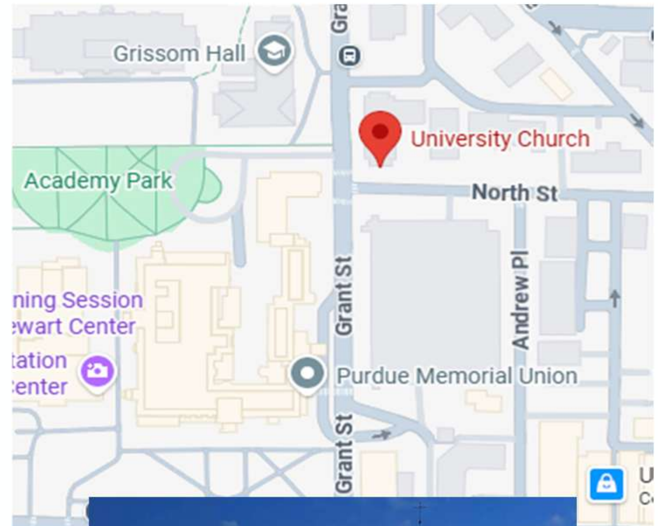
ME 274: Basic Mechanics II

Lecture 30: Rigid Body Kinetics - Work-Energy

PURDUE UNIVERSITY | School of Mechanical Engineering

Announcements

- Exam 2, Thursday, 4/2, 8:00 – 9:30pm
  - Topics covered: Lectures 11-26 (Moving reference frames – Angular impulse momentum)
  - Location: UC 114
  - DRC students – expect an email from Dr. Krousgrill
  - Let me know of any conflicts ASAP!!!
- Review sessions:
  - Pi Tau Sigma: Tuesday, March 31, 6:30-7:30 PM, WTHR 104
  - Wednesday, April 1, 7:00 PM over Zoom
  - Recordings of both sessions will be posted



How to study for your exam:

- Review HW problems
- Review in class examples/additional examples on course blog
- Review conceptual problems (solutions on blog)
- Weeklyjoys past exams
- Take a practice test like you would take the exam!
  - Alone
  - Only using the equation sheet
  - Timed

How is the Work-Energy solution method adapted for rigid bodies?

For a system of particles:

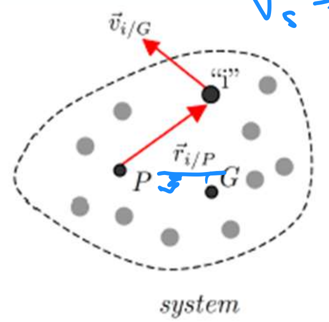
$$T_2 + V_2 = T_1 + V_1 + U_{1\to 2}^{nc}$$

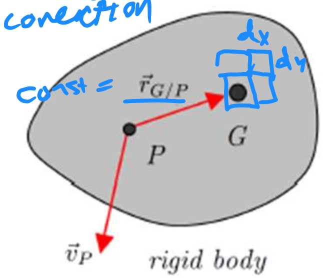
← same for rigid bodies

$V_g \to$ at G

$V_s \to$ use ΔL_{spring} at the point of conection

- Kinetic energy of the system : $T = \frac{1}{2}\sum_i m_i v_i^2 = \sum_i m_i v_i \cdot v_i$
- Potential energy of the system: $V = V_s + V_g$
- Non-Conservative work: $U_{1\to 2}^{nc}$





For a continuous body:

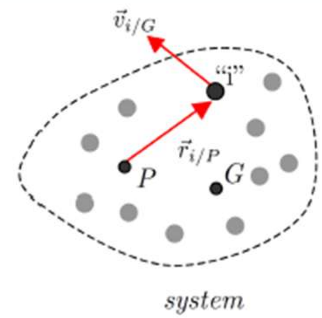
- Think of a rigid body as a continuous collection of particles:

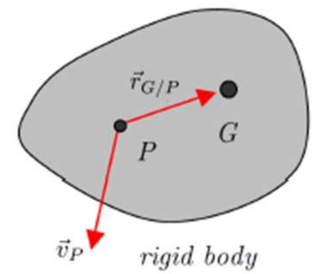
$$\sum_i (\cdot) m_i \to \int_{vol} (\cdot) dm$$

- Note there is no relative motion in a rigid body →use rigid body eqns!
- Calculate your values in terms of the motion of a reference point P and relative motion with respect to that point!

Kinetic Energy

$$T = \frac{1}{2}\sum_i m_i v_i^2 = \sum_i m_i \vec{v_i} \cdot \vec{v}_i$$





For a rigid body, $\vec{v}_i = \vec{v}_P + \vec{\omega} \times \vec{r}_{i/P}$

$$\begin{aligned}\vec{v}_i \bullet \vec{v}_i &= \left(\vec{v}_P + \vec{\omega} \times \vec{r}_{i/P}\right) \bullet \left(\vec{v}_P + \vec{\omega} \times \vec{r}_{i/P}\right) \\ &= \vec{v}_P \bullet \vec{v}_P + 2\vec{v}_P \bullet \left(\vec{\omega} \times \vec{r}_{i/P}\right) + \left(\vec{\omega} \times \vec{r}_{i/P}\right) \bullet \left(\vec{\omega} \times \vec{r}_{i/P}\right) \\ &= v_P^2 + 2\vec{v}_P \bullet \left(\vec{\omega} \times \vec{r}_{i/P}\right) + \left|\vec{\omega} \times \vec{r}_{i/P}\right|^2\end{aligned}$$

$\leftarrow$ in 2D, $= \omega^2 |r_{i/P}|^2$

special cases

1) if P @ G:

$T = \frac{1}{2} m v_P^2 + \frac{1}{2} I_P \omega^2$

2) if P is fixed pt

$T = \frac{1}{2} I_P \omega^2$

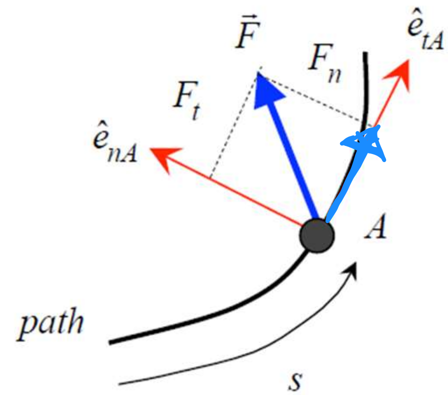
Substituting into the equation for T:

$$\begin{aligned}T &= \frac{1}{2}\sum_i m_i\, v_P^2 + \frac{1}{2}\sum_i m_i \left[2\vec{v}_P \bullet \left(\vec{\omega} \times \vec{r}_{i/P}\right)\right] + \frac{1}{2}\sum_i m_i\, \left|\vec{\omega} \times \vec{r}_{i/P}\right|^2 \\ &= \frac{1}{2}\left(\sum_i m_i\right) v_P^2 + \vec{v}_P \bullet \left(\vec{\omega} \times \sum_i m_i \vec{r}_{i/P}\right) + \frac{1}{2}\left(\sum_i m_i \left|\vec{r}_{i/P}\right|^2\right)\omega^2 \\ &= \underline{\frac{1}{2} m v_P^2 + m\vec{v}_P \bullet \left(\vec{\omega} \times \vec{r}_{G/P}\right) + \frac{1}{2} I_P \omega^2}\end{aligned}$$

$\leftarrow$ full form for kinetic energy

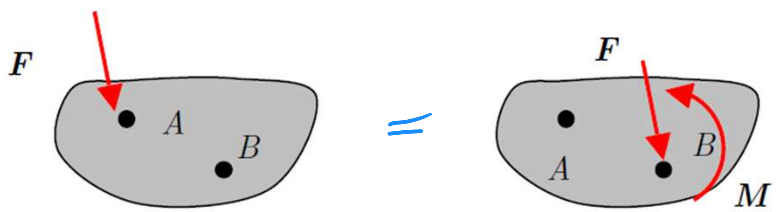
Work from non-conservative forces

Remember - work is the integral of the TANGENTIAL component of force over the path of point at which it acts:



$$U_{1\to2} = \int_1^2 \vec{F} \bullet \hat{e}_{tA}\, ds = \int_1^2 F_t\, ds$$

Often, the path of the point where the force acts is complicated, so we can simplify the path integral by using the concept of a force-couple system to effectively move the force to a more easily described point:



$M = \vec{r}_{AB} \times \vec{F}$

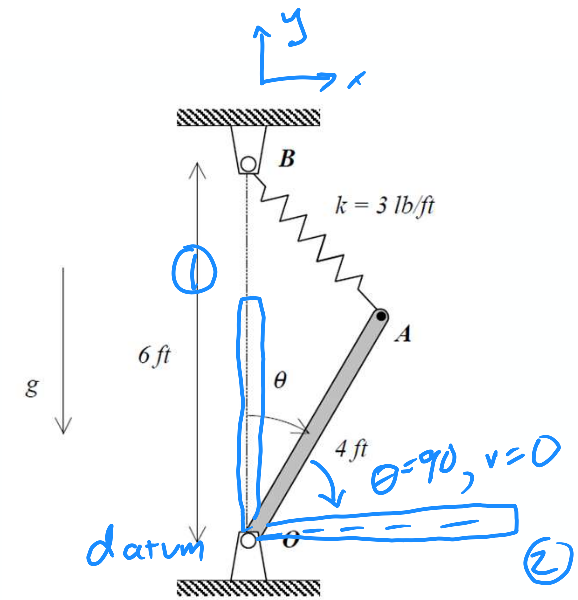
This will change out integral to include a moment integral:

$$U_{1\to2} = \int_1^2 \vec{F} \bullet \hat{e}_{tB} ds + \int_1^2 \vec{M} \bullet d\vec{\theta}$$

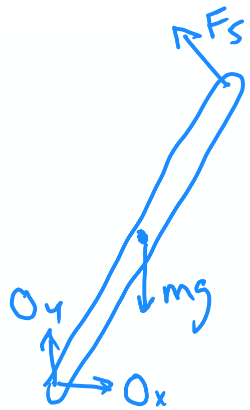
Example 5.B.3

Given: The spring acting on a thin, homogeneous bar (of weight $W = 15$ lb) is unstretched when $\theta = 0$.

Find: Determine the angular speed of the bar at $\theta = 0$, if the bar just reaches $\theta = 90°$ before coming to rest.



1) FBD



2) kinetics : $T_1 + V_1 + \cancel{U^{NC}_{1\to 2}}^{\,0} = \cancel{T_2}^{\,0} + V_2$

$T_1 = \frac{1}{2} I_O \omega_1^2$

$= \frac{1}{2}\left(I_G + m d_{OG}^2\right)\omega_1^2 = \frac{1}{2}\left(\frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2\right)\omega_1^2 = \frac{1}{6} mL^2\omega_1^2$

$V_1 = mg\frac{L}{2}$

$V_2 = \frac{1}{2} k \Delta^2 = \frac{1}{2} k\left(\sqrt{6^2 + L^2} - (6-L)\right)^2$

$T_2 = 0$

$\rightarrow$ solve for ω_1

Exam 2 Review: A quick guide to decision making

Question: What are you given and what are you being asked to find?

Given	Find	Method
Force/acceleration	Force/acceleration	Newton
Change in position	Velocity	Work – Energy
Change in time & external force	Velocity	Linear Impulse Momentum
Impact	Velocity	LIM + restitution
Change in time & non-central force	Angular velocity	Angular Impulse Momentum
Central force	Angular velocity or v_θ	Angular Impulse Momentum
Central force/rotation	Total velocity (v_r & v_θ)	AIM + WE
Position (height, spring ΔL) + moving system / multiple bodies	Velocity of multiple bodies	LIM + WE

Exam 2 Review: A quick guide to decision making

Question: how do I draw my FBD?

Method	System
Newton	Small – individual particles
Work – Energy	Big - Normal forces and tension generally do no work, friction forces are external
Linear Impulse Momentum	Big - internal forces cancel
LIM + restitution	Big - reaction forces are internal
Angular Impulse Momentum	Big, only look at externally applied forces

Reminders:

- Define your coordinate axes, use right hand rule if needed
- Only include external forces acting on your system in your FBD
- Reaction forces often do no work or moment, but must still be included in FBD

Exam 2 Review: A quick guide to decision making

Question: What is being conserved?

Method	What is conserved?	When?
Newton	nothing	-
Work – Energy	Mechanical Energy	If there is no nonconservative work – (no friction, no externally applied NC forces) $T_1 + V_1 = T_2 + V_2$
Linear Impulse Momentum	Linear momentum	No external impulse in that direction – remember you can have a conservation of momentum in x but not in y) $\sum_i m_i v_{i,1x} = \sum_i m_i v_{i,2x}$
LIM + restitution	Linear momentum (along line of impact)	Tangential components unchanged; restitution applies in normal direction $e = -\frac{v_{B2N} - v_{A2N}}{v_{B1N} - v_{A1N}}$
Angular Impulse Momentum	Angular momentum	No external moment about a point $H_{O1} = H_{O2}$