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ME 274 Lecture 32

Rigid body kinetics: Impulse Momentum – Part 1

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04/08/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 31 (5.I and 5.J) due Today!!**

2. Office hours are changing to ME2008B…
   - Second floor of renovated side of ME.

Chapter 5: Planar Rigid Body Kinetics

- LIM and AIM for Planar Rigid Bodies (lectures 32-33)

$$\int_1^2 \sum F\,dt = m\vec{v}_{G2} - m\vec{v}_{G1}$$

$$\int_1^2 \sum \vec{M}_A\,dt = \vec{H}_{A2} - \vec{H}_{A1}$$
$$= I_A\vec{\omega}_2 - I\vec{\omega}_1$$

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A}\times\vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A\cdot(\vec{\omega}\times\vec{r}_{G/A})$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2}\sum \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2}\sum \vec{F}\,dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2}\sum \vec{M}_O\,dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O}\times\vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2}\sum \vec{M}_A\,dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A\vec{\omega}$

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Derivation of Rigid Body Impulse-Momentum Equations

- We start with the Newton-Euler Equations, where G is center of mass of the body and A is either a fixed point/center of mass:

$$\sum \vec{F} = m\vec{a}_G$$

$$\sum \vec{M}_A = \frac{d}{dt}\vec{H}_A = \boxed{I_A \cdot \vec{\alpha}}$$

$$\boxed{\vec{H}_A = I_A \vec{\omega}}$$

- If we integrate the newton-euler equations this gives us the LIM and AIM equations for a rigid body.
  - Where A is a **fixed point** on the body or Center of Mass

$$\int_1^2 \sum \vec{F}\, dt = m\vec{v}_{G2} - m\vec{v}_{G1}$$

$$\int_1^2 \sum \vec{M}_A dt = \vec{H}_{A2} - \vec{H}_{A1} = I_A\vec{\omega}_2 - I_A\vec{\omega}_1$$

[pg. 340-341 content]

- *In this course we will not cover cases where the fixed point is* ***not on the rigid body.***

Example 5.C.1

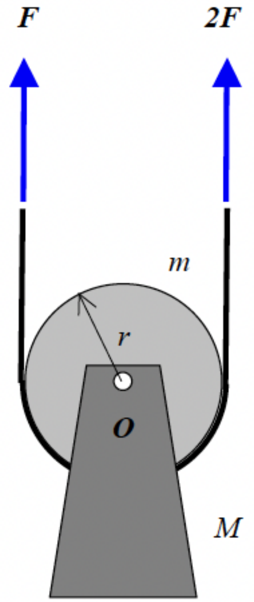
Given: Constant forces of F and $2F$ are applied at the free ends of the hoisting cable as shown. The velocity of O is 4 m/s down and the angular velocity ω_1 of the drum (having a centroidal radius of gyration of $k_O = 0.4$ m) is counterclockwise at time $t = 0$.

Find: Determine, after 10 s:

(a) The velocity of O; and

(b) The angular velocity of the drum.

Use the following parameters in your analysis: $\omega_1 = 2$ rad/s, $r = 0.4$ m, $m = 20$ kg, $M = 30$ kg and $F = 100$ N.



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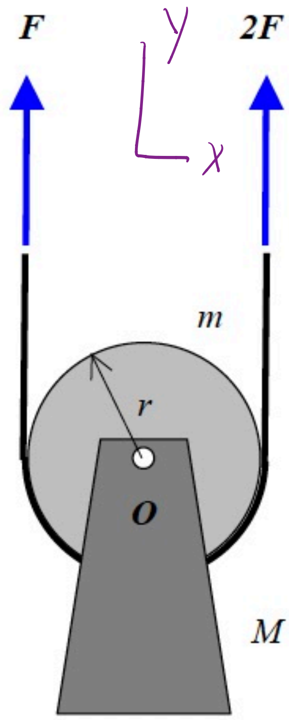
Example 5.C.1 p.342

Given: Constant forces of F and $2F$ are applied at the free ends of the hoisting cable as shown. The velocity of O is 4 m/s down and the angular velocity ω_1 of the drum (having a centroidal radius of gyration of $k_O = 0.4$ m) is counterclockwise at time $t = 0$.

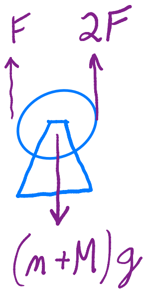
Find: Determine, after 10 s:
(a) The velocity of O; and $\vec{V}_{o2}$?
(b) The angular velocity of the drum. $\vec{\omega}_2$?

Use the following parameters in your analysis: $\omega_1 = 2$ rad/s, $r = 0.4$ m, $m = 20$ kg, $M = 30$ kg and $F = 100$ N.



(1) Velocity & Ang vel. in 2 states → L.I.M. & A.I.M.

(2) FBD



Kinetics

(3) $\overset{+}{\uparrow}$ LIM

$$-(m+M)v_o + \int (3F-(m+M)g)\,dt = (m+M)v_{o2}$$

$$\Rightarrow -(m+M)v_o + [3F-(m+M)g]\Delta t = (m+M)v_{o2}$$

$\Delta t = 10$ secs

$\Rightarrow v_{o2}$

(a) $\Rightarrow \vec{v}_{o2} = v_{o2}\hat{j}$

(4) AIM $\circlearrowleft +$

$$I_O\omega_1 + \int (2Fr - Fr)\,dt = I_O\omega_2$$

$$\Rightarrow m k_o^2 \omega_1 + Fr\Delta t = m k_o^2 \omega_2$$

$\Rightarrow \omega_2$

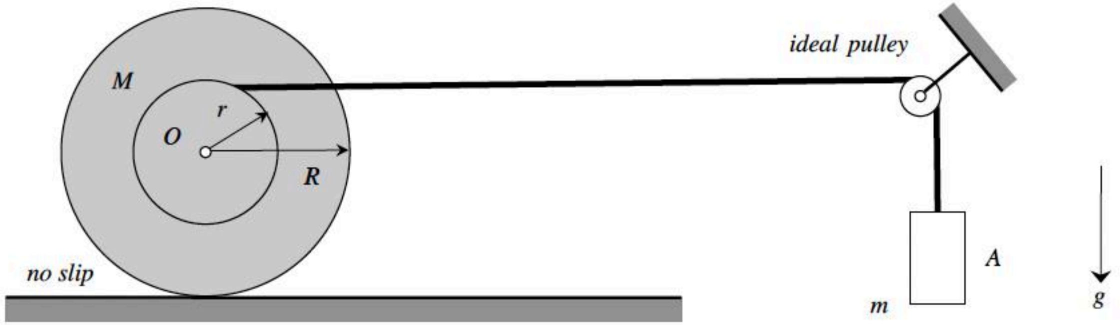
(b) $\Rightarrow \vec{\omega}_2 = \omega_2\hat{k}$

Example 5.C.2

Given: A spool (having a mass of M and radius of gyration about its center O of k_O) rolls without slipping on a rough, horizontal surface. A cable is wrapped around the inner radius of the spool. This cable is pulled over an ideal pulley and is attached to block A. Assume that the cable does not slip on the drum. The system is released from rest.

Find: Find the angular velocity of the spool after 10 s.

Use the following parameters in your analysis: m = 10 kg, M = 20 kg, r = 0.2 m, R = 0.4 m, k_O = 0.25 m.



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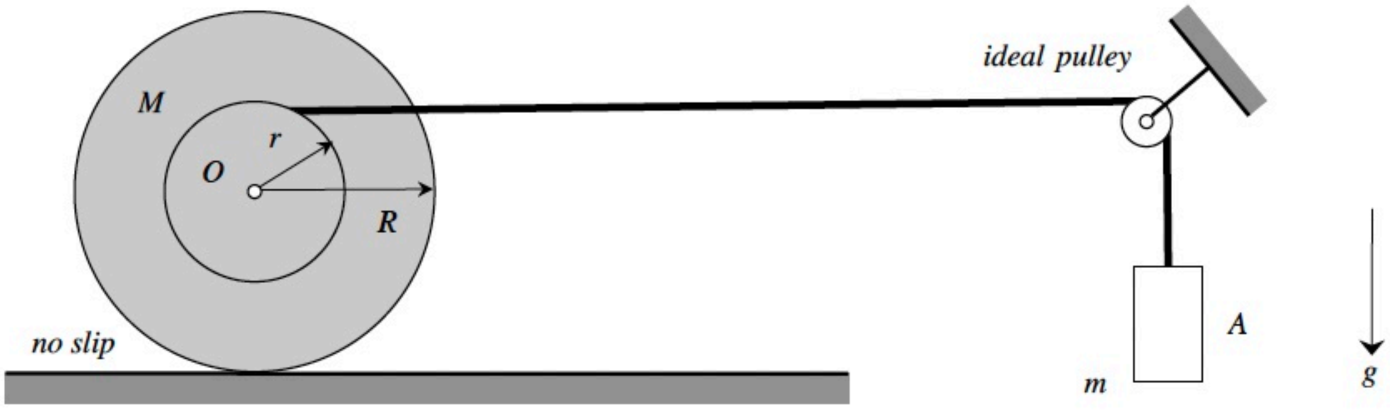
Example 5.C.2 p.343

Given: A spool (having a mass of M and radius of gyration about its center O of k_O) rolls without slipping on a rough, horizontal surface. A cable is wrapped around the inner radius of the spool. This cable is pulled over an ideal pulley and is attached to block A. Assume that the cable does not slip on the drum. The system is released from rest.

RFR

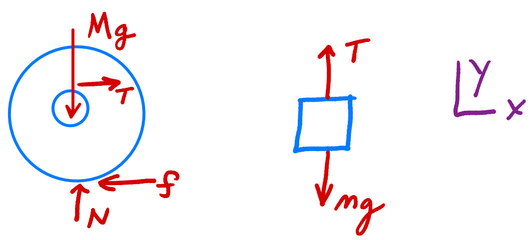
Find: Find the angular velocity of the spool after 10 s. ω_2?

Use the following parameters in your analysis: m = 10 kg, M = 20 kg, r = 0.2 m, R = 0.4 m, k_O = 0.25 m.



1 ω? → AIM.

2 FBD



Kinetics

3 LIM y-dir

$$0+\int(T-mg)dt = -mv_2$$

$$\Rightarrow (T-mg)\Delta t = -mv_2 \quad (1)$$

4 LIM x-dir

$$0+\int(T-f)dt = Mv_{O2}$$

$$\Rightarrow (T-f)\Delta t = Mv_{O2} \quad (2)$$

5 AIM +↺

$$0\int(-Tr-fR)\,dt = Mk_O^2\omega_2$$

$$\Rightarrow Tr\Delta t - fR\Delta t = Mk_O^2\omega_2 \quad (3)$$

6 5 unkns 3 eqns

7 Kinematics

$$v_{O2} = -R\omega_2 \quad (4)$$

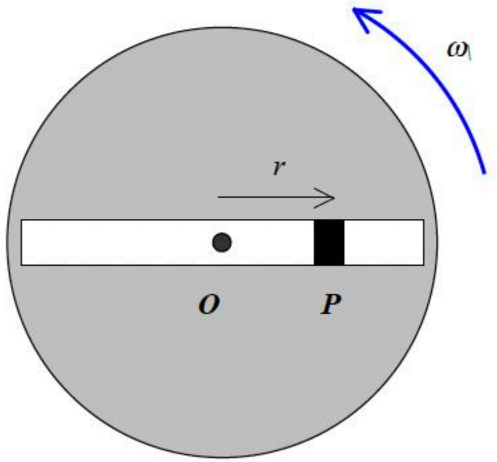
$$v_2 = -(R+r)\omega_2 \quad (5)$$

8 Solve

Example 5.C.3

Given: The small block P of mass m slides in the smooth radial slot of the disk (where the disk has a mass moment of inertia about its center of I_O) while the disk is able to rotate freely about a vertical axis passing through the center of the disk O. The block is displaced slightly from the center of rotation of the shaft when the disk has an angular speed of ω_1.

Find: Determine the velocity of the block as a function of r as the block moves toward the outer radius of the disk.

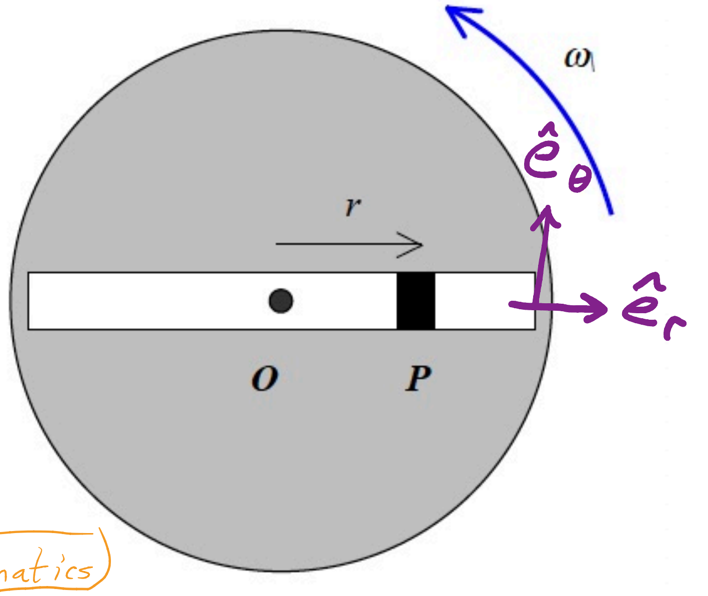


[pg. 344]

Example 5.C.3 p. 344

Given: The small block P of mass m slides in the smooth radial slot of the disk (where the disk has a mass moment of inertia about its center of I_O) while the disk is able to rotate freely about a vertical axis passing through the center of the disk O. The block is displaced slightly from the center of rotation of the shaft when the disk has an angular speed of ω_1. Horizontal Plane

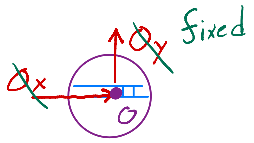
Find: Determine the velocity of the block as a function of r as the block moves toward the outer radius of the disk. $\vec{v}_{P2}$?



(1) Velocity → Δ in position

→ A.I.M

(2) FBD disk + P



Kinetics

(3) AIM ↺. Mom is cons @ O

$\sum \vec{M}_O = \vec{0}$

$\Rightarrow (\vec{H}_O)_1 = (\vec{H}_O)_2$

(4) H_{O1}

$$\vec{H}_{O1} = \underbrace{I_O \omega_1 \hat{k}}_{disk} + \underbrace{m \vec{r}_{P/O} \times \cancel{\vec{v}_{P1}}^{\,0}}_{P;\ particle}$$

$$= I_O \omega_1 \hat{k}$$

(5) H_{O2}

$$\vec{H}_{O2} = \overbrace{I_O \omega_2 \hat{k}}^{disk} + \overbrace{m \vec{r}_{P/O} \times \vec{v}_{P2}}^{P}$$

polar Kinematics ; Loc 3

$$= I_O \omega_2 \hat{k} + m(r_2 \hat{e}_r) \times (\dot{r} \hat{e}_r + r_2 \omega_2 \hat{e}_\theta)$$

$$= [I_O \omega_2 + m r_2^2 \omega_2] \hat{k}$$

(6) $H_{O1} = H_{O2}$

$$I_O \omega_1 = I_O \omega_2 + m r_2^2 \omega_2$$

$$\Rightarrow \vec{\omega}_2 = \left(\frac{I_O}{I_O + m r_2^2}\right) \omega_1 \hat{k} \quad (1)$$

Kinematics

(7) Kinematics eqn for polar rigid bodies.
Take Magnitude.

$$\vec{v}_2 = \dot{r}_2 \hat{e}_r + r_2 \omega_2 \hat{e}_\theta \quad (2)$$

$$v_{P2}^2 = \dot{r}_2^2 + r_2^2 \omega_2^2 \quad (3)$$

(8) 4 unkns 3 eqns

(9) W/E ; Look @ FBD. No non-cons forces !!

$$T_1 + V_1 + U_{1\to2}^{NC} = T_2 + V_2$$

$T_1 = \frac{1}{2} I_O \omega_1^2$; CoM & fixed point

$V_1 = 0$; horizontal plane & no springs

$U_{1-2}^{NC} = 0$; mech. eng. is conserved

$T_2 = \frac{1}{2} I_O \omega_2^2 + \frac{1}{2} m v_{P2}^2$; disk + particle

$V_2 = 0$; horizontal plane & no springs

(10) Plug-in

$$\Rightarrow \frac{1}{2} I_O \omega_1^2 = \frac{1}{2} I_O \omega_2^2 + \frac{1}{2} m v_{P2}^2$$

$$\Rightarrow v_{P2}^2 = \frac{I_O(\omega_1^2 - \omega_2^2)}{m} \quad (4)$$

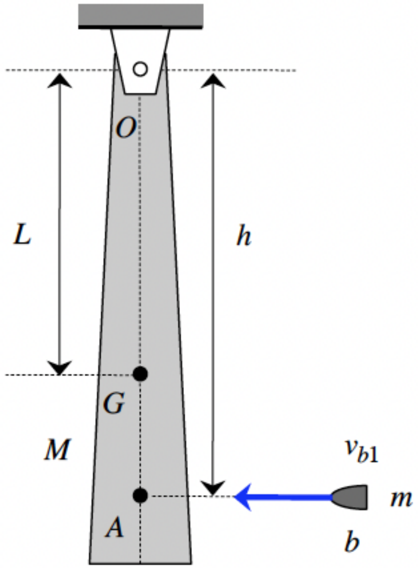
(11) Solve $\vec{v}_2$

Example 5.C.5

Given: A bullet having a mass of m strikes a stationary pendulum at a distance of h from the pivot O with a speed of v_{b1}. The pendulum has a mass of M and has a radius of gyration of k_O about the pivot O. The bullet imbeds itself immediately within the pendulum at A on impact.

Find: Determine the angular speed of the pendulum immediately after impact occurs.

Use the following parameters in your analysis: $v_{b1} = 1500$ ft/s, $m = 0.1$ slugs, $M = 2$ slugs, $h = 4$ ft, $k_O = 3$ ft and $L = 3$ ft.



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Example 5.C.5

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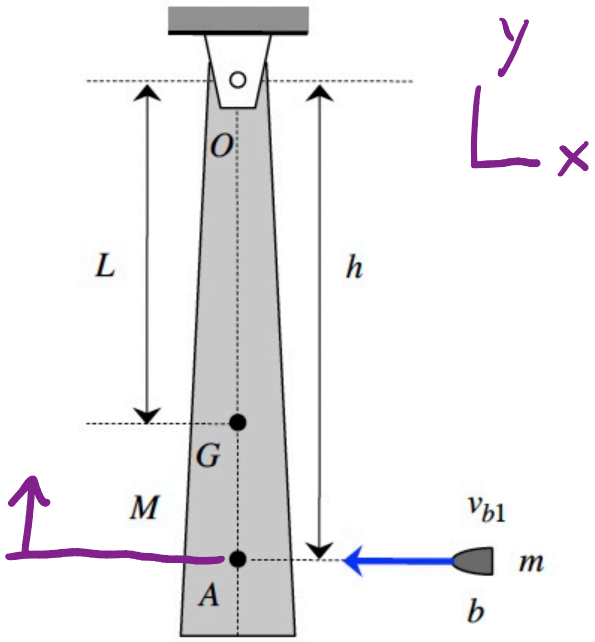
Friday hw has a pendulum problem

Given: A bullet having a mass of m strikes a stationary pendulum at a distance of h from the pivot O with a speed of v_{b1}. The pendulum has a mass of M and has a radius of gyration of k_O about the pivot O. The bullet imbeds itself immediately within the pendulum at A on impact.

Find: Determine the angular speed of the pendulum immediately after impact occurs. $\vec{\omega}_2 = ?$

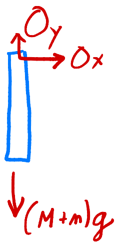
Use the following parameters in your analysis: $v_{b1} = 1500$ ft/s, $m = 0.1$ slugs, $M = 2$ slugs, $h = 4$ ft, $k_O = 3$ ft and $L = 3$ ft.



① ω_2 ? after impact event

$\longrightarrow$ A.I.M.

② FBD bullet + rod



Kinetics

③ A.I.M. @ O

$\Sigma \vec{M}_O = \vec{0}$

$\Rightarrow (\vec{H}_O)_1 = (\vec{H}_O)_2$; Angular Mom is cons

④ H_{O1}

$$(\vec{H}_O)_1 = \underbrace{\vec{0}}_{\substack{\text{bar} \\ \text{@rest}}} + \underbrace{m\vec{r}_1 \times \vec{v}_1}_{\substack{\text{bullet} \\ \text{particle}}}$$

$$= -mh\hat{j} \times (-v_{b1}\hat{i})$$

$$= -mhv_{b1}\hat{k}$$

⑤ H_{O2}

$$(\vec{H}_O)_2 = \underbrace{I_O\omega_2\hat{k}}_{\text{rod}} + \underbrace{m\, r_{A/O} \times b_{b2}}_{\text{bullet}}$$

$$= (MK_O^2\,\omega_2 + m\,h\,V_{b2})\hat{k}$$

⑥ Kinematics

$$\vec{V}_{b2} = \vec{V}_O + \omega_2 \times \vec{r}_{A/O}$$

$$= (\omega_2\hat{k}) \times (-h\hat{j})$$

$$= h\omega_2\,\hat{i} \quad (2)$$

⑦ Solve $H_{O1} = H_{O2}$ (1)

$$\Rightarrow -mh\,V_{b1} = MK_O^2\,\omega_2 + mh(h\,\omega_2)$$

Question C5.2

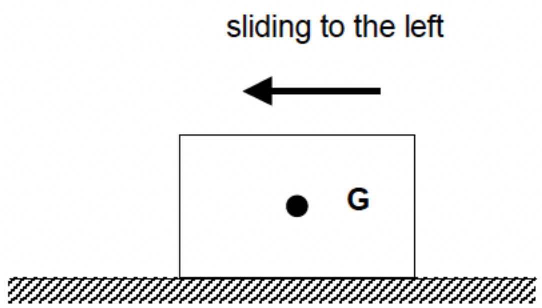
A block with center of mass at G slides to the left on a rough horizontal surface. Circle the answer below that most accurately describes the location of the normal contact force on the block from the ground as the block slides.

(a) The normal force acts at a point to the left of G

(b) The normal force acts at a point to the right of G

(c) The normal force acts at a point directly beneath G

(d) More information is needed to answer this question



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Question C5.2 p.355

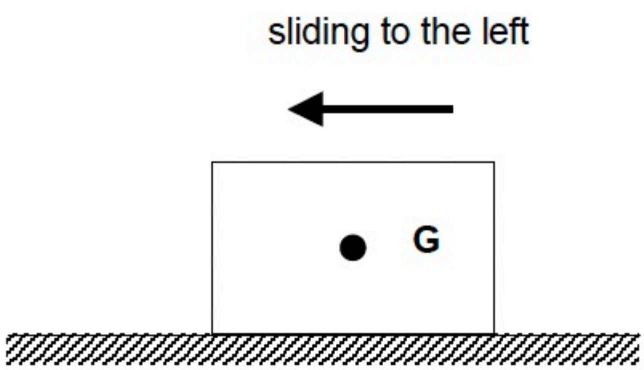
A block with center of mass at G slides to the left on a rough horizontal surface. Circle the answer below that most accurately describes the location of the normal contact force on the block from the ground as the block slides. ← μ_k

(a) The normal force acts at a point to the left of G

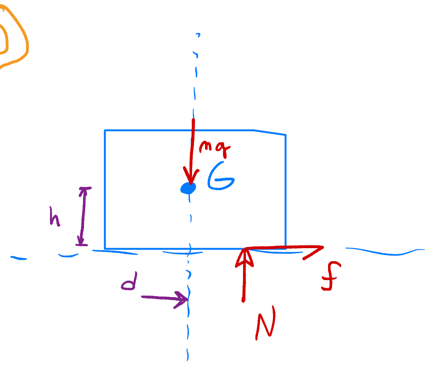
(b) The normal force acts at a point to the right of G

(c) The normal force acts at a point directly beneath G

(d) More information is needed to answer this question



1 FBD



2 Kinetics

0: doesn't slide

$$\Sigma M_G = fh + Nd = I_G \cancel{\alpha}$$

$$\hookrightarrow d = -\frac{f}{N} h$$

$$= -\frac{\mu_k N}{N} h$$

$$= -\mu_k h$$; "−" bc in front of G

$$\neq 0$$ (← not directly under G)

Summary: Impulse/Momentum Equations 1

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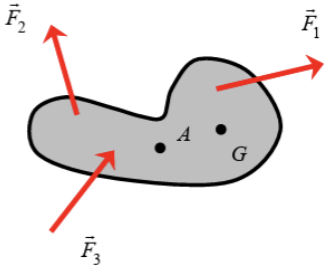
FUNDAMENTAL equations (for your chosen "system"):

$$m\vec{v}_{G2} = m\vec{v}_{G1} + \int_1^2 \vec{F}\,dt$$

$$\vec{H}_{A2} = \vec{H}_{A1} + \int_1^2 \vec{M}_A\,dt$$



for A = c.m. <u>*OR*</u> fixed point, with:

$$\boxed{\begin{aligned} \vec{H}_A &= I_A\vec{\omega} && ;\ \textit{for a RIGID BODY} \\ &= m\vec{r}_{P/A}\times\vec{v}_P && ;\ \textit{for a particle P} \end{aligned}}$$

Lec 32 Short Feedback Form:

SYSTEM CHOICE: Make your system big (make as many forces <u>*internal*</u> as possible to take advantage of conservation).

PARALLEL AXIS THEOREM: You will need to use the PAT if you choose A to be anything other than the c.m.: $I_A = \underbrace{\boxed{I_G}}_{cm} + md^2$