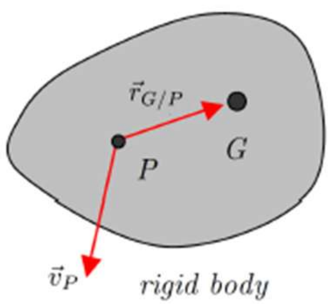
However, each of the components is calculated differently:

Kinetic Energy:

- The general form for calculating kinetic energy is $T = \frac{1}{2}mv_p^2 + m\,\vec{v}_P \cdot (\vec{\omega} \times \vec{r}_{G/P}) + \frac{1}{2}I_P\omega^2$
- P is a chosen reference point.
- Remember, I_P is the mass moment of inertia about P – you can look up I_G in your lecture book and use the parallel axis theorem to calculate

This equation can be simplified by selecting a reference point at G:

Or by selecting a fixed reference point:



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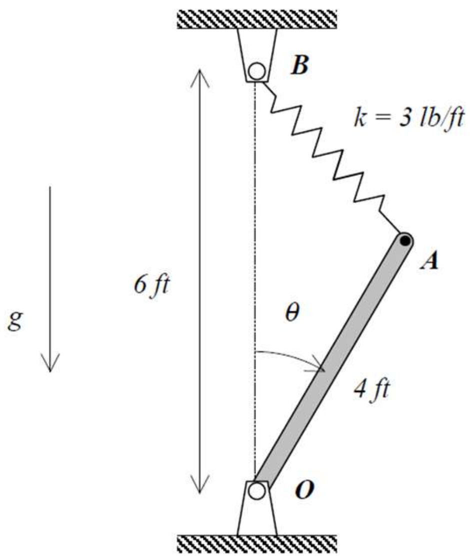
Potential Energy:

- Potential energy is calculated in the same manner as for particle systems: $V = V_g + V_{sp}$
- For gravitational potential, use the height of the center of mass
- For spring potential, use the geometry of the problem to calculate ΔL



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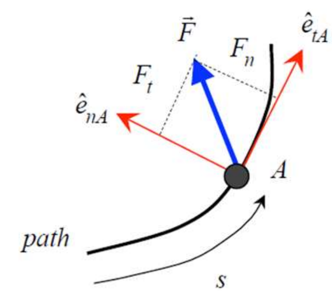
Work:

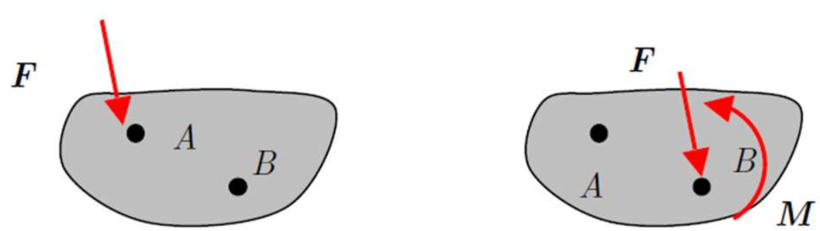
- Work done by a nonconservative force is the integral of the TANGENTIAL component of the force over the path of the point A at which it acts

$$U_{1\to 2} = \int_1^2 \vec{F} \cdot \hat{e}_{tA} ds = \int_1^2 F_t ds$$



- If the point at which the force acts follows a complicated path, an equivalent force couple can be used to simplify the integral:

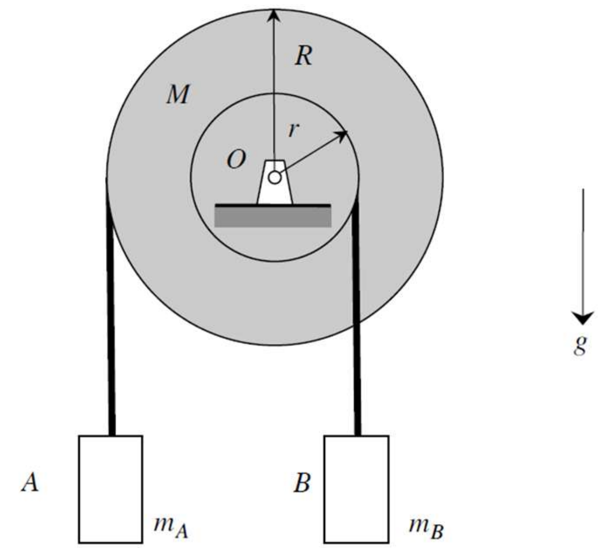
$$U_{1\to 2} = \int_1^2 \vec{F} \cdot \hat{e}_{tB} ds + \int_1^2 \vec{M} \cdot d\theta$$



Example 5.B.1

Given: A stepped drum (having a mass of M and radius of gyration about its center O of k_O) is attached to a smooth shaft passing through its center O. A cable wrapped around the outer radius of the drum is attached to block A. A second cable is wrapped around the inner radius of the drum and is attached to block B. Assume that the cables do not slip on the drum. The system is released from rest.

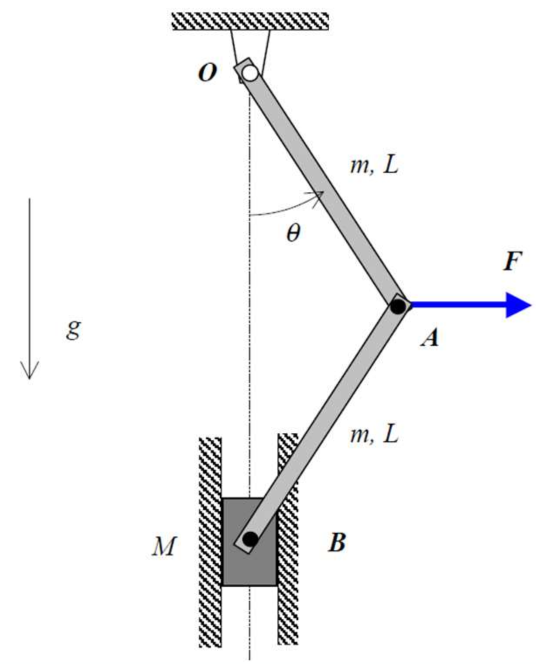
Find: Determine the speed of B after it has dropped 1.5 meters.



Example 5.B.5

Given: The system shown is released from rest when $\theta = 0$. A constant force F acts horizontally at pin B for all time. Both links are homogeneous and have a mass of m. The slider block at B has a mass of M. The system moves in a vertical plane.

Find: Determine the speed of B when $\theta = 53.13°$.



Example 5.B.7

Given: Link OC and the circular disk are released from rest when OC is horizontal. Link OC has a length of $L = 1.2$ m, mass of 10 kg and radius of gyration about O of $k_O = 0.5$ m. The disk has a mass of 8 kg and outer radius of $r = 0.4$ m.

Find: Determine the angular speed of OC when OC has moved to a vertical orientation.