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ME 274 Lecture 30

Rigid body kinetics: Newton/Euler – Part 3

Eugenio “Henny” Frias-Miranda

04/01/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 29 (5.E and 5.F) due Friday!!**

2. Office hours are changing to ME2008B…
   - Second floor of renovated side of ME.

3. Exam 2 Information:
   - Thursday, April 2, 8:00-9:30 PM
   - BHEE129
   - Coverage: Lectures 11-26 (up through angular impulse/momentum for particles)

4. Exam 2 Review sessions both videos to be posted on website
   - Pi Tau Sigma: Tuesday, March 31, 6:30-7:30 PM, WTHR 104 (WL in-person and Indy online)
   - ME 274 Instructor, CK: Wednesday, April 1, 7:00 PM, live on Zoom for both WL and Indy:
     - https://purdue.edu.zoom.us/j/94496659802?pwd=VMHo3NfyaHO3HbbmmLForgikls3PL7.1

5. DRC Students: if you are planning on using your accommodations, please reply to the email you should have received from Prof. Chuck Krousgrill

6. Assigned Seats!!! I will send an email later today

Chapter 5: Planar Rigid Body Kinetics

- Work-Energy (W/E) Equations for Planar Rigid Bodies (lectures 30-31)

$$T_1 + V_1 + U_{1-2}^{NC} = T_2 + V_2$$

where $T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2 + m \vec{v}_A \cdot (\vec{\omega} \times r_{G/A})$

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\,dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\,dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A\vec{\omega}$

[pg. 252 and 295 content]

Derivation of Work-Energy Equation (specifically K.E.) for Rigid Bodies

1. **Start with Kinetic Energy for a system of particles, enforce constraint for a rigid body**
   - **(ie mult. by kinematic velocity equation for a rigid body)**

$$T = \frac{1}{2}\sum_i m_i\, v_i^2 = \boxed{\frac{1}{2}\sum_i m_i\, \vec{v}_i \bullet \vec{v}_i}$$

$$\vec{v}_i = \vec{v}_P + \vec{\omega} \times \vec{r}_{i/P}$$

2. Algebra of this multiplication

$$\begin{aligned}\vec{v}_i \bullet \vec{v}_i &= (\vec{v}_P + \vec{\omega} \times \vec{r}_{i/P}) \bullet (\vec{v}_P + \vec{\omega} \times \vec{r}_{i/P})\\ &= \vec{v}_P \bullet \vec{v}_P + 2\vec{v}_P \bullet (\vec{\omega} \times \vec{r}_{i/P}) + (\vec{\omega} \times \vec{r}_{i/P}) \bullet (\vec{\omega} \times \vec{r}_{i/P})\\ &= v_P^2 + 2\vec{v}_P \bullet (\vec{\omega} \times \vec{r}_{i/P}) + |\vec{\omega} \times \vec{r}_{i/P}|^2\end{aligned}$$

3. Substitute into Kinetic Energy expression from above:

$$\begin{aligned}T &= \frac{1}{2}\sum_i m_i\, v_P^2 + \frac{1}{2}\sum_i m_i\, [2\vec{v}_P \bullet (\vec{\omega} \times \vec{r}_{i/P})] + \frac{1}{2}\sum_i m_i\, |\vec{\omega} \times \vec{r}_{i/P}|^2\\ &= \frac{1}{2}\left(\sum_i m_i\right) v_P^2 + \vec{v}_P \bullet \left(\vec{\omega} \times \sum_i m_i \vec{r}_{i/P}\right) + \frac{1}{2}\left(\sum_i m_i\, |\vec{r}_{i/P}|^2\right)\omega^2\\ &= \boxed{\frac{1}{2} m v_P^2 + m\vec{v}_P \bullet (\vec{\omega} \times \vec{r}_{G/P}) + \frac{1}{2} I_P \omega^2}\end{aligned}$$

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Discussion – Work/Energy Equation for Rigid Bodies

$$\boxed{T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2}$$

$$\text{where} \quad \boxed{T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2 + m\vec{v}_A \cdot \left(\vec{\omega} \times \vec{r}_{G/A}\right)}$$

The point P in the above equation for K.E. can be any point on the rigid body:

- **However, typically we pick one of the two,** since it makes the equation above simpler:

1. Center of Mass, G, for the body. Equation reduces to:

$$\boxed{T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2}$$

2. A fixed point, O. Equation above reduces to:

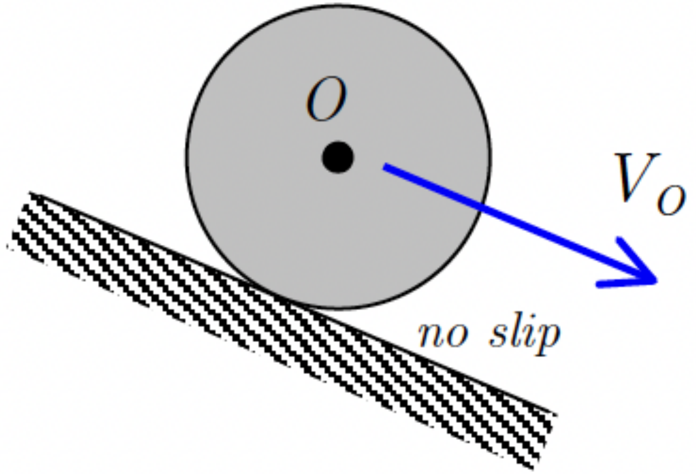
$$\boxed{T = \frac{1}{2} I_O \omega^2}$$

- **Can be the instant center of the body,** we will typically do this
- **Need to use Parallel Axis Theorem,** since its not center of mass, G

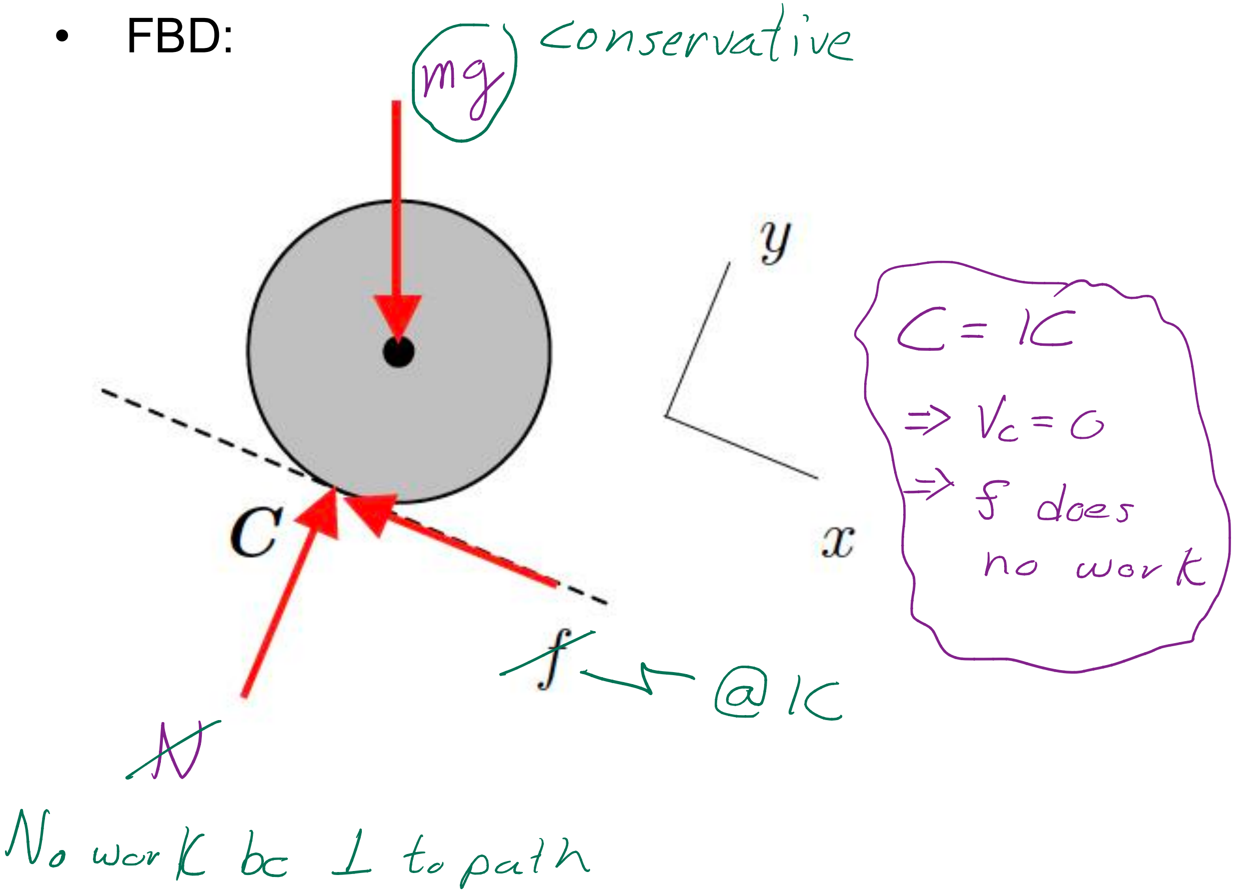
[pg. 329 content]

Important Scenario to keep in mind, Disk rolling w/o slipping

- Let’s make an FBD for the system shown below and classify which forces do work…



- FBD:



- **The friction force** is at the instant center… Therefore, it does no work since its acting at a stationary point

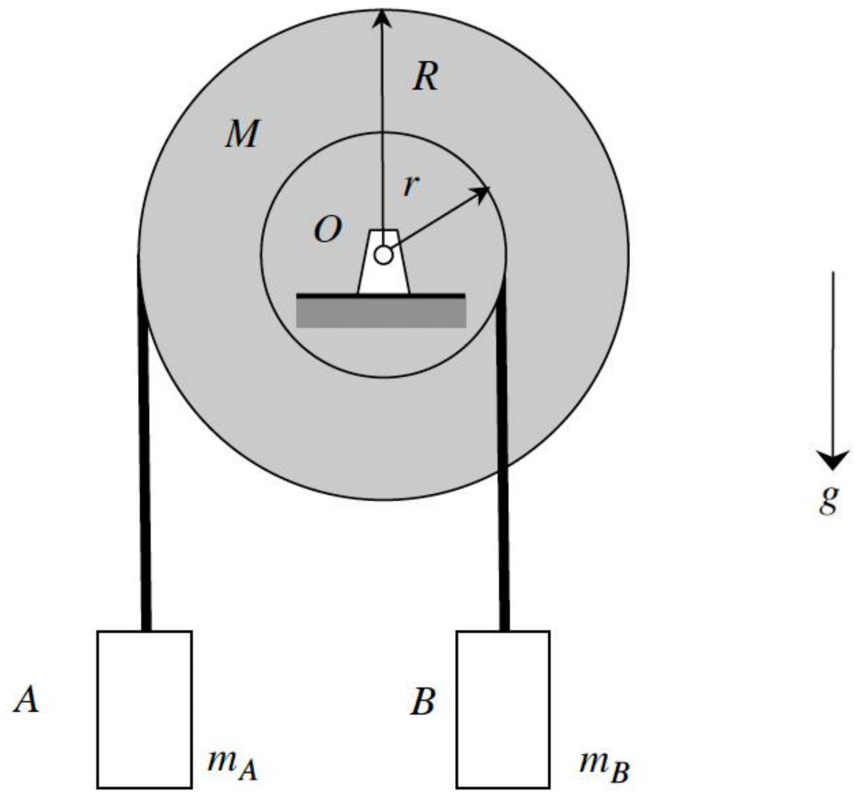
[pg. 329 content]

Example 5.B.1

Given: A stepped drum (having a mass of M and radius of gyration about its center O of k_O) is attached to a smooth shaft passing through its center O. A cable wrapped around the outer radius of the drum is attached to block A. A second cable is wrapped around the inner radius of the drum and is attached to block B. Assume that the cables do not slip on the drum. The system is released from rest.

Find: Determine the speed of B after it has dropped 1.5 meters.

Use the following parameters in your analysis: $m_A = 10$ kg, $m_B = 30$ kg, $M = 20$ kg, $r = 0.2$ m, $R = 0.4$ m, $k_O = 0.25$ m.



[pg. 330]

Example 5.B.1 p.330

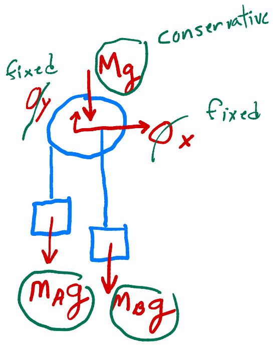
Given: A stepped drum (having a mass of M and radius of gyration about its center O of k_O) is attached to a smooth shaft passing through its center O. A cable wrapped around the outer radius of the drum is attached to block A. A second cable is wrapped around the inner radius of the drum and is attached to block B. Assume that the cables do not slip on the drum. The system is released from rest. RFR

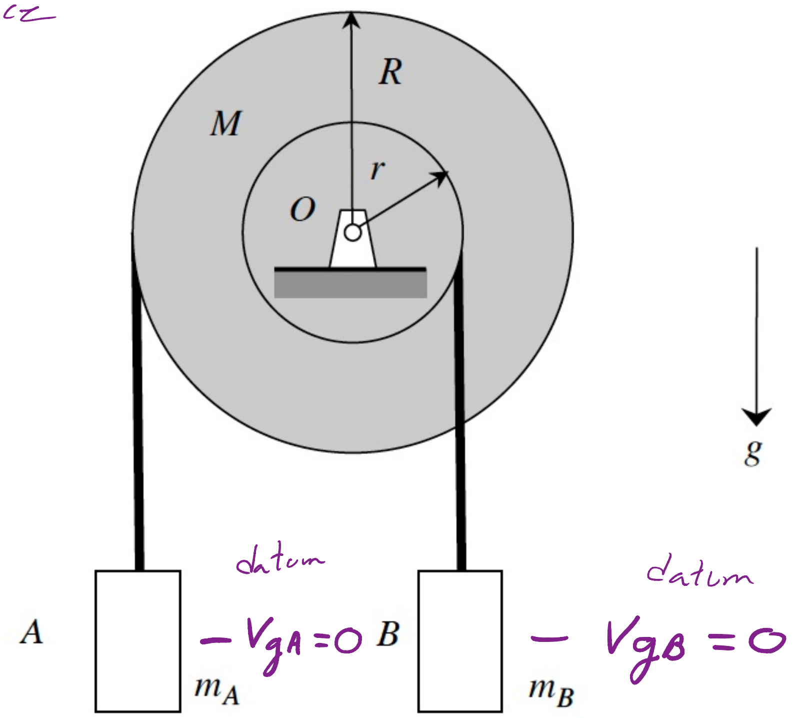
Find: Determine the speed of B after it has dropped 1.5 meters. $v_B = ?$

Use the following parameters in your analysis: $m_A = 10$ kg, $m_B = 30$ kg, $M = 20$ kg, $r = 0.2$ m, $R = 0.4$ m, $k_O = 0.25$ m.

(1) spd as a fcn of distance → W/E

(2) FBD





(3) Kinetics W/E

$$\cancel{T_1}^{0} + \cancel{V_1}^{0} + \cancel{U_{1\to2}^{NC}}^{0} = T_2 + V_2 \quad (1)$$

$T_1 = 0$; RFR

$V_1 = 0$; datum

$U_{1-2}^{NC} = 0$

$$T_2 = \tfrac{1}{2} I_O \omega_2^2 + \underbrace{\tfrac{1}{2} m_A v_A^2}_{A} + \underbrace{\tfrac{1}{2} m_B v_B^2}_{B}$$

$$V_2 = \underbrace{-m_B g d}_{B} + \underbrace{m_A g \Delta_A}_{A} \text{ ; datum}$$

(4) 1 eqn 4unkns

(5) Kinematics Instant Center @ O

$v_B = r\omega_2$

$\Rightarrow \omega_2 = \frac{v_B}{r} \quad (2)$

$v_A = R\omega_2$

$= R\left(\frac{v_B}{r}\right) \quad (3)$

(6) v_B/v_A eqn

$\Delta_A = \frac{R}{r}\Delta_b$

$= \frac{R}{r} d \quad (4)$

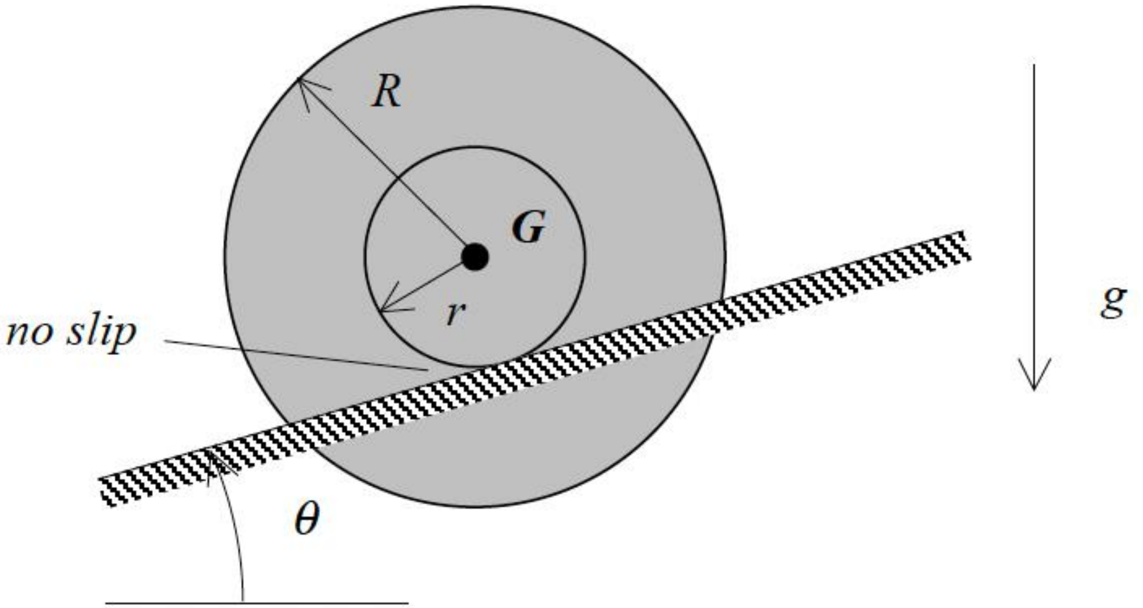
(7) Solve 4eqns 4unkns

$\Rightarrow v_B$

Example 5.B.2

Given: A stepped-wheel (with an outer radius of $R = 1$ ft, an inner radius of $r = 0.4$ ft, a weight of $W = 60$ lb and a centroidal radius of gyration $k_G = 0.6$ ft) is released from rest. Assume that the wheel rolls without slipping on its inner radius.

Find: Determine the velocity of the center G after the wheel has moved a distance of 15 ft down the incline.



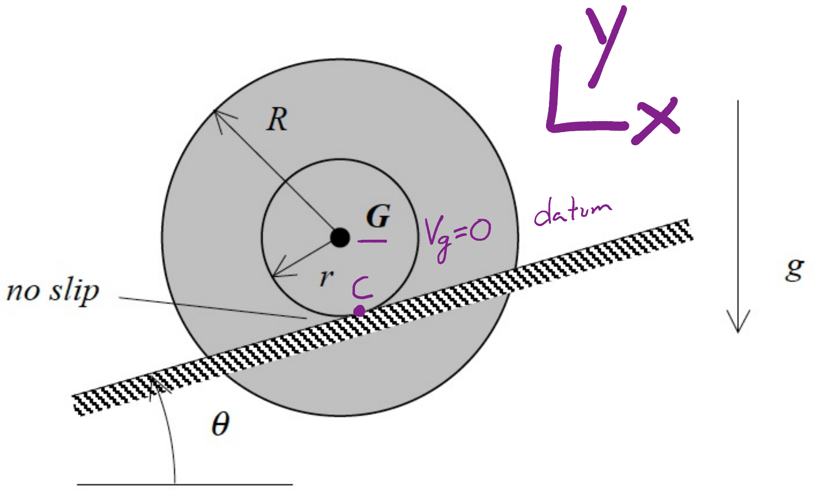
[pg. 331]

Example 5.B.2 p. 331

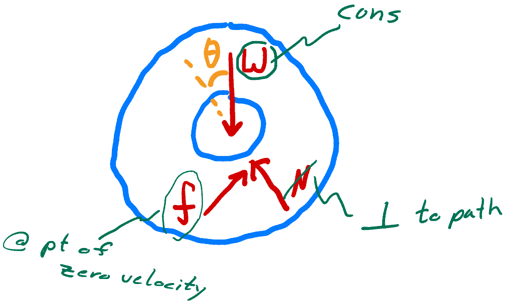
Given: A stepped-wheel (with an outer radius of $R = 1$ ft, an inner radius of $r = 0.4$ ft, a weight of $W = 60$ lb and a centroidal radius of gyration $k_G = 0.6$ ft) is released from rest. Assume that the wheel rolls without slipping on its inner radius.

RFR

Find: Determine the velocity of the center G after the wheel has moved a distance of 15 ft down the incline. $\vec{v}_2$?



(1) FBD



(2) Kinetics W/E

$$\cancel{T_1}^{0} + \cancel{V_1}^{0} + \cancel{U^{NC}_{1\to 2}}^{0} = T_2 + V_2 \quad (1)$$

$T_1 = 0$; RFR

$V_1 = 0$; datum

$U^{NC}_{1-2} = 0$; mech energy is cons

$$T_2 = \tfrac{1}{2} I_C \omega_2^2$$
$$= \tfrac{1}{2}(I_G + r^2 m)\omega_2^2 \quad ; \text{P.A.T.}$$
$$= \tfrac{1}{2}(m k_G^2 + m r^2)\omega_2^2 \quad ; \text{radius of gyration}$$
$$= \tfrac{1}{2}\frac{W}{g}(k_G^2 + r^2)\omega_2^2$$

$V_2 = -W d \sin\theta$; datum

(3) 1 eqn 2 unkn

(4) Kinematics we have ang spd, not velocity

$\omega_2 \Rightarrow v_2 = r\omega_2$; instant center to get v_2

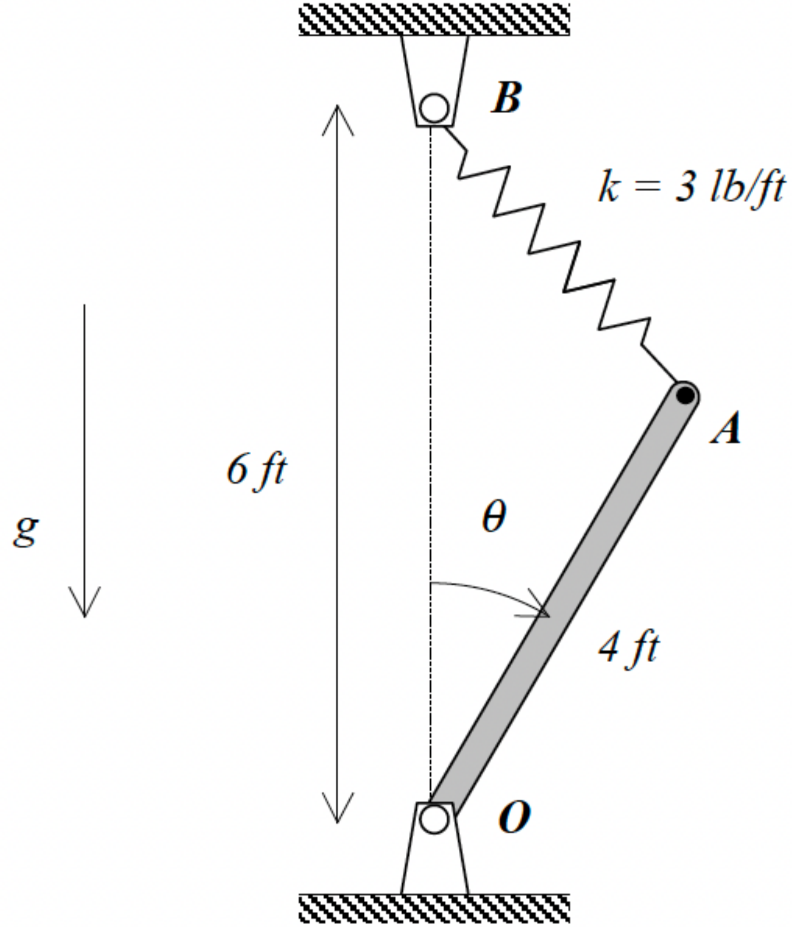
(5) Solve

$\vec{v}_2 = v_2(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$; get $\vec{v}$

Example 5.B.3

Given: The spring acting on a thin, homogeneous bar (of weight $W = 15$ lb) is unstretched when $\theta = 0$.

Find: Determine the angular speed of the bar at $\theta = 0$, if the bar just reaches $\theta = 90°$ before coming to rest.



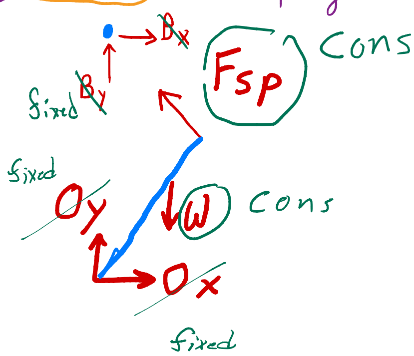
[pg. 332]

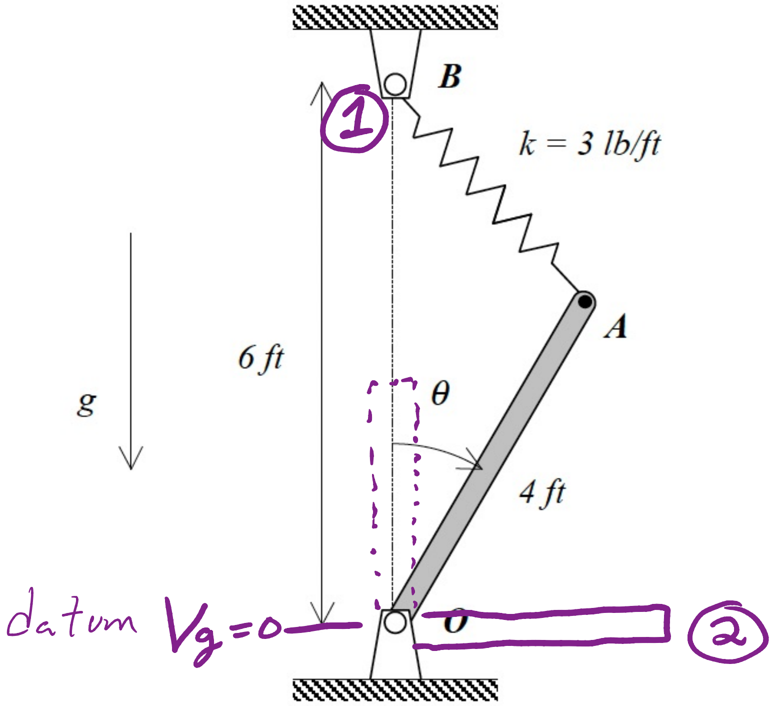
Example 5.B.3 p. 332 H.S. 6 due MONDAY

Given: The spring acting on a thin, homogeneous bar (of weight $W = 15$ lb) is unstretched when $\theta = 0$.

Find: Determine the angular speed of the bar at $\theta = 0$, if the bar just reaches $\theta = 90°$ before coming to rest. $\omega_1 = ?$

① FBD bar + spring





② Kinetics W/E Eqn

$$T_1 + V_1 + \cancel{U^{NC}_{1\to2}}^{\,0} = \cancel{T_2}^{\,0} + V_2$$

$T_1 = \frac{1}{2} I_O \omega_1^2$; pick point O

$= \frac{1}{2}(I_G + m d_{OG}^2)\omega_1^2$; P.A.T

$= \frac{1}{2}\left(\frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2\right)\omega_1^2$; I_G = table

$= \frac{1}{6}mL^2\omega_1^2$

$V_1 = mg\frac{L}{2}$; datum & spring unstretched

$U^{NC}_{1\text{-}2} = 0$; mech energy is cons

$T_2 = 0$; @ rest

$V_2 = \frac{1}{2}k\Delta^2$; datum

$= \frac{1}{2}k\left[\sqrt{6^2+L^2} - (6-L)\right]^2$

③ Kinematics

$\Delta = L - L_o$

$= \sqrt{6^2+L^2} - (6-L)$

④ 1 eqn 1 unkn

⑤ Solve

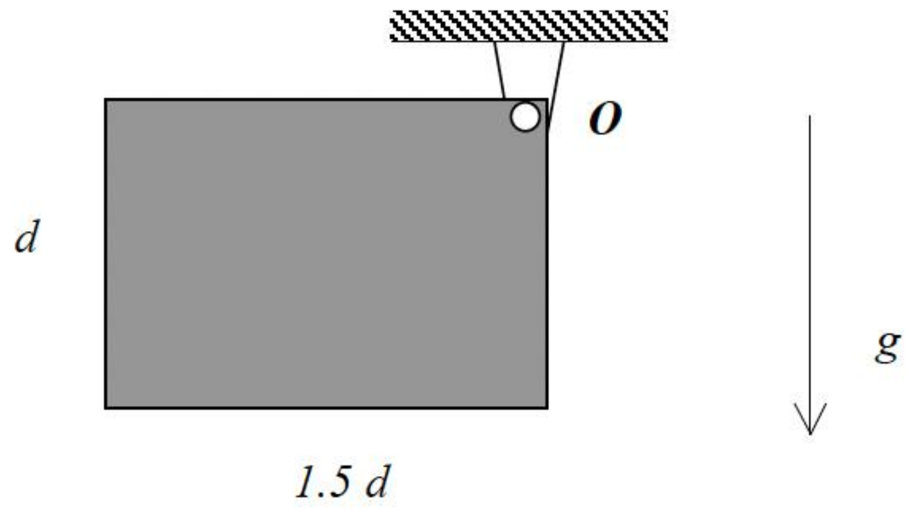
$\Rightarrow \omega_1$

p. 339 (5.B.10) is like H.S.H

Example 5.B.4

Given: A homogeneous, rectangular plate is released from rest with its upper surface being horizontal. Assume the pin at O to be smooth.

Find: Determine the maximum angular velocity of the plate as it rotates about its pinned support at O.



[pg. 333]

Example 5.B.4

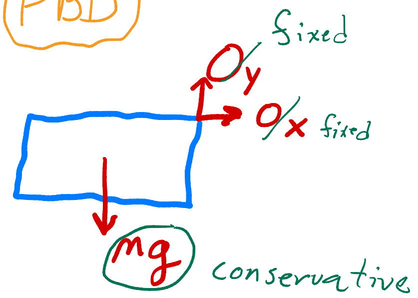
p.333

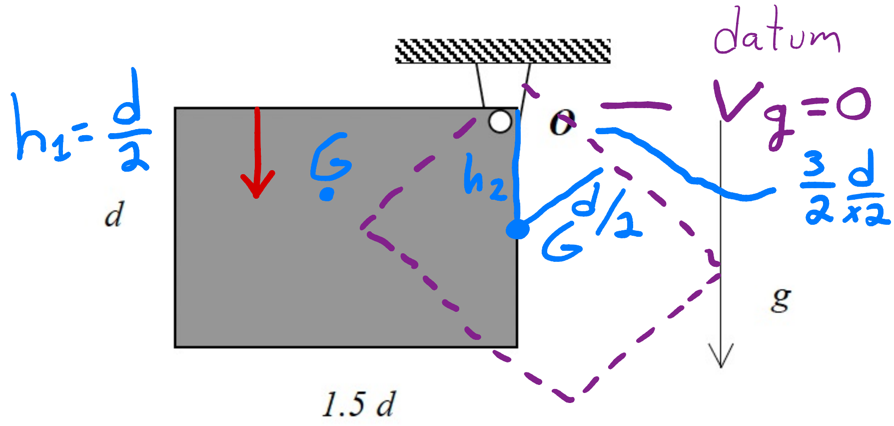
RFR

Given: A homogeneous, rectangular plate is released from rest with its upper surface being horizontal. Assume the pin at O to be smooth.

Find: Determine the maximum angular velocity of the plate as it rotates about its pinned support at O. $\vec{\omega}_2$?

1 FBD





3 Kinematics

$$h_2^2 = \left(\frac{3d}{4}\right)^2 + \left(\frac{d}{2}\right)^2$$

2 Kinetics

$$T_1 + V_1 + U_{1\to 2}^{NC} = T_2 + V_2$$

$T_1 = 0$; R.F.R.

$V_1 = -mg\frac{d}{2}$; datum

$U_{1\to 2}^{NC} = 0$; mech energy is cons

$$T_2 = \frac{1}{2} I_O \omega_2^2$$

$$= \frac{1}{2}(I_G + m h_2^2)\omega_2^2 \quad ; \text{ P.A.T}$$

$$= \frac{1}{2}\left(\frac{1}{12} m\left((1.5d)^2 + d^2\right) + m h_2^2\right)\omega_2^2 \quad ; \text{ use Kinematics / geometry}$$

$V_2 = -mgh_2$; datum

4 Solve

$\Rightarrow \omega_2$

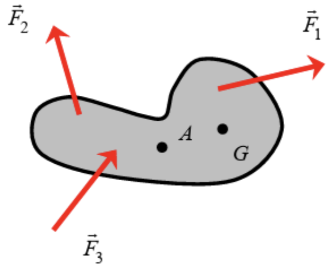
$\Rightarrow \vec{\omega}_2 = \omega_2 \hat{k}$

Summary: Work/Energy Equation 1

[pg. 329]

FUNDAMENTAL equation: $T_2 + V_2 = T_1 + V_1 + U_{1\to2}^{(nc)}$
with:

$$T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \bullet \left(\vec{\omega}\times\vec{r}_{G/A}\right)$$



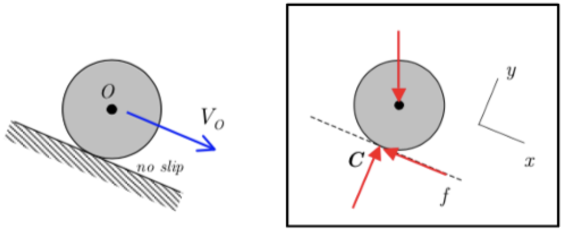
SPECIAL CHOICES FOR POINT "A": If A is *EITHER* the c.m. *OR* a fixed point, then the kinetic energy equation reduces to:

$$T = \underbrace{\boxed{\frac{1}{2}mv_A^2}}_{\text{translation}} + \underbrace{\boxed{\frac{1}{2}I_A\omega^2}}_{\text{rotation}}$$

PARALLEL AXIS THEOREM: As with the Newton/Euler equation, you will need to use the PAT if you choose A to be anything other than the c.m.

SYSTEM CHOICE: Make your system BIG! Include as many components within system to make workless forces INTERNAL (no work on system). Different choice than for Newton/Euler.

ROLLING WITHOUT SLIPPING: The friction force at a no-slip point does not work. Why? Recall that the no-slip point is stationary – no work is done on a stationary point.



Lec 30 Short Feedback Form: