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ME 274 Lecture 31

Rigid body kinetics: Work-Energy – Part 2

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04/06/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 30 (5.G and 5.H) due Today!!**

2. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME.

Chapter 5: Planar Rigid Body Kinetics

- Work-Energy (W/E) Equations for Planar Rigid Bodies (lectures 30-31)

$$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$$

$$\text{where } T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$$

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ $\text{where } T = \frac{1}{2} m v^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ $\text{where } T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_{O2} - \vec{H}_{O1}$ $\text{where } \vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\, dt = \vec{H}_{A2} - \vec{H}_{A1}$ $\text{where } \vec{H}_A = I_A \vec{\omega}$

[pg. 252 and 295 content]

Discussion – Work/Energy Equation for Rigid Bodies

$$\boxed{T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2}$$

where $\boxed{T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2 + m\vec{v}_A \cdot \left(\vec{\omega} \times \vec{r}_{G/A}\right)}$

The point P in the above equation for K.E. can be any point on the rigid body:

- **However, typically we pick one of the two,** since it makes the equation above simpler:

1. Center of Mass, G, for the body. Equation reduces to:

$$\boxed{T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2}$$

2. A fixed point, O. Equation above reduces to:

$$\boxed{T = \frac{1}{2} I_O \omega^2}$$

- **Can be the instant center of the body,** we will typically do this
- **Need to use Parallel Axis Theorem,** since its not center of mass, G

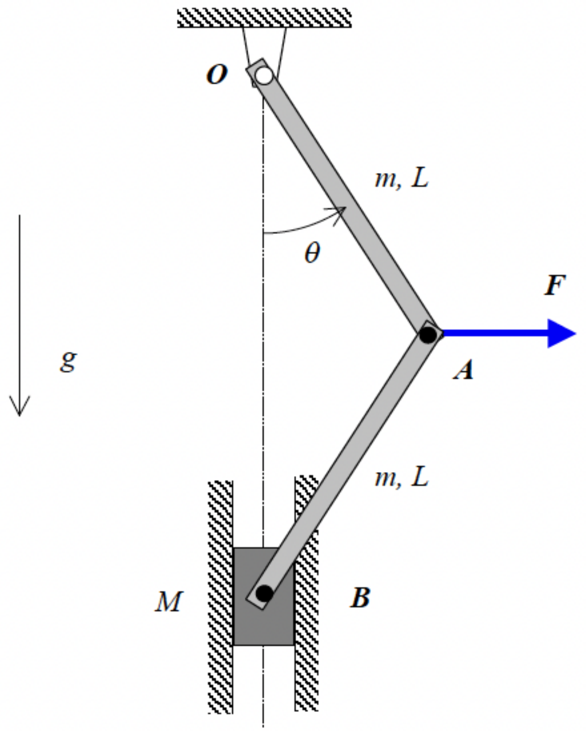
[pg. 329 content]

Example 5.B.5

Given: The system shown is released from rest when $\theta = 0$. A constant force F acts horizontally at pin B for all time. Both links are homogeneous and have a mass of m. The slider block at B has a mass of M. The system moves in a vertical plane.

Find: Determine the speed of B when $\theta = 53.13°$.

Use the following parameters in your analysis: $m = 5$ kg, $M = 10$ kg, $L = 2$ m and $F = 800$ N.



[pg. 334]

Example 5.B.5 p.334 similar to H.S.J due Wed.

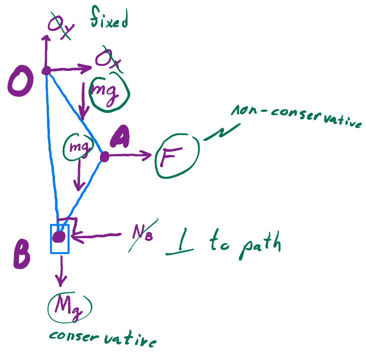
Given: The system shown is released from rest (RFR) when $\theta = 0$. A constant force F acts horizontally at pin B for all time. Both links are homogeneous and have a mass of m. The slider block at B has a mass of M. The system moves in a vertical plane.

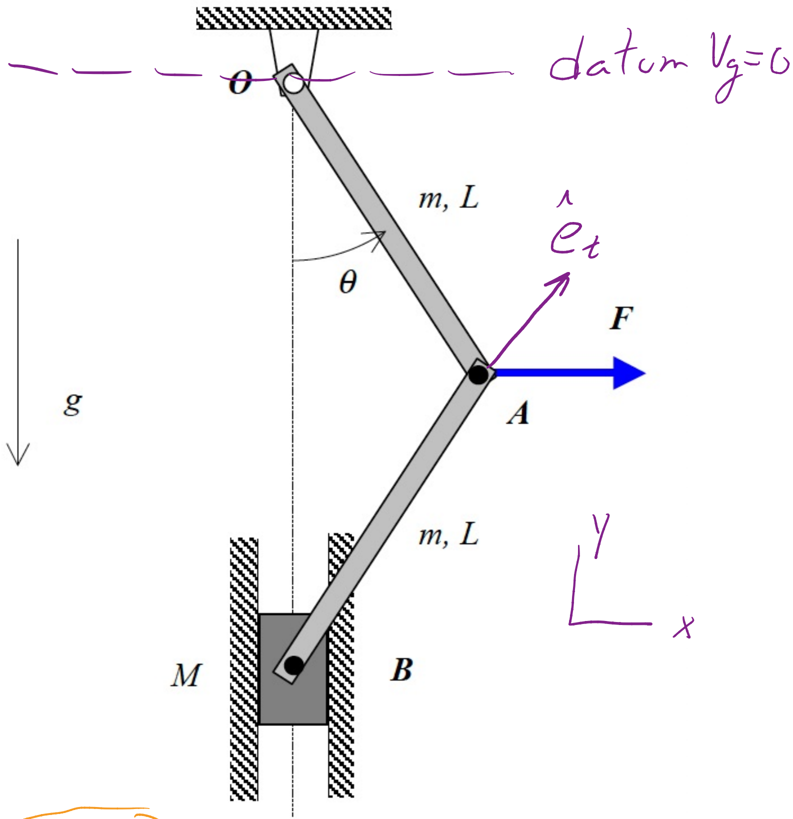
Find: Determine the speed of B when $\theta = 53.13°$.

Use the following parameters in your analysis: $m = 5$ kg, $M = 10$ kg, $L = 2$ m and $F = 800$ N.

① Δ in spd $\rightarrow$ Δ in position
↳ W/E

② FBD Make it big (OA + AB + B)





③ Kinetics W/E

$$T_1 + V_1 + U^{nc}_{1\to 2} = T_2 + V_2$$

$T_1 = 0$; RFR

$$V_1 = \underbrace{-mg\tfrac{L}{2}}_{OA} \underbrace{- mg\tfrac{3L}{2}}_{AB} \underbrace{- Mg(2L)}_{B} \quad ; \text{datum}$$

$$U^{(nc)}_{1\to 2} = \int_1^2 (\vec{F}\cdot\hat{e}_t)\,ds$$

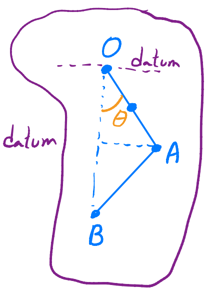
$$= \int_1^2 \overset{\text{constant}}{F_x}\,dx + \int_1^2 \cancel{F_y}^{\,0}\,dy \quad ; \text{pg 211}$$

$$= F\,\Delta x_A \quad ;$$

$$\Rightarrow \Delta x_A = L\sin\theta$$

$$T_2 = \underbrace{\tfrac{1}{2}I_o\omega_{OA}^2}_{\substack{OA \\ \text{fixed pt } O}} + \underbrace{\tfrac{1}{2}mv_G^2 + \tfrac{1}{2}I_G\omega_{AB}^2}_{AB} + \underbrace{\tfrac{1}{2}Mv_B^2}_{\substack{B \\ \text{particle}}}$$

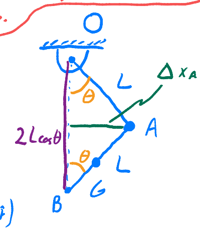
$$V_2 = \underbrace{-mg\tfrac{L}{2}\cos\theta}_{OA} \underbrace{- mg(L\cos\theta + \tfrac{L}{2}\cos\theta)}_{AB} \underbrace{- Mg(2L\cos\theta)}_{B} \quad ; \text{datum}$$



Kinematics

⑤ From figure, ω_{OA} & ω_{AB} change @ same rate in $\angle\theta$

$|\omega_{OA}| = |\omega_{AB}|$ (2) Note: there are 2 other ways ① IC ② Vector; see soln video.



⑥ OA

$$\vec{V}_A = \overset{0;\text{ fixed}}{\cancel{\vec{V}_o}} + \vec{\omega}_{OA} \times \vec{r}_{A/O}$$

⑦ AG

$$\vec{V}_G = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{G/A}$$

$$= (\omega_{OA}\hat{k})\times(L\sin\theta\,\hat{\imath} - L\cos\theta\,\hat{\jmath}) + (\omega_{AB}\hat{k})\times(-\tfrac{L}{2}\sin\theta\,\hat{\imath} - \tfrac{L}{2}\cos\theta\,\hat{\jmath})$$

$$= (L\omega_{OA}\cos\theta + \tfrac{L}{2}\omega_{AB}\cos\theta)\,\hat{\imath} + (L\omega_{OA}\sin\theta - L\omega_{AB}\sin\theta)\,\hat{\jmath}$$

⑧ Take magnitude of $|V_g|$

$$V_G^2 = L^2\cos^2\theta(\omega_{OA} + \omega_{AB})^2 + L^2\sin^2\theta(\omega_{OA} - \omega_{AB})^2 \quad (3)$$

⑨ Take $\frac{d}{dt}$ of OB for V_B

$$V_B = \frac{d}{dt}(2L\cos\theta)$$

$$= -2L\,\omega_{OA}\sin\theta \quad (4)$$

⑩ Find Δx_A using trig

$$\Delta x_A = L\sin\theta \quad (5)$$

⑪ Solve (1)-(5). 5 eqns 5 unkns

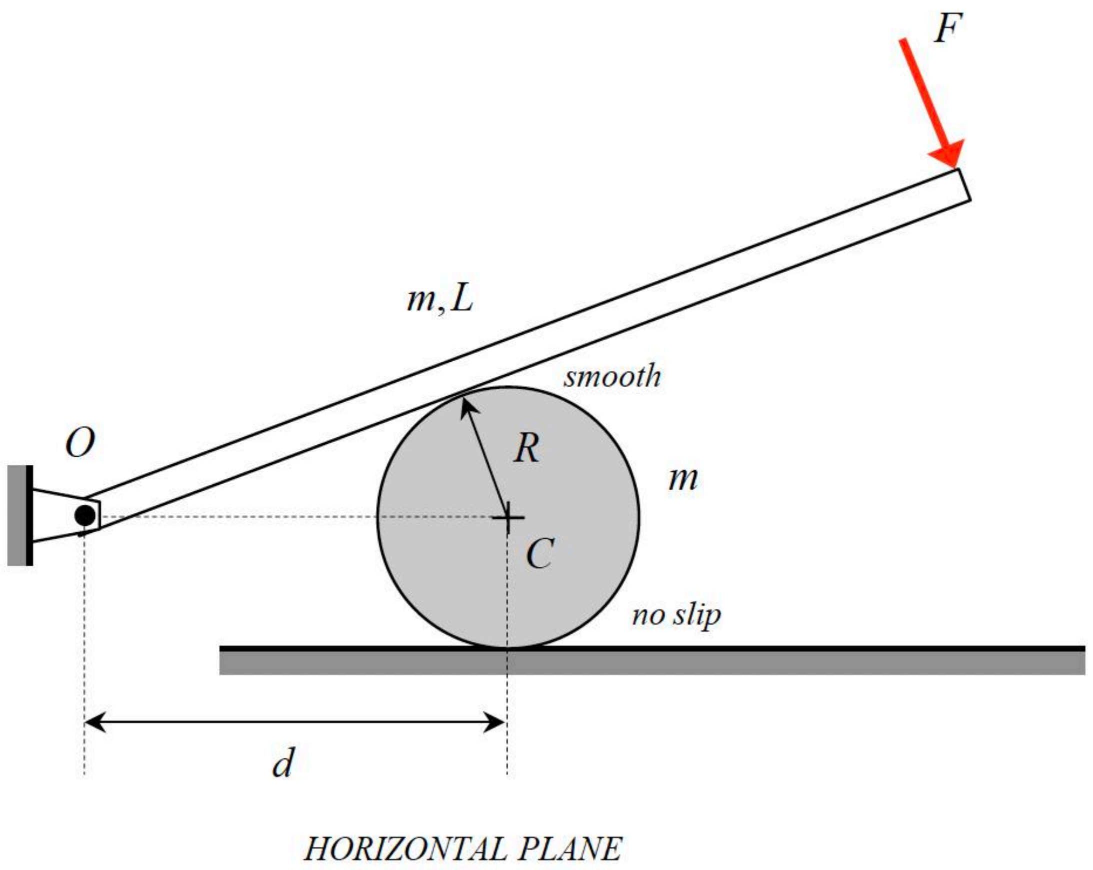
$$\Rightarrow -2mgL - 2MgL + F\Delta x_A = \tfrac{1}{2}I_o\omega_{OA}^2 + \tfrac{1}{2}I_G\omega_{AB}^2 + \tfrac{1}{2}mV_G^2 + \tfrac{1}{2}MV_B^2 - 2mgL\cos\theta - 2MgL\cos\theta \quad (1)$$

④ 5 unkns 1 eqn

Example 5.B.6

Given: A disk of mass m and radius R is able to roll without slipping on a rough surface. A thin, homogeneous bar of mass m and length L is pinned to ground at O and is brought in contact with the disk at a smooth contacting surface. A constant force F is applied to the free end of the bar with F always perpendicular to the bar. The system is released from rest with the center C of the disk at a disk of d from the pin at O. The system moves in a horizontal plane.

Find: Determine the speed of the disk's center C after C has moved an additional distance d to the right.

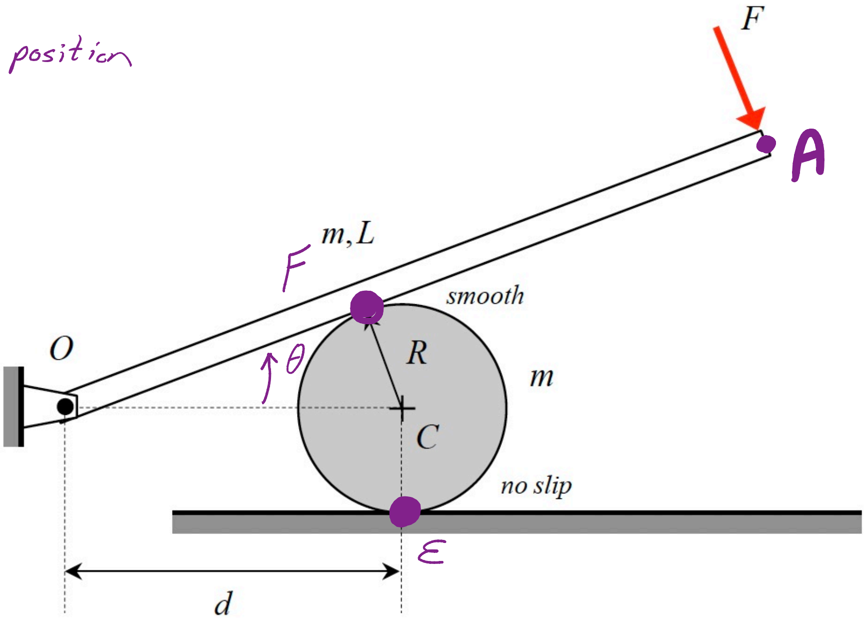


[pg. 335]

Example 5.B.6 p.335

Given: A disk of mass m and radius R is able to roll without slipping on a rough surface. A thin, homogeneous bar of mass m and length L is pinned to ground at O and is brought in contact with the disk at a smooth contacting surface. A constant force F is applied to the free end of the bar with F always perpendicular to the bar. The system is released from rest with the center C of the disk at a disk of d from the pin at O. The system moves in a horizontal plane. RFR

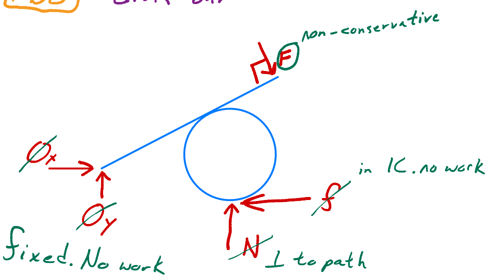
Find: Determine the speed of the disk's center C after C has moved an additional distance d to the right. v_c?



HORIZONTAL PLANE

① Change in spd – change in position
↳ W/E Eqn

② FBD disk + bar



③ Kinetics W/E

$$T_1 + V_1 + U_{1\to2}^{(nc)} = V_2 + T_2 \quad (1)$$

$T_1 = 0$; RFR

$V_1 = 0$; horizontal plane

$$U_{1\to2}^{(nc)} = \int_1^2 (\vec{F}\cdot\hat{e}_t)\, ds_A$$

$$= F\Delta s_A$$

$$= +FL(\theta_1 - \theta_2)$$

$V_2 = 0$; horizontal plane

$$T_2 = \underbrace{\tfrac{1}{2} I_O \omega_{OA}^2}_{\text{bar}} + \underbrace{\tfrac{1}{2} I_E \omega_d^2}_{\text{disk}}$$; use: ① O bc fixed pt ② E bc no slip pt on disk

w/ $I_O = \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2$

$= \frac{1}{3} mL^2 \quad (2)$

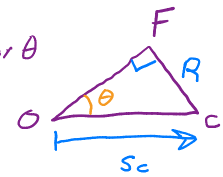
$I_E = \frac{1}{2} mR^2 + mR^2$

$= \frac{3}{2} mR^2 \quad (3)$

P.A.T. (x 2)

Kinematics

④ Draw Triangle. Solve for θ



$\sin\theta = \frac{R}{s_c}$

$\Rightarrow R = s_c \sin\theta$

$\Rightarrow \theta = \sin^{-1} \frac{s_c}{R}$

@ state 1 & 2 $\Rightarrow$ $\begin{cases} \theta_1 = \sin^{-1} \frac{d}{R} & (4) \\ \theta_2 = \sin^{-1} \frac{2d}{R} & (5) \end{cases}$

⑤ take $\frac{d}{dt}$ of R

$\cancel{\dot{R}}$ 0; not changing $= \dot{s}_c \sin\theta + s_c \dot{\theta}\cos\theta$; product rule

$\Rightarrow \dot{\theta} = \frac{-\dot{s}_c}{s_c}\tan\theta$; $\dot{\theta} = \omega_{OA}$; $\dot{s}_c = v_c$

$\omega_{OA} = -\frac{v_c}{s_c}\tan\theta \quad (6)$

⑥ Use disk roll w/o slip

Since E = no slip

$\Rightarrow v_c = R\omega_d$

$\Rightarrow \omega_d = \frac{v_c}{R} \quad (7)$

⑦ Solve Plugin (1), (6), (7) ∴ 5 eqns. 5 unkns.

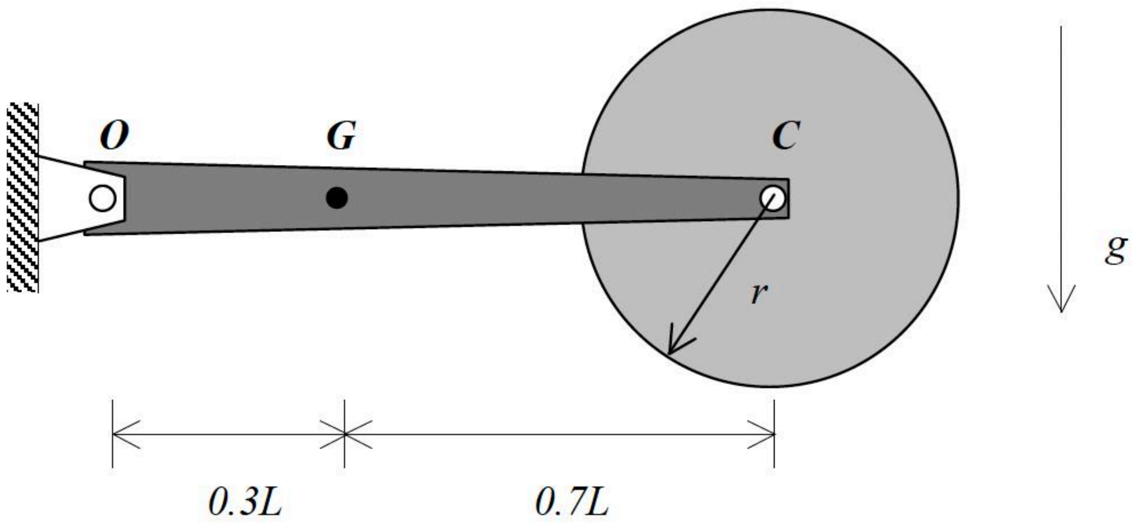
$$FL(\theta_1 - \theta_2) = \tfrac{1}{2} I_O \left(-\frac{v_c}{2d}\tan\theta_2\right)^2 + \tfrac{1}{2} I_E \left(\frac{v_c}{R}\right)^2$$

↳ $v_c =$ ______

Example 5.B.7

Given: Link OC and the circular disk are released from rest when OC is horizontal. Link OC has a length of $L = 1.2$ m, mass of 10 kg and radius of gyration about O of $k_O = 0.5$ m. The disk has a mass of 8 kg and outer radius of $r = 0.4$ m.

Find: Determine the angular speed of OC when OC has moved to a vertical orientation.



[pg. 336]

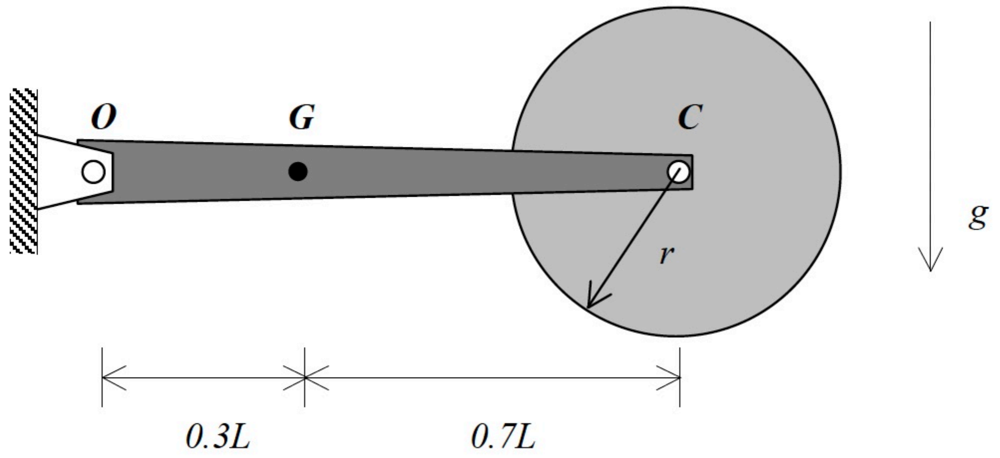
Example 5.B.7 p. 336 RFR Similar to H.S.I due Wed

Given: Link OC and the circular disk are released from rest when OC is horizontal. Link OC has a length of $L = 1.2$ m, mass of 10 kg and radius of gyration about O of $k_O = 0.5$ m. The disk has a mass of 8 kg and outer radius of $r = 0.4$ m.

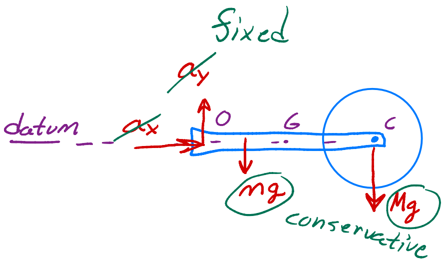
Find: Determine the angular speed of OC when OC has moved to a vertical orientation.

ω_{oc}?

① Method = Δ in spd for Δ in position ∴ W/E



② FBD Whole system (disk + bar)



③ Kinetics W/E

$$\cancel{T_1}^{0} + \cancel{V_1}^{0} + \cancel{U_{1\to 2}^{(nc)}}^{0} = T_2 + V_2$$

$T_1 = 0$; RFR

$V_1 = 0$; @ datum

$U_{1\to 2}^{(nc)} = 0$; mech eng is conserved

$$T_2 = \underbrace{\tfrac{1}{2} I_O \omega_{oc}^2}_{\text{bar}} + \underbrace{\tfrac{1}{2} M v_c^2 + \tfrac{1}{2} I_c \omega_d^2}_{\text{disk}}$$; Use fixed pt O & pt C

w/ $\begin{cases} I_O = m k_O^2 \text{ ; } k_O \text{ is given, so use radius of gyration} \\ I_c = \tfrac{1}{2} M r^2 \text{ ; @ center} \end{cases}$

$V_2 = -mg(0.3L) - MgL$

$$\Rightarrow 0 = \tfrac{1}{2} I_O \omega_{oc}^2 + \tfrac{1}{2} M v_c^2 + \tfrac{1}{2} I_c \omega_d^2 - 0.3mgL - MgL \quad (1)$$

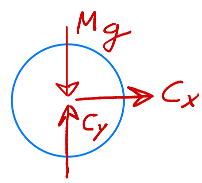
④ 1 eqn 3 unkns

Kinematics

⑤ Since C is on OC & disk...

$v_c = L\,\omega_{oc}$ (2)

⑥ Draw FBD of disk. All forces act @ center. Do ΣM_c.



$\Sigma M_c = 0$

$\Rightarrow \alpha_d = 0$

$\Rightarrow \omega_d = \text{const}$

$= 0$ (3) ; since it starts from rest

⑦ Solve 3 eqns 3 unkns

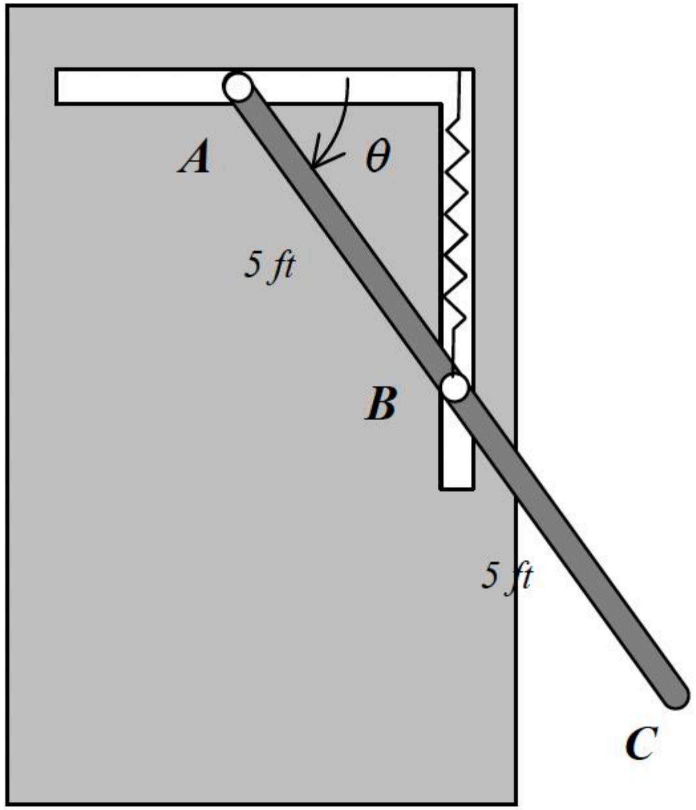
$$\tfrac{1}{2} I_O \omega_{oc}^2 + \tfrac{1}{2} M (L\,\omega_{oc})^2 = +(0.3m + M)gL$$

Solve for $\omega_{oc} =$ ___

Example 5.B.8

Given: End A of a garage door is constrained to move along a horizontal guide, whereas the midpoint B of the door moves along a vertical guide. The spring has a stiffness of $k = 40$ lb/ft and is unstretched when $\theta = 30°$. The door has a weight of $W = 200$ lb. The door is released from rest when $\theta = 30°$. Assume all surfaces to be smooth.

Find: Find the speed of end A of the door as the door goes vertical.



[pg. 337]

Example 5.B.8

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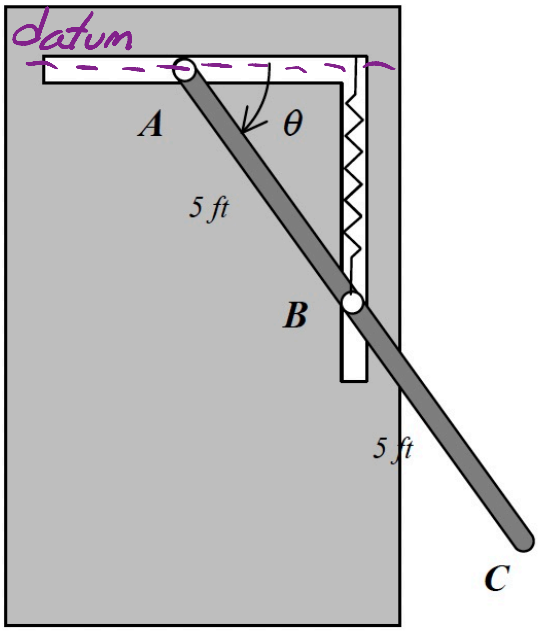
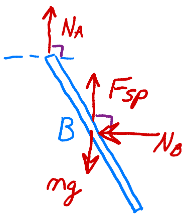
Find: Find the speed of end A of the door as the door goes vertical.

v_A ?



① FBD



② Kinetics W/E

$$\cancel{T_1}^{0} + V_1 + \cancel{U_{1\to2}^{(nc)}}^{0} = T_2 + V_2$$

$T_1 = 0$; I.A.R.

$V_1 = -mg\frac{L}{2}\sin\theta$

$U_{1\to2}^{(nc)} = \vec{0}$

$T_2 = \frac{1}{2} m v_B^2 + \frac{1}{2} I_B \omega^2$

$V_2 = -mg\frac{L}{2} + \frac{1}{2}k\Delta^2$; Δ = stretch in spring

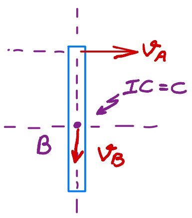
$$\Rightarrow -mg\frac{L}{2}\sin\theta = \frac{1}{2} m \cancel{v_B^2}^{0} + \frac{1}{2} I_B \omega^2 - mg\frac{L}{2} + \frac{1}{2}k\Delta^2$$

③ Kinematics

Since $B = IC_{door}$

$\Rightarrow v_B = 0$

$\Rightarrow v_A = \frac{L}{2}\omega$



Also: $\Delta = L_{sp} - L_o$

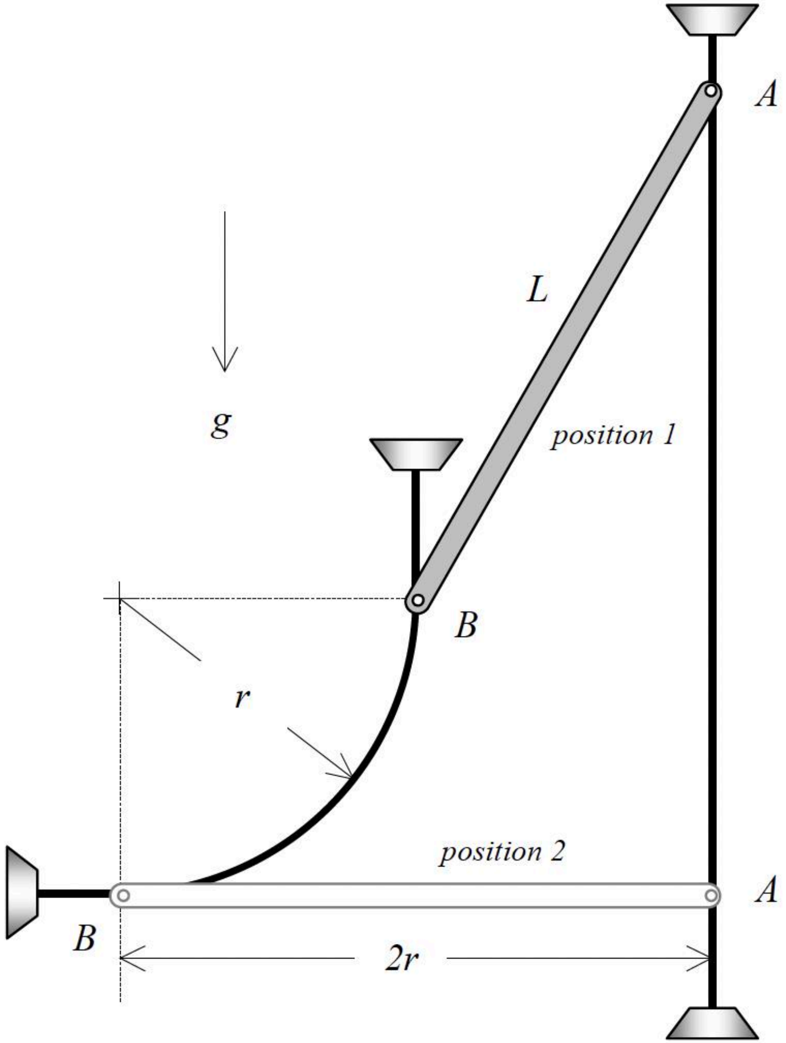
w/ $L_o = \frac{L}{2}\sin 30°$, $L_{sp} = \frac{L}{2}$ } $\boxed{\Delta = \frac{L}{2} - \frac{L}{2}\sin 30°}$

④ Solve Solve for v_A from W/E eqn

Example 5.B.9

Given: Ends A and B of the thin, homogeneous bar AB (having a mass of m and a length of $L = 2r$) are constrained as follows: A moves along a smooth vertical guide and B moves along a smooth semi-circular guide of radius r. The bar is released from rest at position 1.

Find: Find the speed of end A when the bar is horizontal (i.e. is at position 2).



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Example 5.B.9

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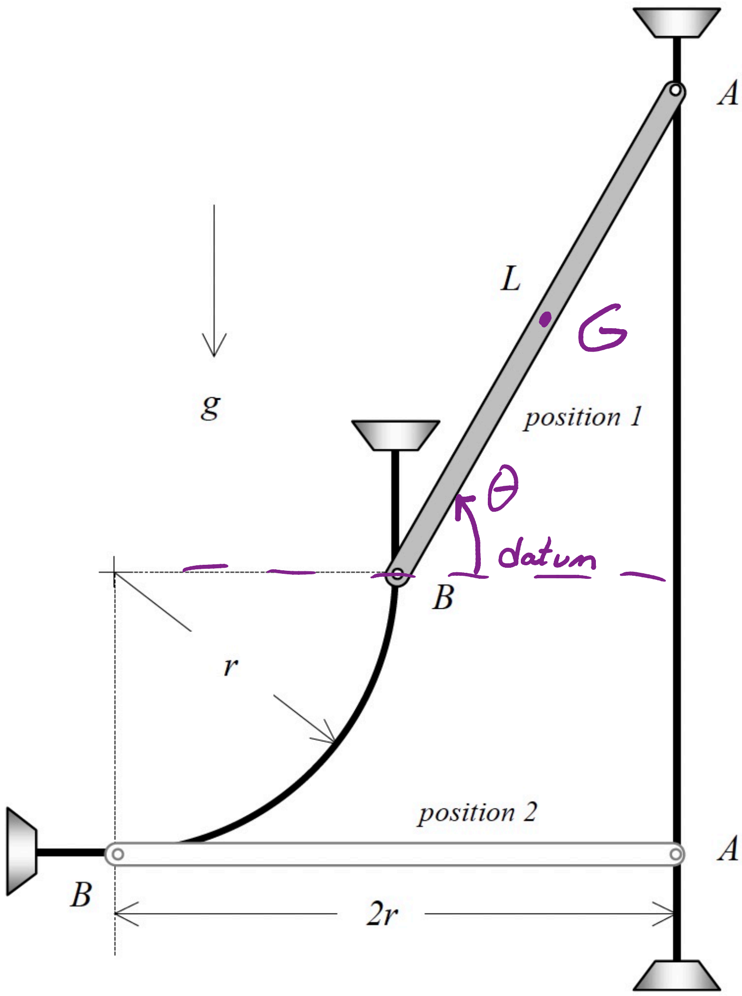
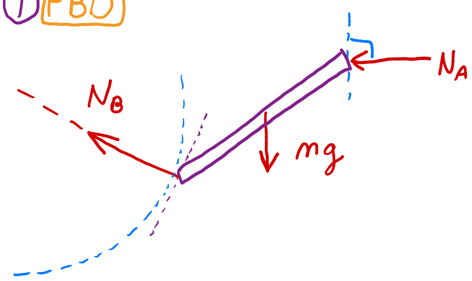
v_A ?



① FBD



② Kinetics W/E

$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{(nc)}_{1\to 2}}^{\,0} = T_2 + V_2$

$T_1 = 0$; I.A.R.

$V_1 = mg\frac{L}{2}\sin\theta$

$U^{(nc)}_{1\to 2} = 0$

$T_2 = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

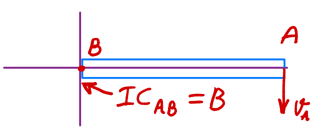
$V_2 = -mgr$

$\Rightarrow mg\frac{L}{2}\sin\theta = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2 - mgr$

③ Kinematics

$v_B = 0$

$\Rightarrow v_G = \frac{L}{2}\omega$



④ Solve

$mg\frac{L}{2}\sin\theta = \frac{1}{2}m\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega^2 - mgr$

$\hookrightarrow \omega =$ ___

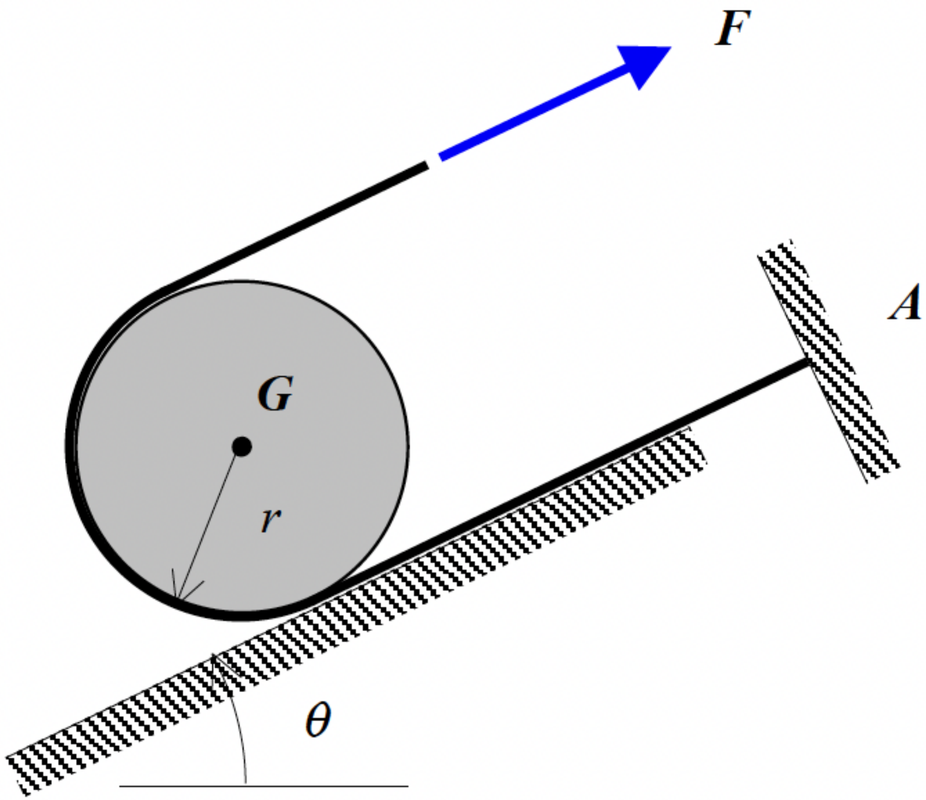
$\hookrightarrow v_A = \frac{L}{2}\omega$ ___ ←

Example 5.B.10

Given: The motion of a 25 lb wheel with radius of gyration of $k_G = 4$ in about its center of mass G is constrained by a band, which is secured at A and wrapped around the wheel. The wheel is released from rest with a constant $F = 20$ lb force applied to the band end.

Find: Determine the speed of the center G of the wheel when it has rolled 0.8 ft up the incline.

Use the following parameters in your analysis: $r = 0.5$ ft and $\theta = 36.87°$.



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Example 5.B.10

p. 339

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Find: Determine the speed of the center G of the wheel when it has rolled 0.8 ft up the incline.

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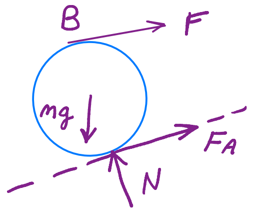
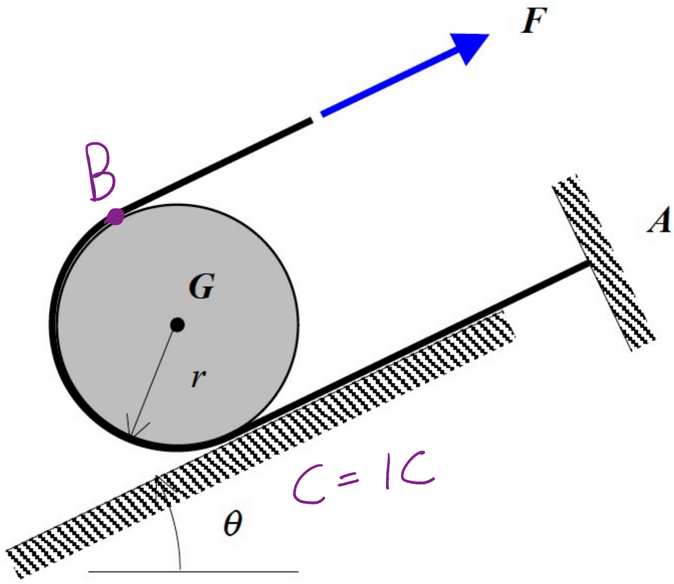
v_G?

(1) FBD





(2) Kinetics W/E

$T_1 = 0$; D.A.R.

$V_1 = 0$; @ datum

$$U_{1\to2}^{(nc)} = \int_1^2 \underbrace{(\vec{F}\cdot\hat{e}_t)}_{F} ds_B = F\,\Delta s_B$$

$$T_2 = \frac{1}{2}m v_G^2 + \frac{1}{2}I_G\omega^2 \quad \underline{\underline{or}} \quad \frac{1}{2}I_C\omega^2$$

w/ $I_C = I_G + mR^2$; P.A.T.

$\quad = \frac{3}{2}mR^2$

$$V_2 = +mg\,\Delta s_G \sin\theta$$

(3) Kinematics

$C = IC \Rightarrow \begin{cases} v_G = r\omega \\ v_B = 2r\omega \end{cases}$

$\hookrightarrow v_B = 2v_G$

$\hookrightarrow \Delta s_B = 2\,\Delta s_G$

(4) Solve

$$0 + 0 + F(2\Delta s_G) = \frac{1}{2}\left(\frac{3}{2}mr^2\right)\omega^2 + mg\,\Delta s_G \sin\theta$$

$\hookrightarrow \omega = \underline{\checkmark}$

$\hookrightarrow v_G = r\omega = \underline{\checkmark}$

Summary: Work/Energy Equation 2

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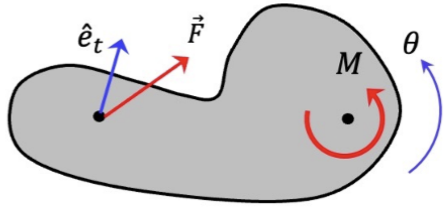
FUNDAMENTAL equations: $T_2 + V_2 = T_1 + V_1 + U_{1\to 2}^{(nc)}$ with:

$$T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2$$

where A = ***EITHER*** c.m. ***OR*** fixed point, and:

$$U_{1\to 2}^{(nc)} = \int_1^2 (\vec{F} \cdot \vec{e}_t) ds + M\Delta\theta$$



Lec 31 Short Feedback Form:

SOLUTION METHOD: the four-step plan

Kinetics: Four-Step Problem Solving Method

The suggested plan of action for solving kinetics problems:

1. ***Free body diagram(s)***. Draw appropriate free body diagrams (FBDs) for the problem. Your choice of FBDs is problem dependent. For some problems, you will draw an FBD for each body; for others, you will draw an FBD for the entire system. An integral part of your FBDs is your choice of coordinate systems. For each FBD, draw the unit vectors corresponding to your coordinate choice.

2. ***Kinetics equations***. At this point, you will need to choose what solution method(s) that you will need to use for the particular problem at hand. In this section of the course we will study four basic methods: Newton/Euler, work/energy, linear impulse/momentum and angular impulse/momentum. Based on your choice of method(s), write down the appropriate equations from your FBD(s) from Step 1.

3. ***Kinematics***. Perform the needed kinematic analysis. A study of the equations in Step 2 above will guide you in deciding what kinematics are needed to find a solution to the problem.

4. ***Solve***. Count the number of unknowns and the number of equations from above. If you do not have enough equations to solve for your unknowns, then you either: (i) need to draw more FBDs, OR (ii) need to do more kinematic analysis. When you have sufficient equations for the number of unknowns, solve for the desired unknowns from the above equations.

Draw free-body diagram of entire system for work/energy.

Be sure to use the correct mass moment of inertia for your choice of point "A". Use PAT if necessary.

Typically the most difficult step. Recall the rigid body kinematics from Chapter 2.

If you are short equations, go back to Step 3 – Kinematics.