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ME 274 Lecture 33

Rigid body kinetics: Impulse Momentum – Part 2

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04/10/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 32 (5.K and 5.L) due Today!!**

2. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME.

3. Quiz 06 at end of class, hand in hard copy before you leave class.

Chapter 5: Planar Rigid Body Kinetics

- LIM and AIM for Planar Rigid Bodies (lectures 32-33)
  - Where A is a **fixed point on the body or center of mass**

$$\boxed{\int_1^2 \sum \vec{F}\,dt = m\vec{v}_{G2} - m\vec{v}_{G1}}$$

$$\boxed{\int_1^2 \sum \vec{M}_A dt = \vec{H}_{A2} - \vec{H}_{A1} = I_A\vec{\omega}_2 - I_A\vec{\omega}_1}$$

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $\text{where } T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $\text{where } T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\,dt = \vec{H}_{O2} - \vec{H}_{O1}$ $\text{where } \vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\,dt = \vec{H}_{A2} - \vec{H}_{A1}$ $\text{where } \vec{H}_A = I_A\vec{\omega}$

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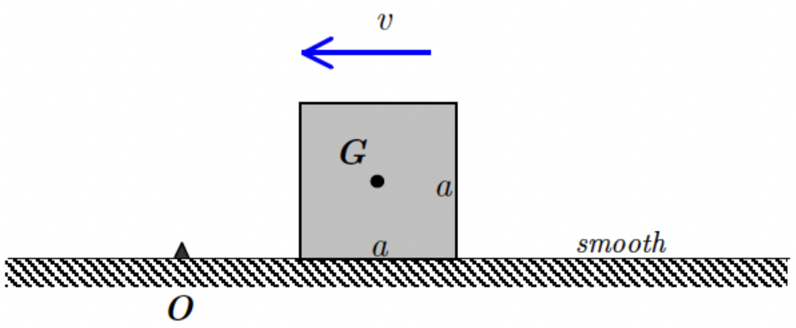
Example 5.C.6

Given: A square block of mass m strikes a small step at O while traveling to the left with a speed of v.

Find: Determine the value of v required to have the block just reach a standing position with zero velocity after impact, assuming it sticks to the step.



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Example 5.C.6

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Given: A square block of mass m strikes a small step at O while traveling to the left with a speed of v.

Find: Determine the value of v $\boxed{\text{required to}}$ have the block just reach a standing position with zero velocity after impact, assuming it sticks to the step.

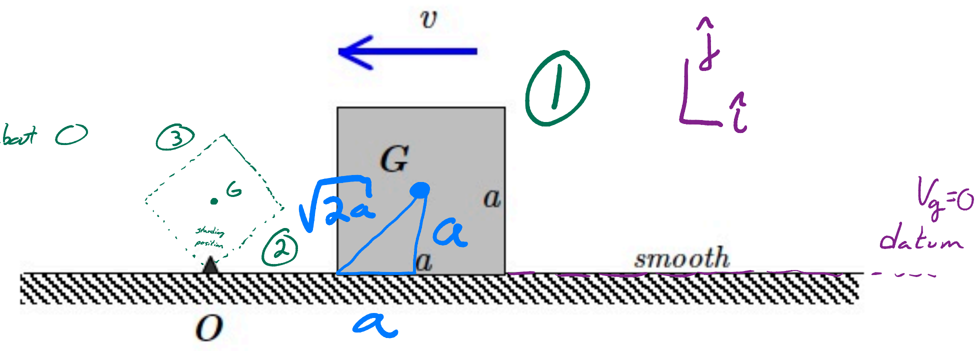
v_1 ?

① States → rotation → AIM

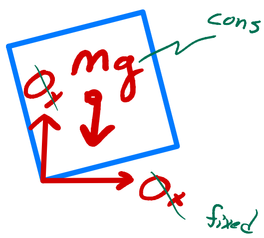
state ① before impact

state ② imm. after impact, slight rot about O

state ③ standing position



② FBD



Kinetics

③ A.I.M.

$\Sigma M_o = 0$

$\Rightarrow (\vec{H}_o)_1 = (\vec{H}_o)_2$

④ H_{o1}

$(\vec{H}_o)_1 = \vec{r}_1 \times m\vec{v}_1$

$= \left(\frac{a}{2}\hat{i} + \frac{a}{2}\hat{j}\right) \times m(-v_1\hat{i})$

$= \frac{m v_1 a}{2}\hat{k}$

⑤ H_{o2}

$(\vec{H}_o)_2 = I_o \omega_2 \hat{k}$

$= (I_G + m d_{OG}^2)\omega_2 \hat{k}$

$= \left(\frac{1}{12}m(a^2+a^2) + m2a^2\right)\omega_2 \hat{k}$ (1)

⑥ $H_{o1} = H_{o2}$; eqn w/ ω_2 & v_1

$\frac{m v_1 a}{2} = \left(\frac{1}{12}m(a^2+a^2) + m2a^2\right)\omega_2$ (1)

⑦ 2 unkns 1 eqn

⑧ W/E ② → ③

$T_2 + V_2 + U_{2\to3}^{NC} = T_3 + V_3$

$T_2 = \frac{1}{2} I_o \omega_2^2$

$V_2 = mg\frac{a}{2}$; datum

$U_{2-3}^{NC} = 0$; mech en. cons.

$T_3 = 0$

$V_3 = mg\, d_{OG}$; $d_{OG} = \sqrt{2}\, a$, datum

$\frac{1}{2} I_o \omega_2^2 + mg\frac{a}{2} + 0 = 0 + mg\, d_{OG}$ (2)

$\Rightarrow \omega_2$

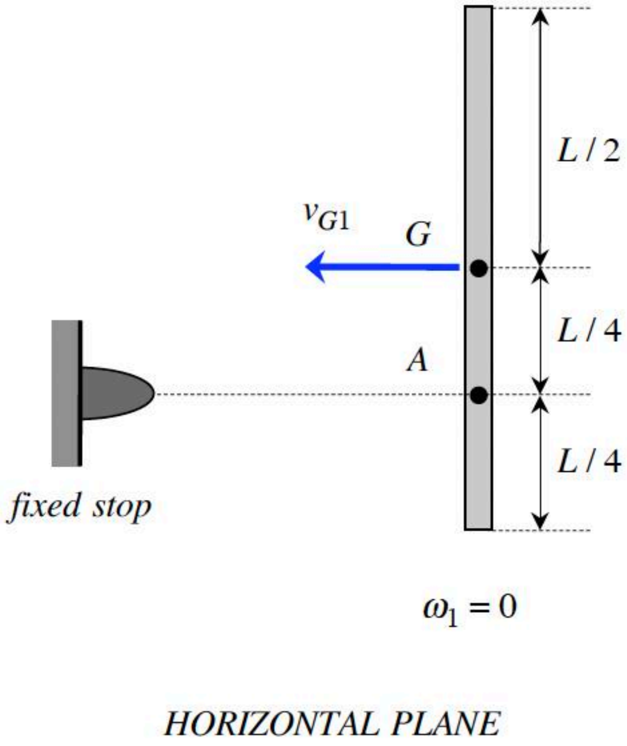
Example 5.C.7

Given: A thin homogeneous bar of mass m and length L translates along a smooth horizontal surface with a speed of v_{G1}. The bar strikes a fixed stop at location A on the bar, where the coefficient of restitution for this impact is e, where $0 < e < 1$.

Find: Determine:

(a) The speed of the center of mass G for the bar after impacting the stop; and

(b) The angular velocity of the bar after impacting the stop.



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Example 5.C.7

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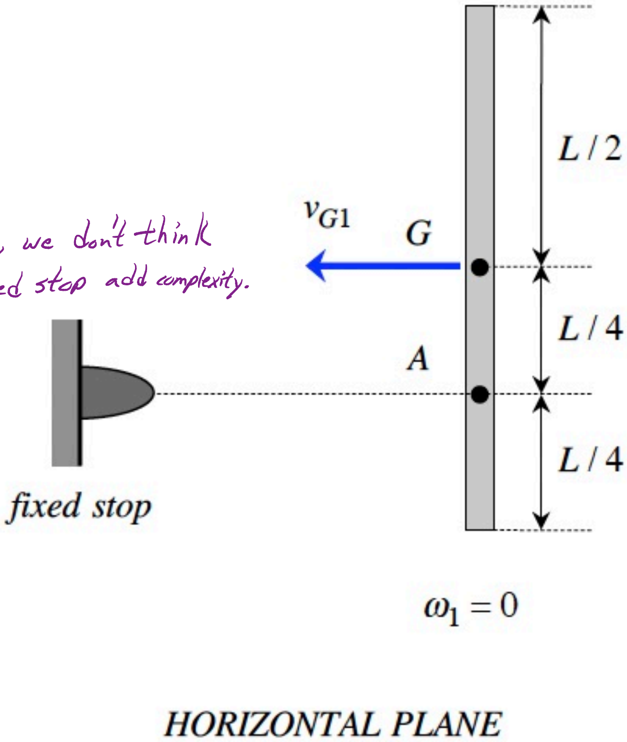
Given: A thin homogeneous bar of mass m and length L translates along a smooth horizontal surface with a speed of v_{G1}. The bar strikes a fixed stop at location A on the bar, where the coefficient of restitution for this impact is e, where $0 < e < 1$.

Find: Determine:

(a) The speed of the center of mass G for the bar after impacting the stop; and v_{G2}?

(b) The angular velocity of the bar after impacting the stop. ω_2?

① spd & ω? for rigid body } LIM/AIM



② FBD Just the bar. For this example, we don't think big in FBD bc the forces of fixed stop add complexity.

Kinetics

③ LIM x-dir

$-mv_{G1} + \int F dt = m v_{G2}$ (1)

④ AIM$_G$

$0 + \int F \frac{L}{4} dt = I_G \omega_2$

$\Rightarrow \frac{L}{4} \int F dt = \frac{1}{12} m L^2 \omega_2$ (2)

Kinematics

⑤ "impact rule" = "coef. of rest eqn"

$e = \dfrac{v_{Abx2} - \cancel{v_{Awx2}}^{\,0}}{\cancel{v_{Awx1}}_{\,0} - v_{Abx1}}$; w = wall, b = bar

doesn't move

$e = \dfrac{-v_{Abx2}}{v_{Abx1}}$

$\Rightarrow e = \dfrac{+v_{Ax2}}{+v_{G1}}$ (3)

⑥ 3 eqns 4 unkns

⑦ @ state 2

$\vec{v}_A = \vec{v}_G + \vec{\omega} \times \vec{r}_{A/G}$

$v_{Ax2}\hat{i} + v_{Ay2}\hat{j} = v_{G2x}\hat{i} + v_{G2y}\hat{j} + \omega_2 \hat{k} \times -\frac{L}{4}\hat{j}$

$= v_{G2x}\hat{i} + v_{G2y}\hat{j} + \frac{\omega_2 L}{4}\hat{i}$

$\hat{i}$: $v_{Ax2} = v_{G2} + \frac{\omega_2 L}{4}$ (4)

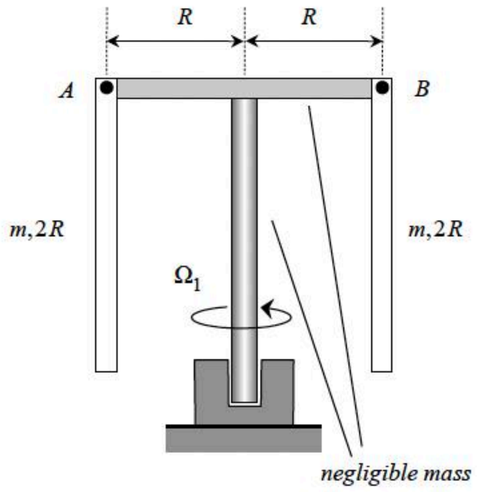
⑧ Solve for $\vec{\omega}_2$; $\vec{v}_{G2}$

Example 5.C.8

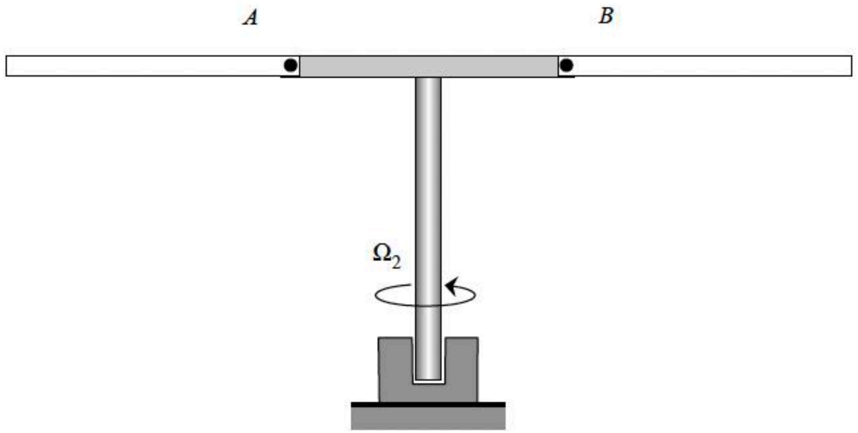
Given: Two thin, homogeneous bars, each of length $2R$ and of mass m are pinned to a frame of negligible mass at a distance of R from the fixed vertical rotation axis of the frame. Initially, the frame is rotating at a rate of Ω_1 with the two bars hanging straight down and not moving relative to the frame (Position 1). The bars are then slowly rotated up to Position 2 shown where the bars are perpendicular to the rotation axis of the frame, as shown in the figure.

Find: Determine the angular speed Ω_2 of the frame at Position 2.

Use the following parameters in your analysis: $m = 10$ kg, $\omega_1 = 8$ rad/s, $R = 0.5$ m.



Position 1



Position 2

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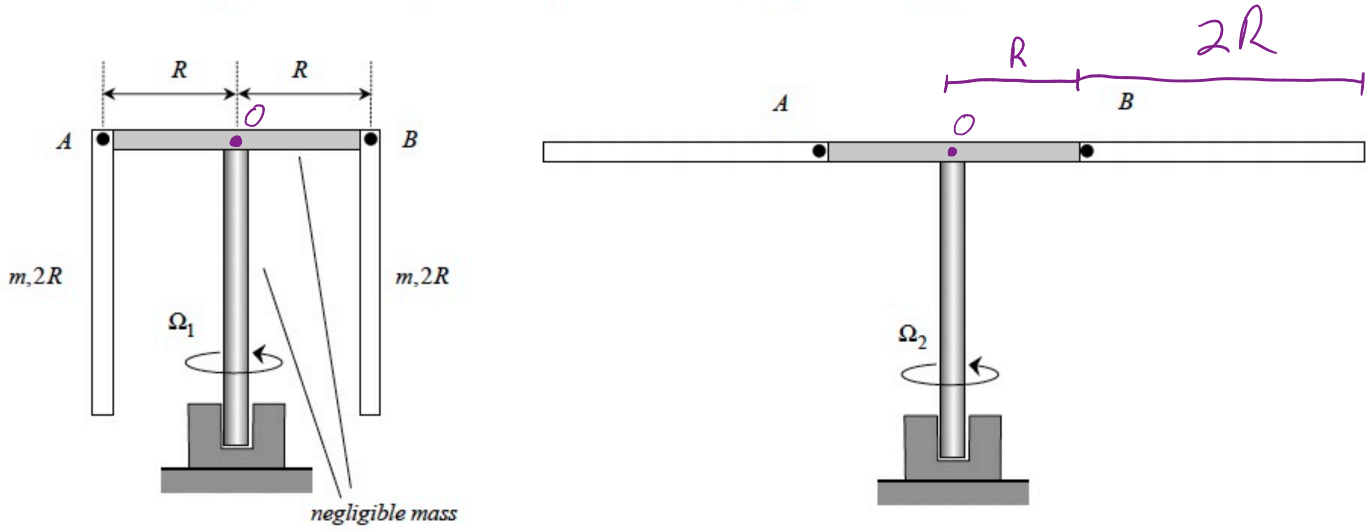
Example 5.C.8 p. 349

Given: Two thin, homogeneous bars, each of length $2R$ and of mass m are pinned to a frame of negligible mass at a distance of R from the fixed vertical rotation axis of the frame. Initially, the frame is rotating at a rate of Ω_1 with the two bars hanging straight down and not moving relative to the frame (Position 1). The bars are then slowly rotated up to Position 2 shown where the bars are perpendicular to the rotation axis of the frame, as shown in the figure.

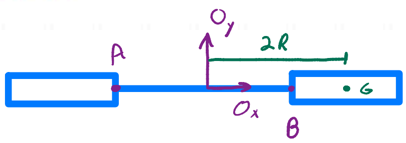
Find: Determine the angular speed Ω_2 of the frame at Position 2. Ω_2?

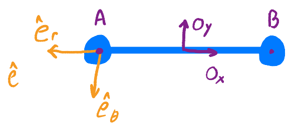
Use the following parameters in your analysis: $m = 10$ kg, $\omega_1 = 8$ rad/s, $R = 0.5$ m.



Position 1 *Position 2*

① FBD





Top View of position 1

Top View of position 2

⑤ Solve $H_{o1} = H_{o2}$

$$2mR^2\Omega_1 = 2\left(\tfrac{4}{3}mR^2\right)\Omega_2$$

$$\hookrightarrow \Omega_2 = \tfrac{3}{4}\Omega_1$$

Kinetics

② AIM

$\Omega_1 = \omega_1$
$\Omega_2 = \omega_2$

$$\Sigma \vec{M}_o = \vec{0}$$

$$(\vec{H}_o)_1 = (\vec{H}_o)_2$$

③ H_{o1}

$$(\vec{H}_o)_1 = \overset{A\,\&\,B}{2}\left(m\,\vec{r}_{A/o} \times \vec{V}_A\right)$$

$$= 2m\left(R\hat{e}_r\right) \times \left(\dot{R}\hat{e}_r + R\Omega_1\hat{e}_\theta\right)$$

$$= 2mR^2\Omega_1\,\hat{k}$$

④ H_{o2}

$$(\vec{H}_o)_2 = \overset{A+B}{2}\left(I_o\,\Omega_2\,\hat{k}\right) \;;\; I_o = I_G + mR^2$$

$$= \tfrac{1}{12}m(2R)^2 + mR^2$$

$$= \tfrac{4}{3}mR^2$$

Ice Skating Example:

① arms extended ⇒ I ↑ ⇒ ω ↓

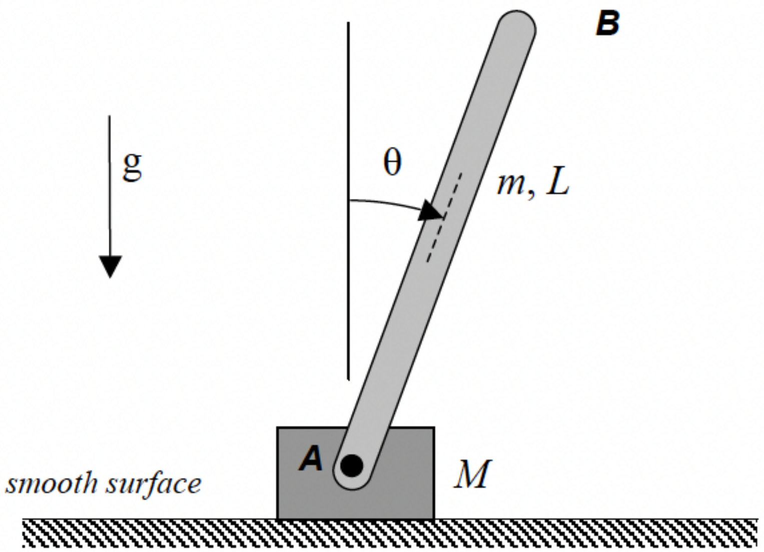
② arms in ⇒ I ↓ ⇒ ω ↑

Example 5.C.9

Given: A rod of mass m and length L is pinned to a block of mass M. The block can slide on a frictionless horizontal surface. The rod is released from rest when $\theta = 30°$.

Find: Determine:
(a) The angular velocity of the rod when $\theta = 90°$; and
(b) The velocity of the block when $\theta = 90°$.

Consider using both the work-energy and linear impulse-momentum equations in your solution. Use the following parameters in your analysis: $m = 10$ kg, $M = 30$ kg and $L = 2$ m.



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[pg. ~~XXX~~]

Example 5.C.9 p. 350 Similar to hw due Mon.

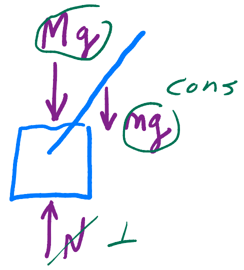
Given: A rod of mass m and length L is pinned to a block of mass M. The block can slide on a frictionless horizontal surface. The rod is released from rest when $\theta = 30°$.
RFR

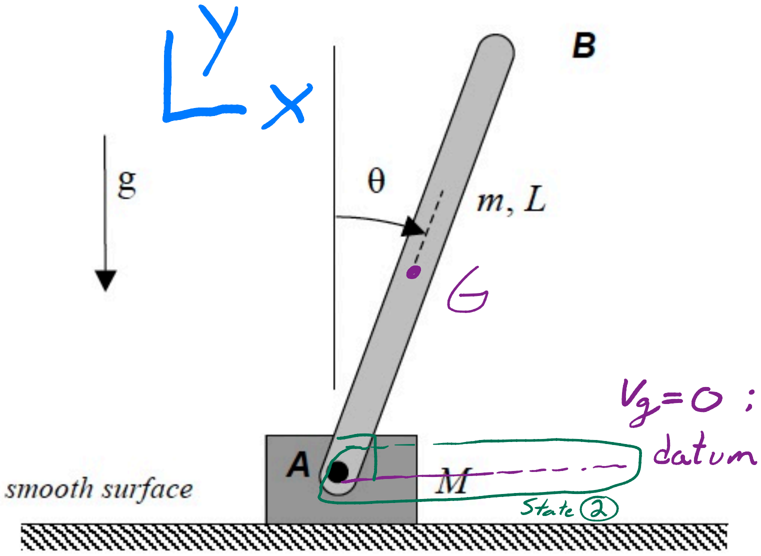
Find: Determine:
(a) The angular velocity of the rod when $\theta = 90°$; and ω_2?
(b) The velocity of the block when $\theta = 90°$. V_{A2}?

Consider using both the work-energy and linear impulse-momentum equations in your solution. Use the following parameters in your analysis: $m = 10$ kg, $M = 30$ kg and $L = 2$ m.

(1) ω_2 & v_2? Δ in position. $\{$ W/E, IM

(2) FBD AB + Block





Kinetics

(3) LIM x-dir

$\cancel{M V_{A1}}^{RFR} + \cancel{m V_{G1x}}^{RFR} = M V_{A2} + m V_{G2x}$

$0 = M v_{A_2} + m v_{Gx_2} \quad (1)$

(4) W/E

$T_1 + V_1 + U^{NC}_{1\to 2} = T_2 + V_2$

$T_1 = 0$; RFR

$V_1 = mg\frac{L}{2}\cos\theta$; datum

$U^{NC}_{1-2} = 0$: No non-cons forces

$T_2 = \underbrace{\frac{1}{2}M v_{A2}^2}_{block} + \underbrace{\frac{1}{2}m v_{G2}^2 + \frac{1}{2}I_G\omega_2^2}_{bar}$

$V_2 = 0$; datum

$0 + mg\frac{L}{2}\cos\theta + 0 = \frac{1}{2}M v_{A2}^2 + \frac{1}{2}m v_{G2}^2 + \frac{1}{2}I_G\omega_2^2 + 0$

$mg\frac{L}{2}\cos\theta = \frac{1}{2}M v_{A2}^2 + \frac{1}{2}m v_{G2}^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2 \quad (2)$

(5) 2 eqns 4 unkns

Kinematics

(6) AG @ state (2)

$\vec{v}_{G_2} = \vec{v}_{A2} + \vec{\omega} \times \vec{r}_{G/A}$

$\Rightarrow V_{G2x}\hat{\imath} + v_{G2y}\hat{\jmath} = v_{A2}\hat{\imath} + \omega\hat{k} \times \frac{L}{2}\hat{\imath}$

$= v_{A2}\hat{\imath} + \frac{\omega L}{2}\hat{\jmath}$

$\hat{\imath}: v_{G2x} = v_{A2} \quad (3)$

$\hat{\jmath}: v_{G2y} = \frac{\omega L}{2} \quad (4)$

(7) 4 eqns 5 unkns. Use $|v_g|$.

$v_{G2}^2 = v_{G2x}^2 + v_{G2y}^2 \quad (5)$

(8) Solve (1)–(5). ω_2 & V_{A2}

Summary: Impulse/Momentum Equations 2

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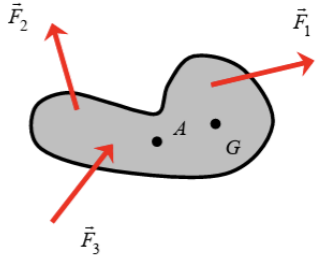
FUNDAMENTAL equations (for A = cm *OR* fixed point):

$$m\vec{v}_{G2} = m\vec{v}_{G1} + \int_1^2 \vec{F}\,dt$$

$$\vec{H}_{A2} = \vec{H}_{A1} + \int_1^2 \vec{M}_A\,dt$$

$$\boxed{\begin{aligned}\vec{H}_A &= I_A\vec{\omega} &&;\ \textit{for a RIGID BODY}\\ &= m\vec{r}_{P/A}\times\vec{v}_P &&;\ \textit{for a particle P}\end{aligned}}$$



Lec 33 Short Feedback Form:

SOLUTION METHOD: the four-step plan

Kinetics: Four-Step Problem Solving Method

The suggested plan of action for solving kinetics problems:

1. ***Free body diagram(s)***. Draw appropriate free body diagrams (FBDs) for the problem. Your choice of FBDs is problem dependent. For some problems, you will draw an FBD for each body; for others, you will draw an FBD for the entire system. An integral part of your FBDs is your choice of coordinate systems. For each FBD, draw the unit vectors corresponding to your coordinate choice.

2. ***Kinetics equations***. At this point, you will need to choose what solution method(s) that you will need to use for the particular problem at hand. In this section of the course we will study four basic methods: Newton/Euler, work/energy, linear impulse/momentum and angular impulse/momentum. Based on your choice of method(s), write down the appropriate equations from your FBD(s) from Step 1.

3. ***Kinematics***. Perform the needed kinematic analysis. A study of the equations in Step 2 above will guide you in deciding what kinematics are needed to find a solution to the problem.

4. ***Solve***. Count the number of unknowns and the number of equations from above. If you do not have enough equations to solve for your unknowns, then you either: (i) need to draw more FBDs, OR (ii) need to do more kinematic analysis. When you have sufficient equations for the number of unknowns, solve for the desired unknowns from the above equations.

Draw free-body diagram of entire system for impulse momentum.

Be sure to use the correct mass moment of inertia for your choice of point "A". Use PAT if necessary.

Typically the most difficult step. Recall the rigid body kinematics from Chapter 2.

If you are short equations, go back to Step 3 – Kinematics.