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ME 274 Lecture 35

Vibrations: Equations Of Motion (EOMs)

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04/15/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 34 (5.O and 5.P) due Today!!**

2. Office hours are on ME2008. Located on second floor of renovated side of ME.

Chapter 6: Vibrations

$$\boxed{\begin{aligned} \sum \vec{F} &= m\vec{a}_G \\ \sum \vec{M}_A &= I_A \vec{\alpha} \end{aligned}}$$

$$\boxed{M\ddot{x} + C\dot{x} + Kx = f(t)}$$

Goals for Chapter 6:

- *We are used* to solving acceleration/velocity/position for a given **instant in time** (using Newton-Euler Equations, upper left).

- In this chapter, we will study the response of systems whose motion is *oscillatory in nature* (~vibrations).

- We will focus on understanding the change in position as a **general function of time** (ie. **Deriving the EOMs of a system**, upper right).

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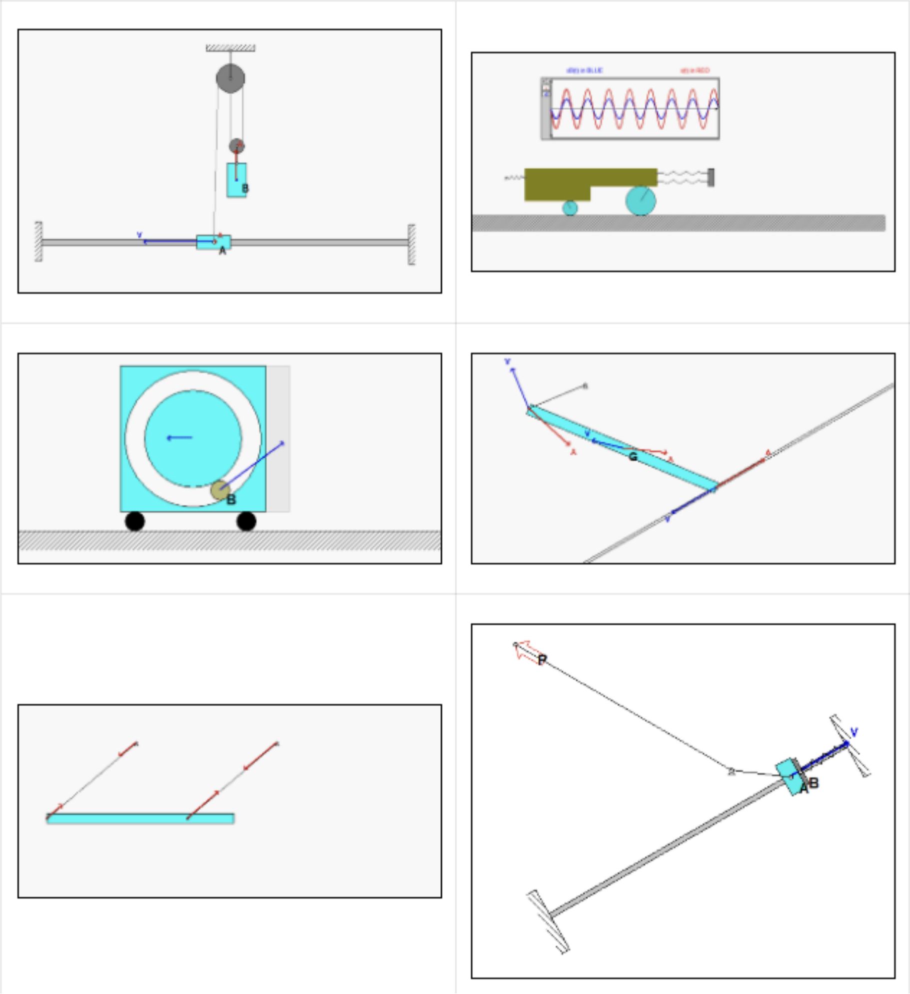
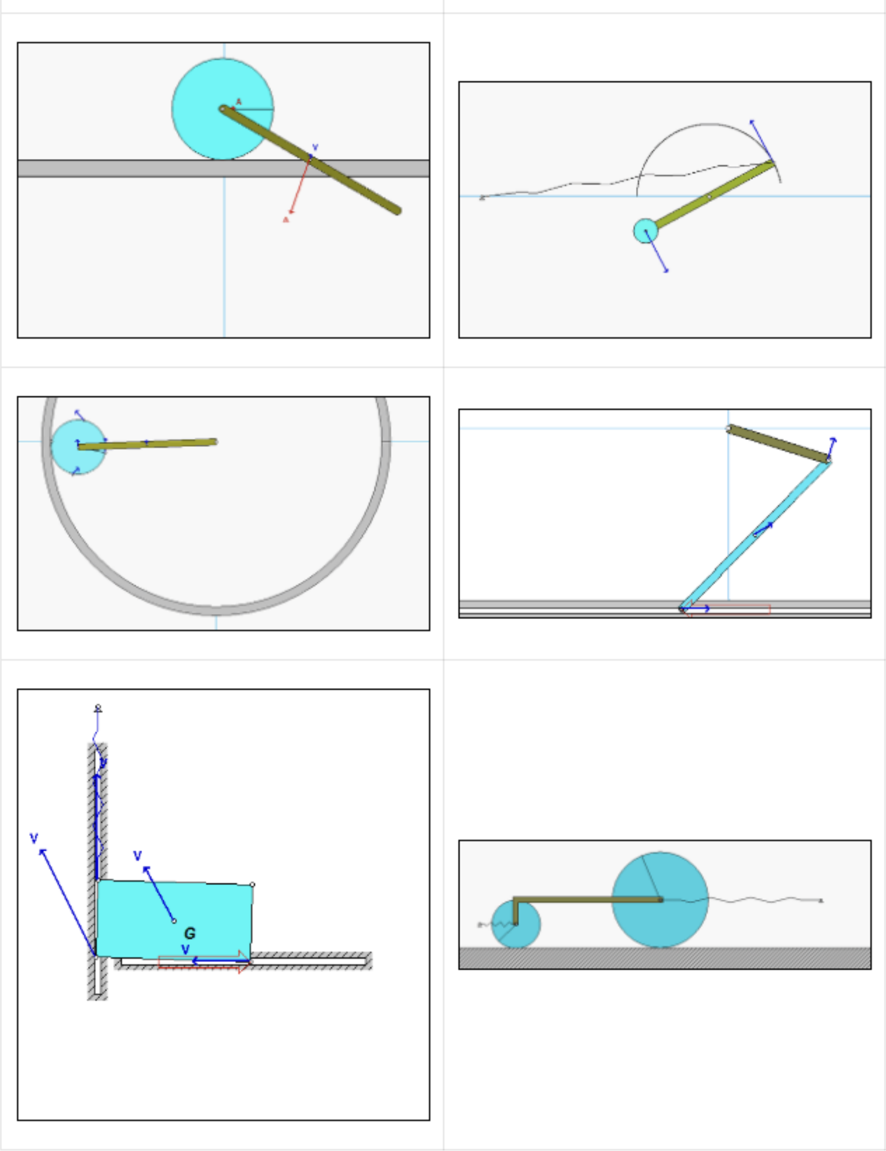
All of the problems below, we have already done, and are vibrations problems!

We were just evaluating them at an instant or discrete time intervals





Derivation of Equations of Motion (EOMs)

An equation that represents the change in position as a function of time of a given system is known as that systems equation of motion.

The equations of motion for the vibrational systems studied in this course are typically composed of the following components:

1. Particles / Rigid Bodies
2. Dashpots (Damper)
3. Springs
4. External Forcing

$$\boxed{M\ddot{x}} + \boxed{C\dot{x}} + \boxed{Kx} = \boxed{f(t)}$$

Additional discussion is needed related to types of forces that are important in the study of vibrations:

1. Spring forces
2. Dashpot/Damper forces

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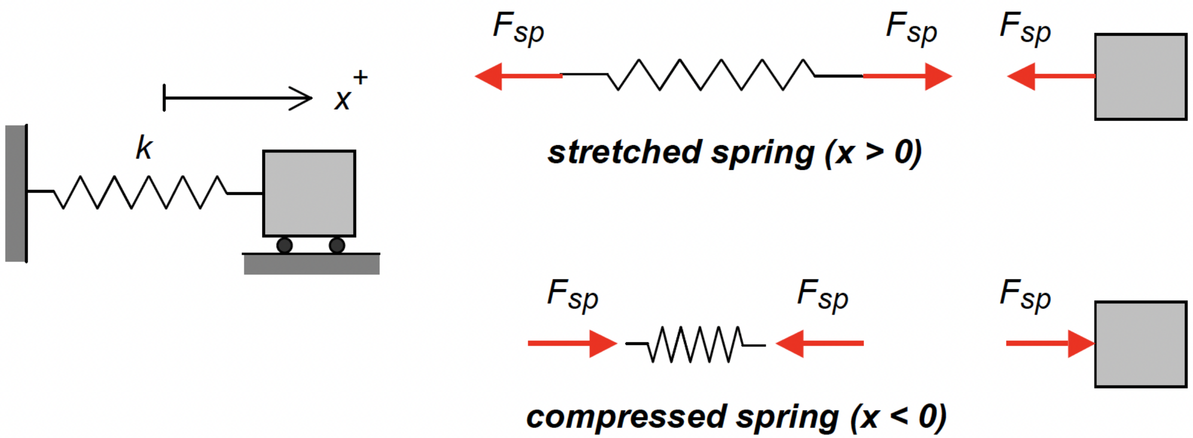
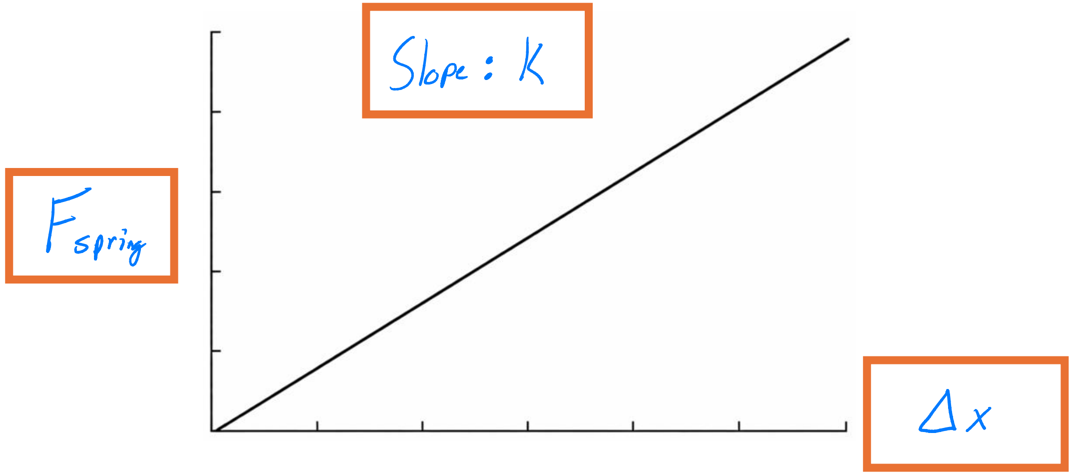
Linear Springs and Dampers

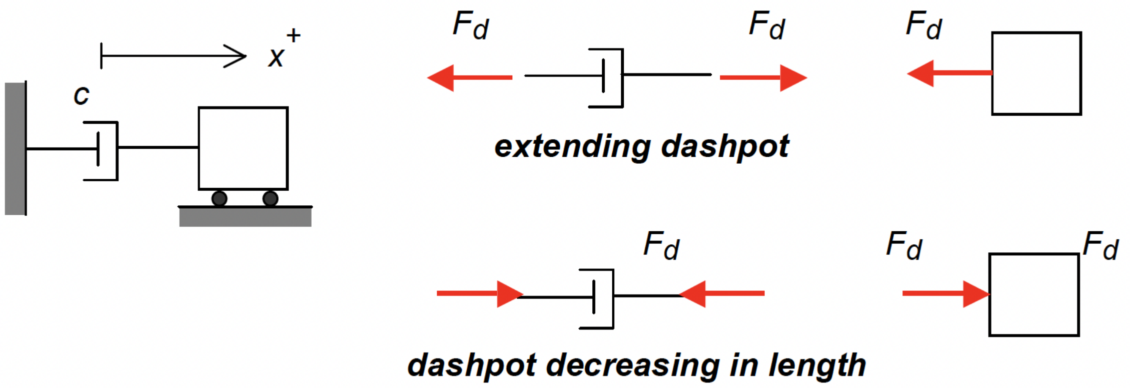
[pg. 379-380]

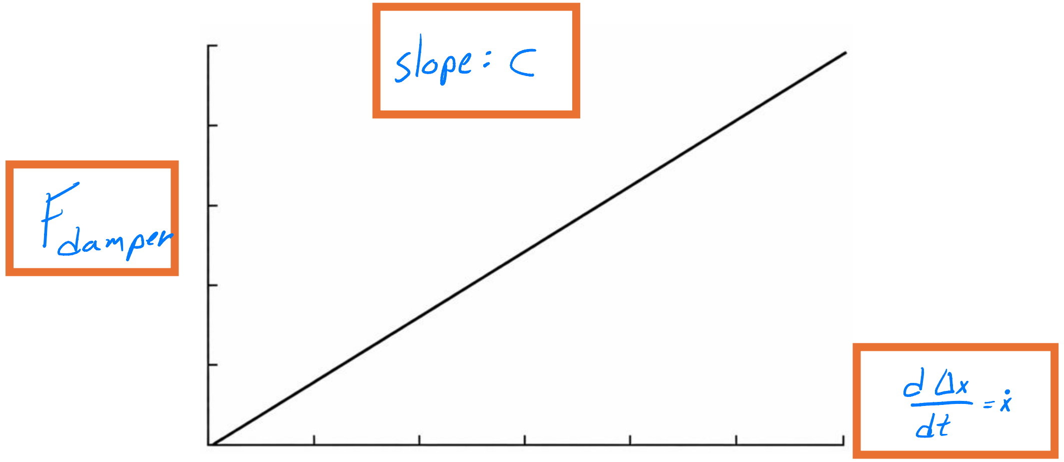
Consider a **spring** of stiffness, k:
- When it is stretched, it **pulls** on the mass to the left.
- When it compressed, it **pushes** on the mass to the right





Consider a **damper** with a damping constant, c:
- When it extends, it **pulls** on the mass to the left.
- When it shortens, it **pushes** on the mass to the right.





Steps to deriving an EOM of a given problem

The process of deriving the EOM for a given problem is summarized by the following steps (these steps are very similar to those used throughout our study of kinetics in this course…):

1. Define a set of coordinates that you will use to describe the motion of the system.
    - Draw an FBD of each body in the system.
    - For spring and dashpot forces, determine the directions of these forces.

2. Based on your FBDs in step 2, use the Newton-Euler Equations to write down the dynamical equations (ie sum of forces in x,y and sum of moments) for these bodies.

3. Write down the needed kinematic equations needed to relate the coordinates used.

4. Use the results of steps 2 and 3 above to reduce the set of equations to a single, differential equation of motion.

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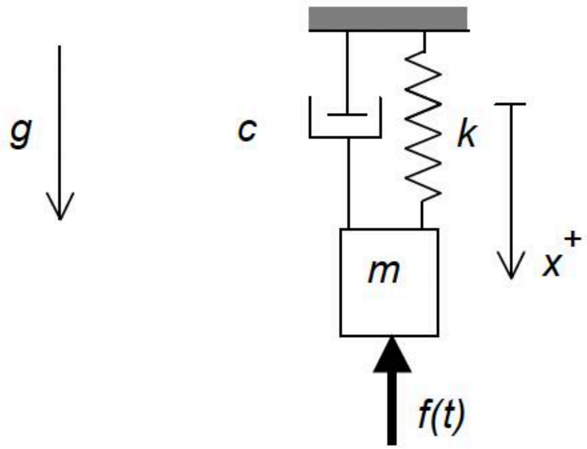
Example 6.A.1

Given: A particle of mass m is supported by a spring of stiffness k and a dashpot with damping constant c. A vertical force $f(t)$ acts on the particle as shown. Let x describe the position of the particle, where x is measured from the position of the particle when the spring is unstretched.

Find: Determine the EOM of this system in terms of the coordinate x.



[pg. 382]

Example 6.A.1 p. 382

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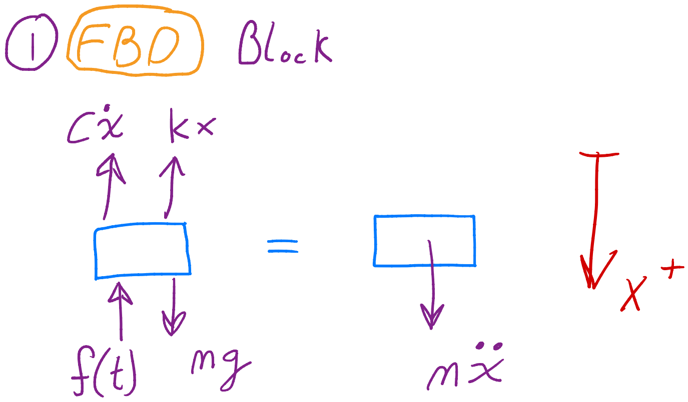
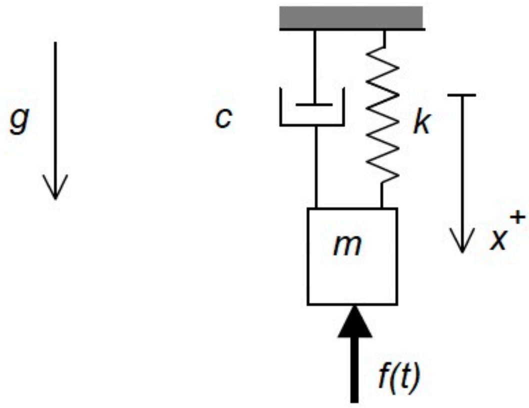
Find: Determine the EOM of this system in terms of the coordinate x.

1 FBD Block





2 Kinetics N/E

$$+\downarrow \sum F_x = m\ddot{x}$$

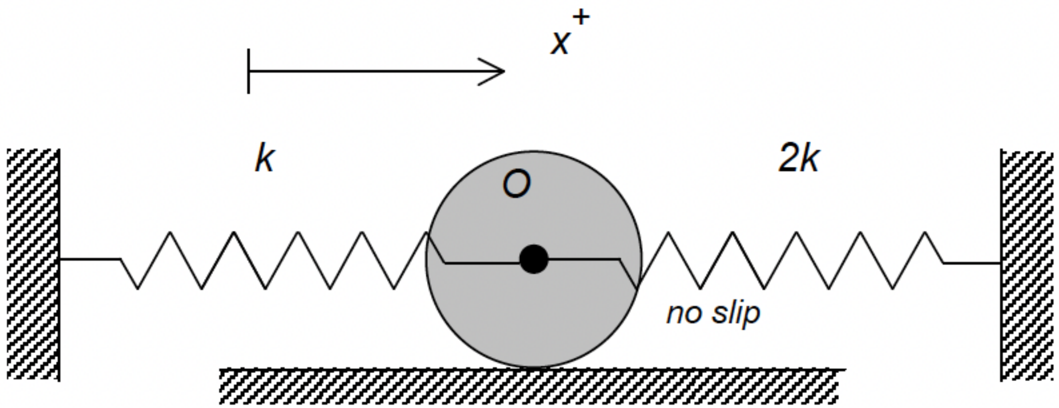
$$\Rightarrow -c\dot{x} - kx - f(t) + mg = m\ddot{x}$$

$$\Rightarrow m\ddot{x} + c\dot{x} + kx = mg - f(t) \quad \text{(EOM)}$$

Example 6.A.2

Given: A homogeneous disk of mass m and radius r rolls without slipping on a rough horizontal surface. Two springs, having stiffnesses of k and $2k$, are attached between the disk center O and ground, as shown below. Let x describe the position of O, where the springs are unstretched when $x = 0$.

Find: Determine the EOM for the disk in terms of the coordinate x.



[pg. 383]

Example 6.A.2 p. 383

Given: A homogeneous disk of mass m and radius r rolls without slipping on a rough horizontal surface. Two springs, having stiffnesses of k and $2k$, are attached between the disk center O and ground, as shown below. Let x describe the position of O, where the springs are unstretched when $x = 0$.

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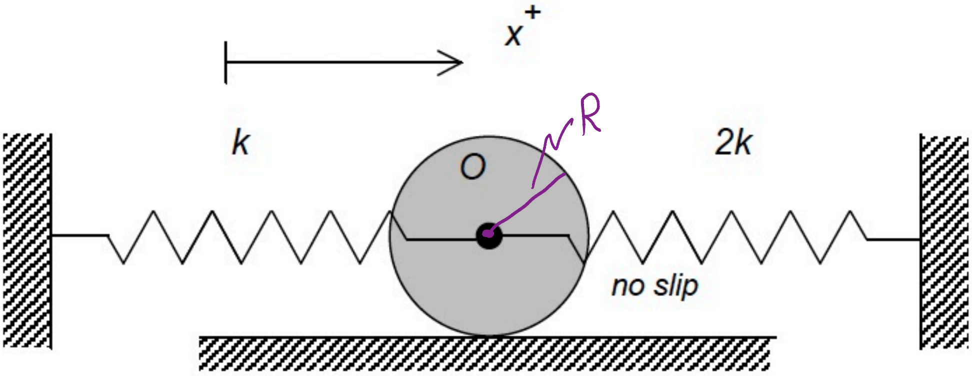
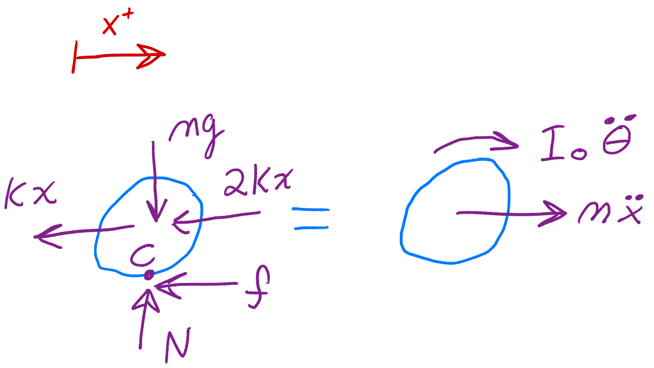


① FBD



friction to right or Left?
For no slip, f is a reaction force.
(ie you do not need to know the direction)

② Kinetics N/E

$\circlearrowleft + \sum M_C = I_C\ddot{\theta}$

P.A.T.

$$\Rightarrow -kx\cancel{R} - 2kx\cancel{R} = \underbrace{(I_O + mR^2)}_{\text{P.A.T.}}\ddot{\theta}$$

$$= \frac{3}{2}mR^{\cancel{2}}\ddot{\theta}$$

$$\Rightarrow -3kx = \frac{3}{2}mR\ddot{\theta} \quad (1)$$

dependent variables (ie vary in time)

③ 1 eqn 2 unkns

④ Kinematics Instant Center

$C = IC \therefore \Rightarrow \dot{x} = V_o = +r\dot{\theta} \Rightarrow x = R\theta, \ \ddot{x} = R\ddot{\theta} \quad (2)$

$$(1) \Rightarrow -3kx = \frac{3}{2}m\ddot{x}$$

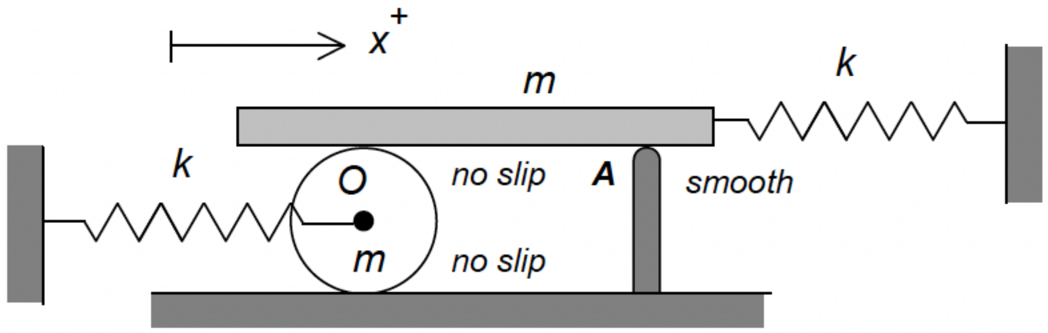
$$\Rightarrow \frac{\cancel{3}^{1}}{2}m\ddot{x} + \cancel{3}^{1}kx = 0$$

$$\Rightarrow \frac{1}{2}m\ddot{x} + kx = 0 \quad \text{(EOM)}$$

Example 6.A.3

Given: A homogeneous disk of mass of m and radius r rolls without slipping on a rough horizontal surface. A spring, having a stiffness of k, is attached between the disk center O and ground, as shown below. A block, also of mass m, is in *no-slip* contact with the top surface of the disk and with a smooth vertical support at A. A second spring of stiffness k is connected between the block and ground. Let x describe the position of O, where the springs are unstretched when $x = 0$.

Find: Determine the EOM for the disk in terms of the coordinate x.



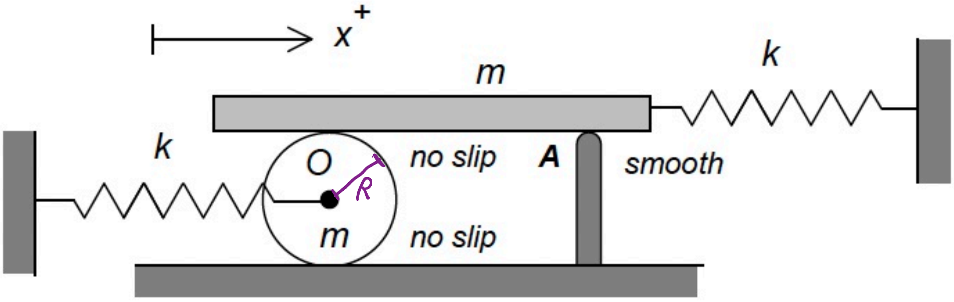
[pg. 384]

Example 6.A.3 p. 384

hw problem similar

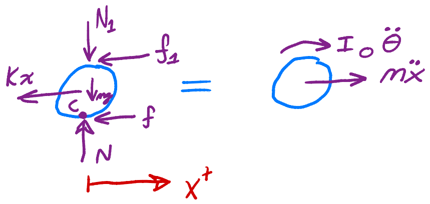
Given: A homogeneous disk of mass of m and radius r rolls without slipping on a rough horizontal surface. A spring, having a stiffness of k, is attached between the disk center O and ground, as shown below. A block, also of mass m, is in *no-slip* contact with the top surface of the disk and with a smooth vertical support at A. A second spring of stiffness k is connected between the block and ground. Let x describe the position of O, where the springs are unstretched when $x = 0$.

Find: Determine the EOM for the disk in terms of the coordinate x.

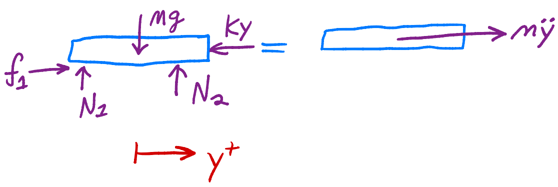


① FBD individual

disk:



Bar:



Kinetics N/E

② Disk

$\circlearrowleft + \sum M_C = I_C \ddot\theta$

$\Rightarrow -kx\cancel{R} - f_1(2\cancel{R}) = (I_O + mR^2)\ddot\theta$

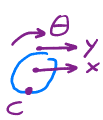
$= \frac{3}{2} mR^{\cancel{2}}\ddot\theta$

$-kx - 2f_1 = \frac{3}{2} mR\ddot\theta$ (1)

③ Bar

$\xrightarrow{+} \sum F_y = m\ddot y$

$f_1 - ky = m\ddot y$ (2)



Kinematics

④ IC's @ C

$y = 2x$ (3)

$V_O = \dot x = R\dot\theta$

$\Rightarrow \dot\theta = \frac{\dot x}{R}$; $\ddot\theta = \frac{\ddot x}{R}$ (4)

Solve

⑤ (2) & (3)

(2) $\Rightarrow f_1 = ky + m\ddot y$

$= 2kx + 2m\ddot x$

⑥ (1) & (2) & (4)

(1) $\Rightarrow -kx - 4kx - 4m\ddot x = \frac{3}{2} m\ddot x$

$\Rightarrow \frac{11}{2} m\ddot x + 5kx = 0$ (EOM)

Summary: *Vibrations 1 (EOMs)*

[pg. 381]

1. *GOAL*? To derive the differential equation of motion (EOM):

$$m\ddot{x} + c\dot{x} + kx = f\left(t\right)$$

2. *HOW*?
   Good news: You already know how to do this! The four-step plan:
   1) *FBD*; 2) *Newton/Euler*; 3) *Kinematics*; 4) *Solve* (here "solve" means combining together into a single EOM.

3. *THE CATCH*: You *EOM must describe motion for all time*, not just a single instant in time. Define your coordinate(s) at the start, and stick with them throughout. If you do, you are all set.

4. *WHAT's NEXT*? We will spend the rest of the semester solving our differential EOMs. You already know how to do that also! Woo Hoo!

Lec 35 Short Feedback Form: