ME 274: Basic Mechanics II

Lecture 16: Particle Kinetics – Newton's Laws

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In our study of **kinematics**, the focus has been on describing **how bodies are moving**

$$\textit{Cartesian description}: \vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} \qquad \textit{Path description}: \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

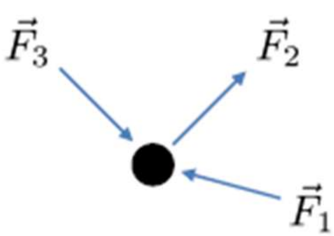
$$\textit{Polar description}: \vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

In our study of **kinetics**, we will now look at why bodies are moving – i.e. what external forces are acting on the system, and how they impact the motion we describe with our kinematic descriptions.

The fundamental principles that form the basis of our analysis are Newton's Laws of motion.

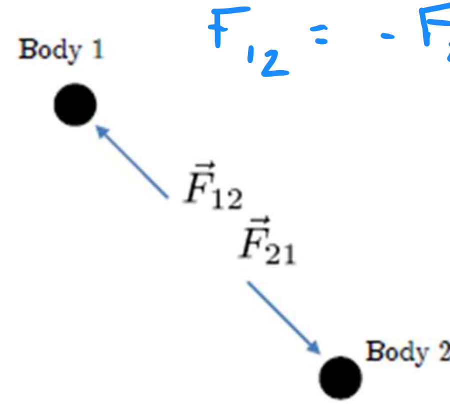
Newton's 2nd Law: The vector resultant of forces on a body equals the time rate of change of linear momentum. $= m\vec{v}$

$\Sigma F = m\vec{a}$



Newton's 3rd Law: Two interacting bodies will exert equal and opposite forces on each other.

$\vec{F}_{12} = -\vec{F}_{21}$



Newton's 2nd Law in 3 kinematic descriptions:

Cartesian Coordinates

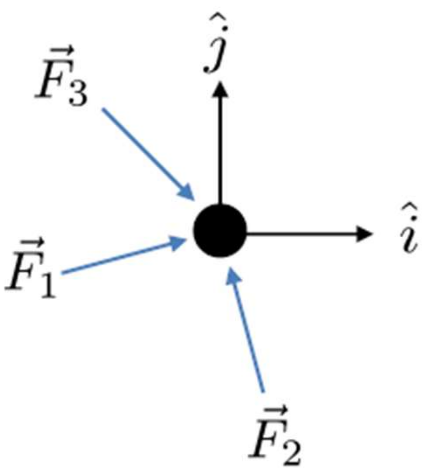
$$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$

Sum forces: $\sum \vec{F} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:

$$\Sigma F_x = m\ddot{x}$$

$$\Sigma F_y = m a_y = m\ddot{y}$$



Path Coordinates

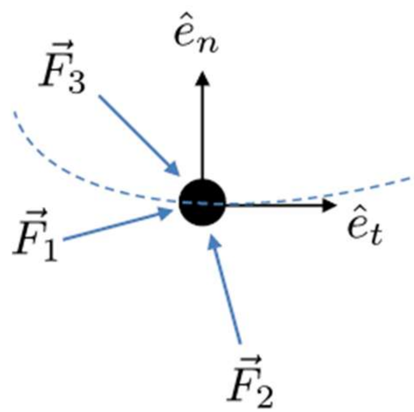
$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

Sum forces: $\sum \vec{F} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:

$$\Sigma F_t = m a_t = m\dot{v}$$

$$\Sigma F_n = m a_n = m\frac{v^2}{\rho}$$



Polar Coordinates

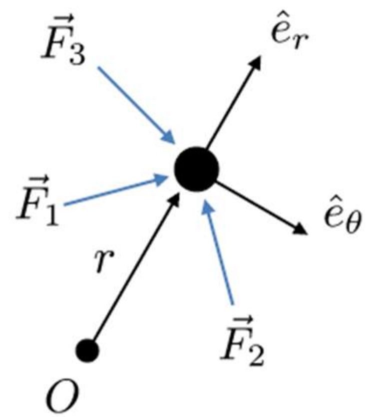
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\,\dot{r}\dot{\theta})\hat{e}_\theta$$

Sum forces: $\sum \vec{F} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:

$$\Sigma F_r = m a_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$



Kinetics: Four-Step Problem Solving Method

Suggested plan of action for solving kinetics problems:

1. ***Free body diagram(s).*** Draw the appropriate free body diagrams (FBDs) for the problem. Your choice of FBDs is problem dependent. For some problems, you will draw an FBD for each body; for others, you will draw an FBD for the entire system. An integral part of your FBDs is your choice of coordinate system. For each FBD, draw the unit vectors corresponding to your coordinate choice.

2. ***Kinetics equations.*** At this point, you will need to choose what solution method(s) that you will need to use for the particular problem at hand. In this section of the course we will study four basic methods: Newton/Euler, work/energy, linear impulse/momentum and angular impulse/momentum. Based on your choice of method(s), write down the appropriate equations from your FBD(s) from Step 1. $F = m\vec{a}$

3. ***Kinematics.*** Perform the needed kinematic analysis. A study of the equations in Step 2 above will guide you in deciding what kinematics are needed for a solution of the problem.

4. ***Solve.*** Count the number of unknowns and the number of equations from above. If you do not have enough equations to solve for your unknowns, then you either: (i) need to draw more FBDs, OR (ii) you need to do more kinematic analysis. When you have sufficient equations for the number of unknowns, solve for the desired unknowns from the above equations.

ODE, system of eqn.

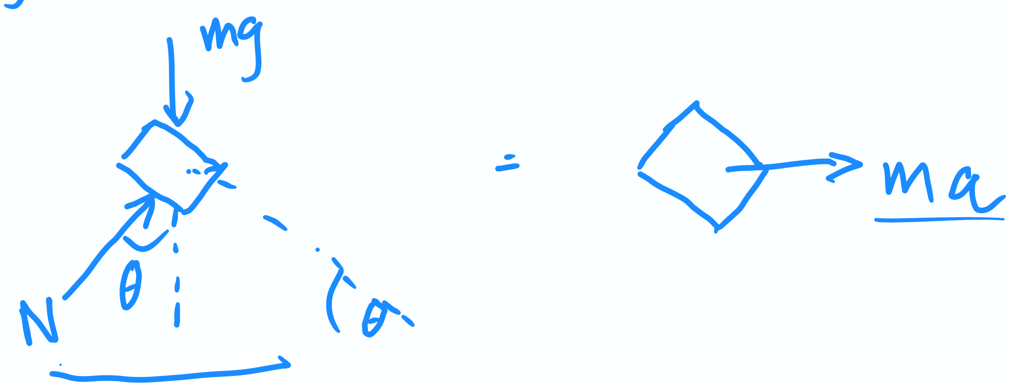
Example 4.A.4 x y - coordinates

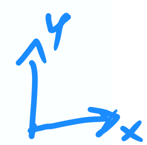
Given: A collar P of mass m is free to slide along a smooth rod that is mounted at angle of $\theta = 36.87°$ in a frame. The frame is constrained to move along a horizontal surface, as shown, with a constant acceleration of a.

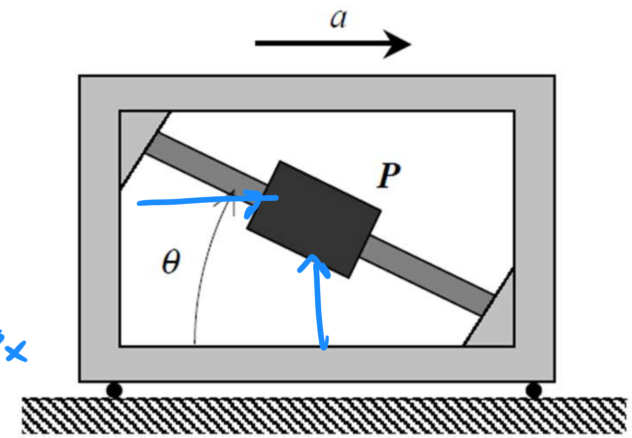
no friction

Find: Determine the value of a that is required for the collar to not slide along the rod as the frame accelerates to the right.

1) FBD







① $\sum F_x = ma = N \sin\theta$

② $\sum F_y = 0 = N\cos\theta - mg \rightarrow N = \dfrac{mg}{\cos\theta}$

$$\cancel{m}a = \frac{\cancel{m}g}{\cos\theta}\sin\theta$$

$$a = g\tan\theta$$

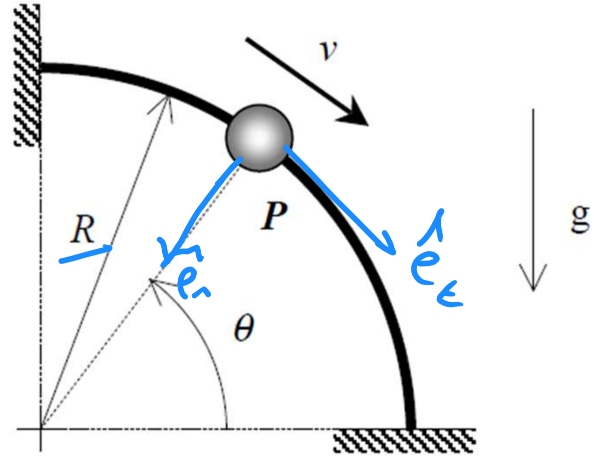
Example 4.A.7 path coordinates

no friction

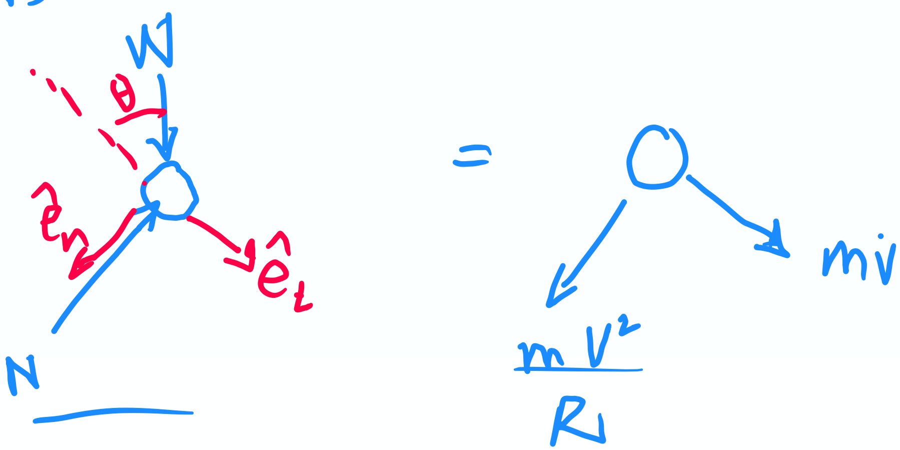
Given: Particle P, having a weight of $W = 5$ lb, slides along a smooth, curved rod where $R = 3$ ft. At the position where $\theta = 53.13°$, the speed of P is known to be 20 ft/s.

Find: Determine:

(a) The normal force acting on P by the rod at the instant shown; and
(b) The rate of change of speed of P at the instant shown.



1) FBD



$$m = \frac{W}{g}$$

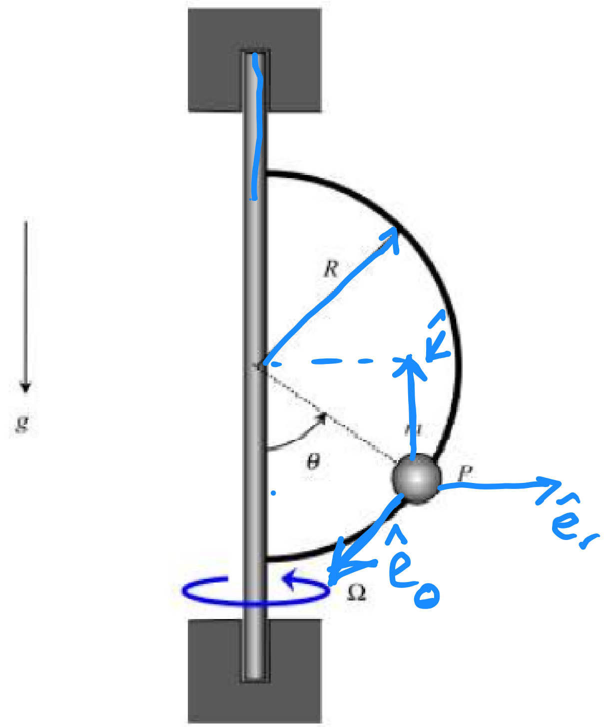
① $\sum F_t = W\cos\theta = m\dot{v} \rightarrow \dot{v} = \frac{W\cos\theta}{m} \Rightarrow \dot{v} = g\cos\theta$

② $\sum F_n = W\sin\theta - N = \frac{mv^2}{R}$ $\quad \vec{N} = \left(W\sin\theta - \frac{Wv^2}{gR}\right)\hat{e}_n$

Homework 4.A.17

Given: A rigid semi-circular guide of radius R is attached to a vertical shaft, with the shaft rotating with a constant rate of Ω. Particle P (of mass m) is able to slide on the guide.

Find: If the guide is smooth, determine the angle θ at which the particle will not slide on the guide.



ΣF_r

ΣF_θ

ΣF_z

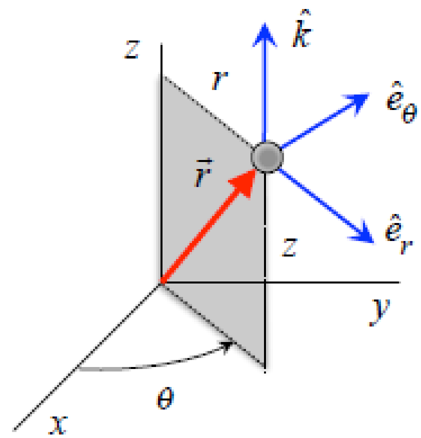
Please leave your answer in terms of, at most, g, m, R and Ω.

Cylindrical Kinematics

For the cylindrical description, the following set of unit vectors will be used:

- unit vector $\hat{k}$ is aligned with the z-axis and is stationary
- unit vector $\hat{e}_r$ is perpendicular to $\hat{k}$ and is aligned with the projection of the position vector $\vec{r}$ onto the xy plane (see the figure below)
- unit vector $\hat{e}_\theta$ is found from $\hat{e}_\theta = \hat{k} \times \hat{e}_r$.

r is the radial distance from the z axis to the particle. θ is measured from the positive x-axis.



The projection of the $\vec{r}$ vector onto the xy plane from above shows the same view as for the 2D polar description discussed earlier. From that work we saw that:

$$\frac{d\hat{e}_r}{dt} = \dot{\theta}\hat{e}_\theta$$

$$\frac{d\hat{e}_\theta}{dt} = -\dot{\theta}\hat{e}_r$$

In 3D, we write the position vector of P as:

$$\vec{r} = r\hat{e}_r + z\hat{k}$$

From this, the velocity of the particle is:

$$\vec{v} = \frac{d}{dt}(r\hat{e}_r) + \dot{z}\hat{k} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + \dot{z}\hat{k}$$

and the acceleration is:

$$\vec{a} = \frac{d}{dt}(\dot{r}\hat{e}_r) + \frac{d}{dt}\left(r\dot{\theta}\hat{e}_\theta\right) + \ddot{z}\hat{k} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta + \ddot{z}\hat{k}$$

From this, we see that the velocity and acceleration equations for the cylindrical description are the same as those for the polar description with the addition of $\dot{z}\hat{k}$ and $\ddot{z}\hat{k}$, respectively.

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Lecture 17: Particle Kinetics – Newton's Laws

School of Mechanical Engineering

Announcements:

- Hw 4.A, 4.B extended to Tuesday at 11:59 PM!
- Thursday office hours canceled this week – see me by appointment if needed!

Newton's 2nd Law in 3 kinematic descriptions:

Cartesian Coordinates

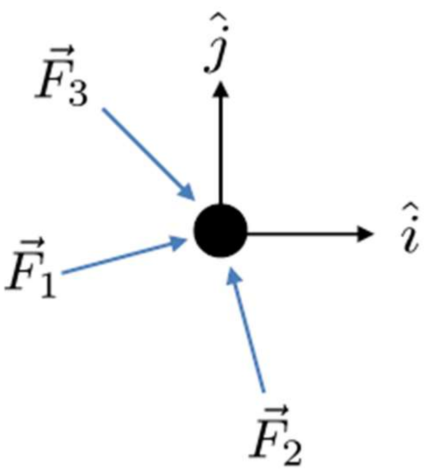
$$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$

Sum forces: $\sum \vec{\boldsymbol{F}} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:

$$\sum F_x = m\ddot{x}$$

$$\sum F_y = m\ddot{y}$$



Path Coordinates

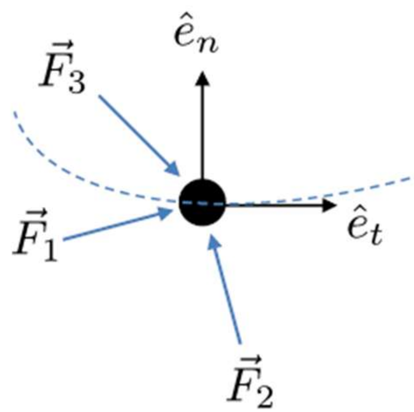
$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

Sum forces: $\sum \vec{\boldsymbol{F}} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:

$$\sum F_t = m\dot{v}$$

$$\sum F_n = m\frac{v^2}{\rho}$$



Polar Coordinates

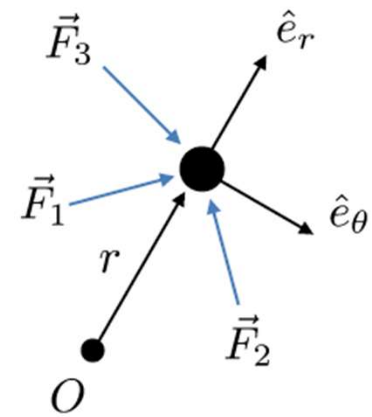
$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\,\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

Sum forces: $\sum \vec{\boldsymbol{F}} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:

$$\sum F_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$\sum F_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$



Kinetics: Four-Step Problem Solving Method

1) FBD
    - Draw appropriate FBD (note- you may have to draw more than 1)
    - Choose coordinate system (cartesian, path, polar)
2) Kinetics
    - Sum forces and break into components
    - Choose appropriate method to solve the problem – we will learn these in upcoming sections!
        - Newton/Euler
        - work/energy
        - linear impulse/momentum
        - angular impulse/momentum)
3) Kinematics
    - Perform a kinematic analysis of the system using techniques we have developed in the previous chapters.
    - Use your kinetics equations from step 2 to determine what information you need to solve the problem
4) Solve
    - Count the number of equations and unknowns. Do they match?
    - If not:
        - Draw more FBDs
        - Do additional kinematic analysis

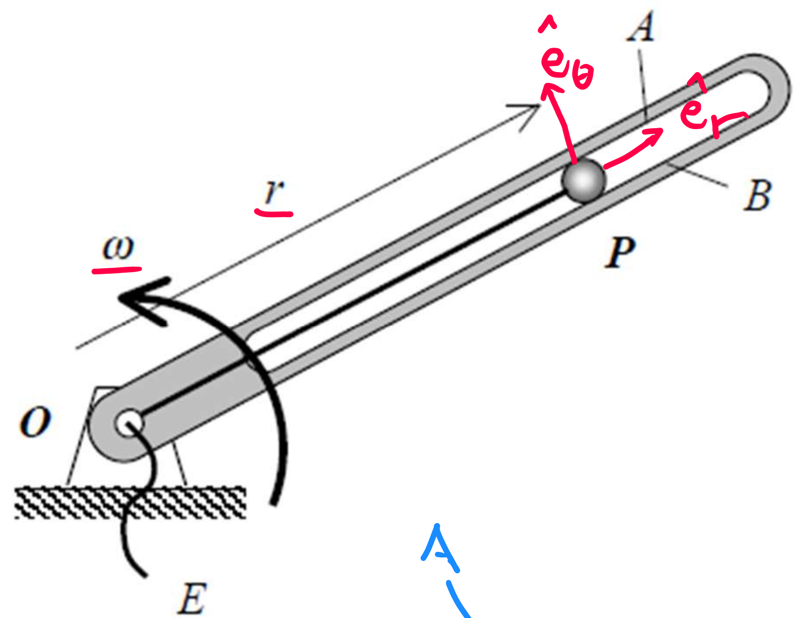
Example 4.A.10

Given: Particle P (weighing W) is able to slide within a straight slot cut into an arm. The arm is rotating within a horizontal plane about end O at a constant rate of ω. The slider is being pulled toward O at a constant rate of $\dot{r}$.

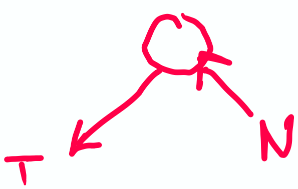
Find: Determine:

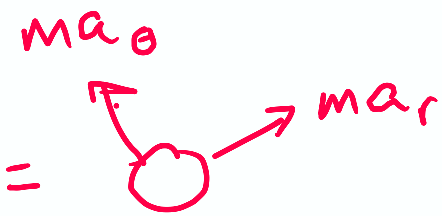
(a) The tension in the cord;
(b) The normal contact force of the slot on P; and
(c) Which side of the slot (A or B) is P in contact with?

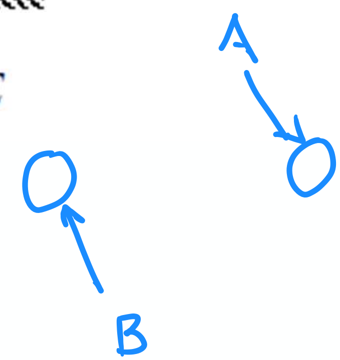
Use the following parameters in your analysis:
$W = 5$ lb, $\omega = 5$ rad/s, $r = 0.75$ ft and $\dot{r} =$ -0.5 ft/s.



1) FBD







2) Kinetics

$$\Sigma F_r = -T = m a_r$$

$$\Sigma F_\theta = N = m a_\theta$$

3) kinematics

polar form : $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta$

$\underbrace{\ddot{r} - r\dot{\theta}^2}_{a_r}$ $\quad \underbrace{2\dot{r}\dot{\theta} + r\ddot{\theta}}_{a_\theta}$

① $-T = m(\ddot{r} - r\dot{\theta}^2)$

② $N = m(2\dot{r}\dot{\theta} + \underline{r\ddot{\theta}})$

4) solving → 2 eqn, 2 unknown

$r = 0.75\text{ ft}$ $\quad \dot{\theta} = \omega = \text{const.}$ $\quad m = \frac{W}{g}$

$\dot{r} = \underline{-0.5\text{ft}} = \text{const.}$ $\quad \ddot{\theta} = 0$

$\ddot{r} = 0$

$$\boxed{\begin{aligned} T &= -\frac{W}{g}(r\omega^2) \\ N &= \frac{W}{g}(2\dot{r}\omega) \end{aligned}}$$

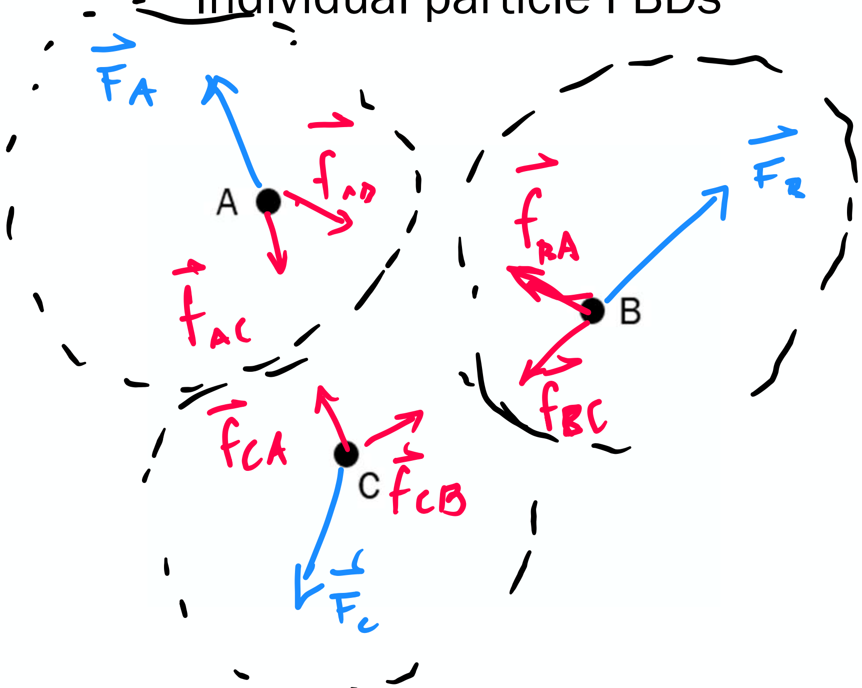
→ $N < 0$ → switch direction from FBD → side A

So far we have seen problems including a single particle where $\sum \boldsymbol{F}_{particle} = \boldsymbol{m}_{particle}\boldsymbol{a}_{particle}$

How does our kinetic analysis change when we are analyzing a system with multiple interacting bodies?

Consider these 3 interacting particles:

Individual particle FBDs

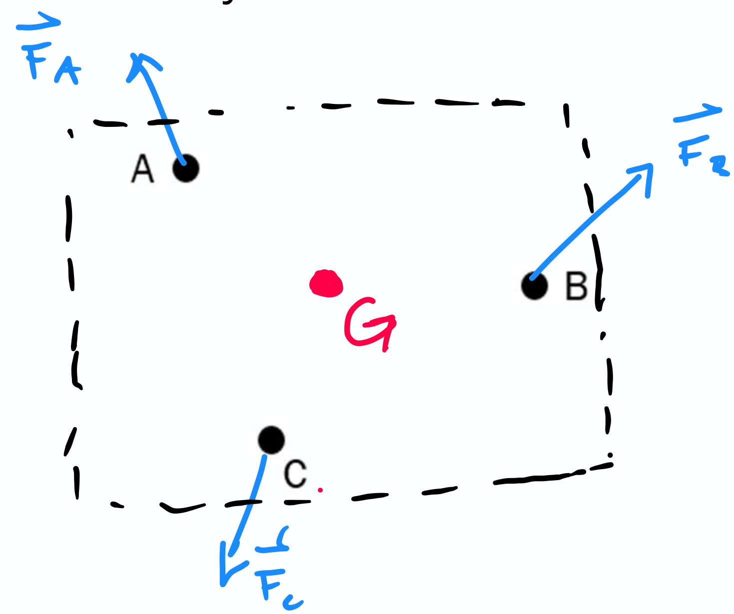


$$m_A\vec{a}_A = \vec{F}_A + \vec{f}_{AB} + \vec{f}_{AC}$$

$$m_B\vec{a}_B = \vec{F}_B + \vec{f}_{BA} + \vec{f}_{BC}$$

$$m_C\vec{a}_C = \vec{F}_C + \vec{f}_{CA} + \vec{f}_{CB}$$

System FBDs



$$m_A\vec{a}_A + m_B\vec{a}_B + m_C\vec{a}_C$$

$$= \vec{F}_A + \cancel{\vec{f}_{AB}} + \cancel{\vec{f}_{AC}} + \vec{F}_B + \cancel{\vec{f}_{BA}} + \cancel{\vec{f}_{BC}}$$

$$+ \vec{F}_C + \cancel{\vec{f}_{CA}} + \cancel{\vec{f}_{CB}}$$

$$\vec{f}_{AB} = -\vec{f}_{BA} \ldots$$

$$\vec{m}_A\vec{a}_A + \vec{m}_B\vec{a}_B + \vec{m}_C\vec{a}_C = \vec{F}_A + \vec{F}_B + \vec{F}_C$$

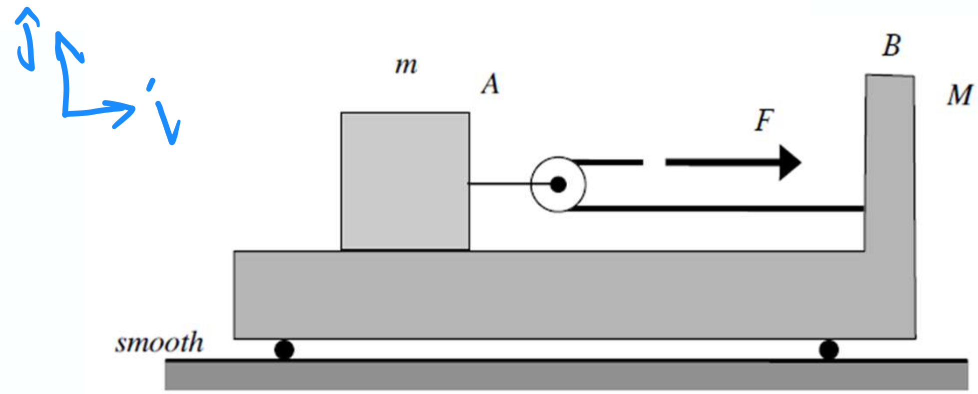
$$\sum_j m_j \vec{a}_j = \sum_j \vec{F}_j^{ext}$$

$$M\vec{a}_G = \sum F^{ext}$$

Example 4.A.6 → pulley

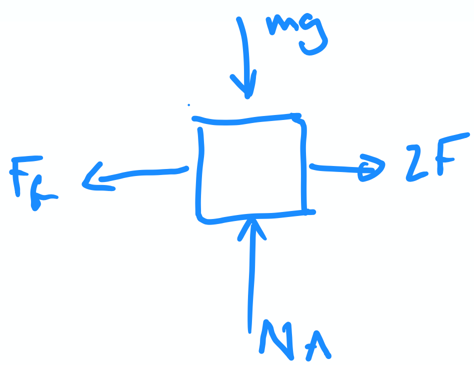
Given: The system of blocks A and B shown below is initially at rest when a force F is applied to the free end of the cable.

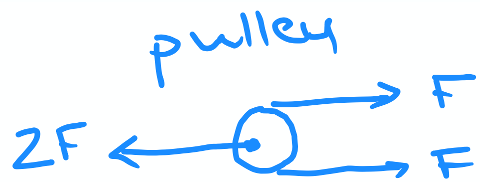
Find: Determine the accelerations of blocks A and B: (a) if the surface between A and B is smooth, and (b) if the surface between A and B is rough with a coefficient of kinetic friction being μ_k, and where it is known that block A slips on block B.

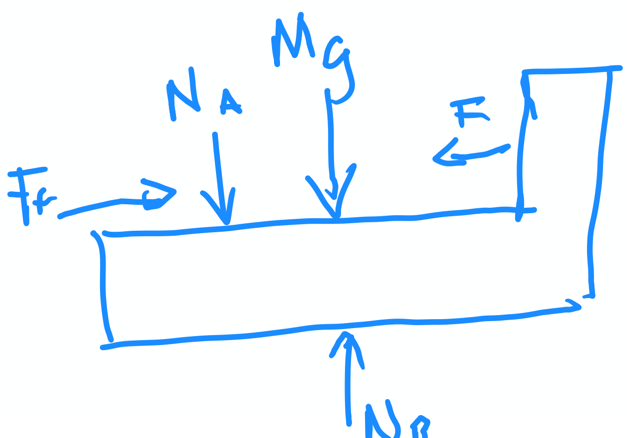


1) FBD

Block A







2) Kinetics

① $\sum F_{A,x} = 2F - F_f = m a_{Ax}$ $\quad$ $\sum F_{Ay} = 0 = N_A - mg$

② $\sum F_{B,x} = F_f - F = M a_{Bx}$ $\quad$ $\sum F_{By} = 0$

3) Kinematics $\quad a_x = \ddot{x} \quad a_y = 0$

4) solve

a) smooth $\rightarrow F_f = 0$

① $2F = m a_{Ax} \rightarrow a_{Ax} = \frac{2F}{m}$

② $-F = M a_{Bx} \rightarrow a_{Bx} = -\frac{F}{M}$

b) rough $\rightarrow F_f = \mu_k N_A \quad N_A = mg$

① $2F - \mu_k mg = m a_x \rightarrow a_A = \frac{2F - \mu_k mg}{m} \hat{\imath}$

② $\mu_k mg - F = M a_B \Rightarrow a_B = \frac{\mu_k mg - F}{M} \hat{\imath}$

Example 4.A.8 rigid 2Force Member

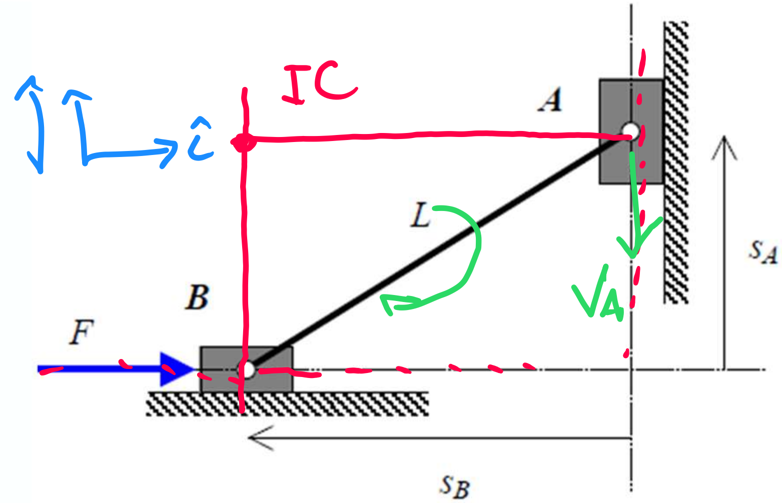
Given: Blocks A and B (having masses of 10 kg and 5 kg, respectively) are constrained to move along smooth, vertical and horizontal guides, as shown in the figure. A and B are connected by a lightweight rod of length $L = 2.5$ m. A force of $F = 50$ N acts to the right on block B. At the position where $s_A = 1.5$ m, A is moving downward with a speed of 4 m/s.

smooth = no friction

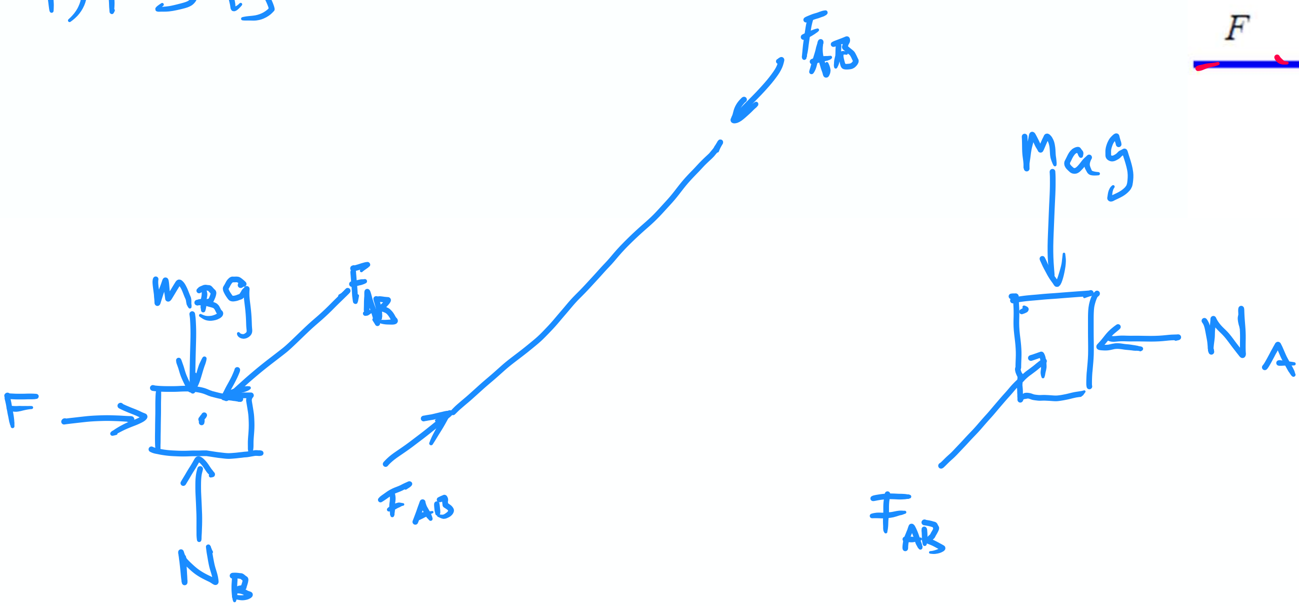
Find: Determine:

(a) The acceleration of A and B at this instant; and

(b) The force in rod AB at this instant.



1) FBD



Kinetics

Block B

$$\sum F_{x,B} = F - F_{AB}\frac{s_B}{L} = m_b a_{bx} \qquad (1)$$

$$\sum F_{y,B} = N_B - m_B g - F_{AB}\frac{s_A}{L} = \cancel{m_b a_{by}} \qquad (2)$$

Block A

$$\sum F_{x,A} = F_{AB}\frac{s_B}{L} - N_A = \cancel{m_A a_{Ax}} \qquad (3)$$

$$\sum F_{y,A} = F_{AB}\frac{s_A}{L} - m_A g = m_a a_{Ay} \qquad (4)$$

Kinematics

$$a_{Ax} = 0, \quad a_{By} = 0$$

rigid body eqn $\rightarrow$ Link AB

relate $\underline{V_A}$, ω_{AB} using IC ↑ see above

Given

$$\omega_{AB} = \frac{V_A}{|r_{AC}|} = \frac{V_A}{S_B}$$

relate acceleration of pts A & B

$$\vec{a}_A = \vec{a}_B + \vec{\alpha}_{AB} \times \vec{r}_{A/B} - \omega^2 r_{AB}$$

$$a_A \hat{j} = a_B \hat{i} + \alpha_{AB} \hat{k} \times (S_B \hat{i} + S_A \hat{j}) - \left(\frac{V_A}{S_B}\right)^2 (S_B \hat{i} + S_A \hat{j})$$

$$\hat{i}: \quad 0 = a_B - S_A \alpha_{AB} - \frac{V_A^2}{S_B}$$

$$\hat{j}: \quad a_A = \alpha_{AB} S_B - \left(\frac{V_A}{S_B}\right)^2 S_A$$

Treating AB as an inextensible cable :-

$$L^2 = s_A^2 + s_B^2$$

$$\frac{d}{dt}(L^2) = 2s_A\dot{s}_A + 2s_B\dot{s}_B = 0 \rightarrow \dot{s}_B = -\frac{s_A}{s_B}\dot{s}_A$$

$$\frac{d}{dt}(s_A\dot{s}_A + s_B\dot{s}_B) = s_A\ddot{s}_A + \dot{s}_A^2 + s_B\ddot{s}_B + \dot{s}_B^2$$

$$\ddot{s}_A = \frac{-(\dot{s}_A^2 + \dot{s}_B^2 + s_B\ddot{s}_B)}{s_A} \quad 5$$

$$\ddot{s}_A = a_A, \quad \ddot{s}_B = a_B$$

5 eqn, 5 unknown

→ Solve!

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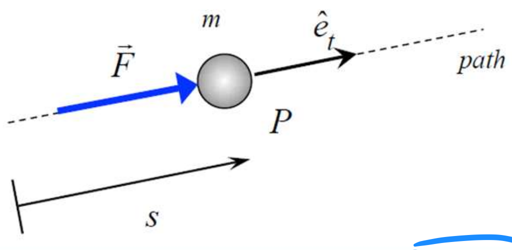
    - Use your kinetics equations from step 2 to determine what information you need to solve the problem
4) Solve
    - Count the number of equations and unknowns. Do they match?
    - If not:
        - Draw more FBDs
        - Do additional kinematic analysis

Kinetics for the rectilinear Motion of Particles – Net force dependent on time

For a particle of mass m traveling along a straight path and experiencing a force $\vec{F} = F\hat{e}_t$



$$\sum F_t = ma_t \quad \Rightarrow \quad F = m\frac{dv}{dt}$$

already in terms of t

If force is a function of time: $F = F(t)$

$$F = m\frac{dv}{dt}$$

$$F(t)\,dt = m\,dv$$

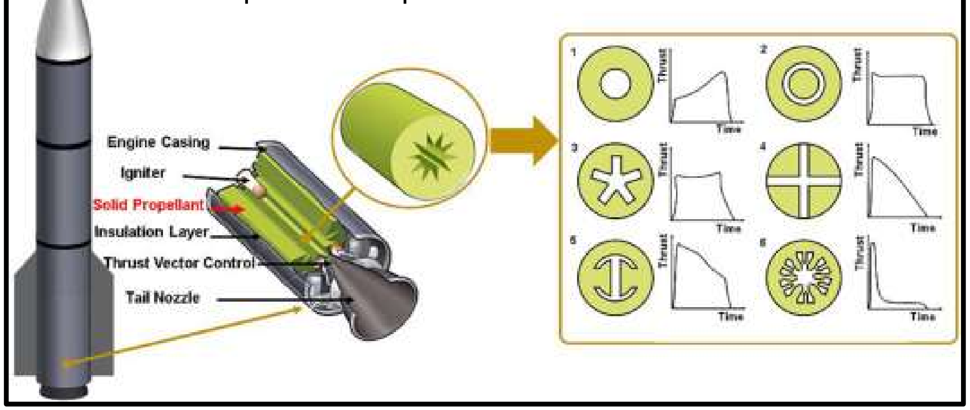
$$\int_{t_1}^{t_2} F(t)\,dt = \int_{v_1}^{v_2} m\,dv$$

$$\int_{t_1}^{t_2} F(t)\,dt = m(v_2 - v_1)$$

impulse - momentum

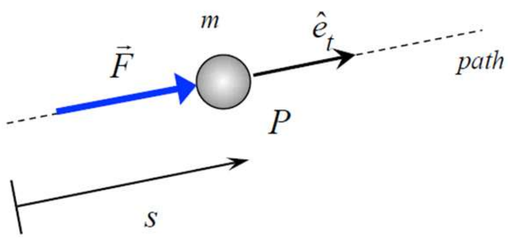
tells how speed changes with time

Example: thrust profiles of solid rocket motors



Kinetics for the rectilinear Motion of Particles – Net force dependent on **position**

For a particle of mass m traveling along a straight path and experiencing a force $\vec{F} = F\hat{e}_t$



$$\sum F_t = ma_t \quad \Rightarrow \quad F = m\boxed{\frac{dv}{dt}}$$ ← need to write in terms of s

If force is a function of position: $F = F(s)$

$F = m\frac{dv}{dt}$ → using chain rule

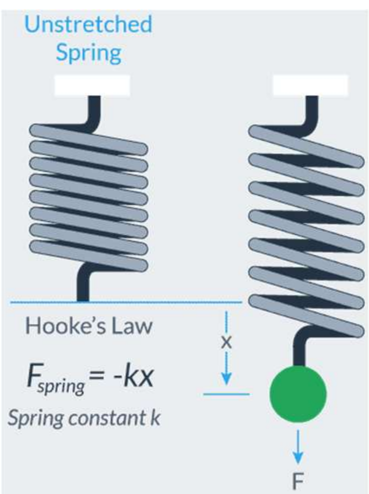
$= m\frac{dv}{ds}\frac{ds}{dt}$ ← velocity!

$F(s) = mv\frac{dv}{ds}$

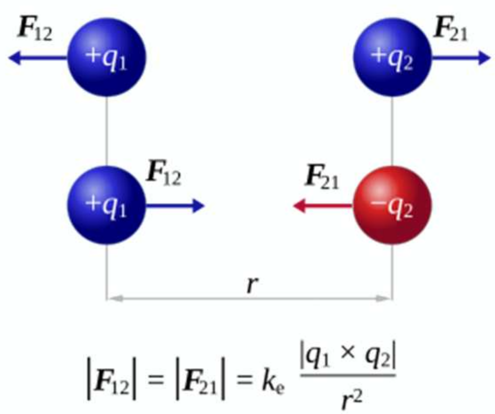
$\int_{s_1}^{s_2} F(s) = \int_{v_1}^{v_2} mv$

$$\underbrace{\int_{s_1}^{s_2} F(s)}_{\text{work}} = \frac{mv^2}{2}\Big|_{v_1}^{v_2} = \underbrace{\tfrac{1}{2}m(v_2^2 - v_1^2)}_{\text{energy}}$$

Ex: spring forces

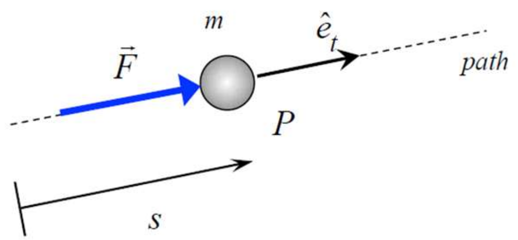


Ex: magnetic forces



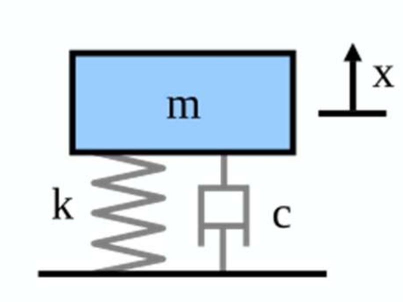
Kinetics for the rectilinear Motion of Particles – Net force dependent on **velocity**

For a particle of mass m traveling along a straight path and experiencing a force $\vec{F} = F\hat{e}_t$



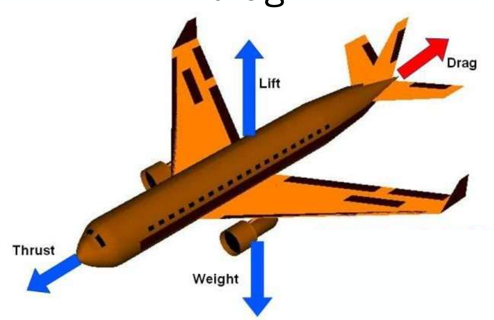
$$\sum F_t = ma_t \quad \Rightarrow \quad F = m\frac{dv}{dt}$$

Ex: dampers



$$F_{damper} = c\dot{x}$$

Ex: drag



$$F_D = \frac{1}{2}\rho v^2 C_D A$$

If force is a function of velocity: $F = F(v)$

$$F(v) = m\frac{dv}{dt}$$

$$\int_{t_1}^{t_2} dt = \int_{v_1}^{v_2} m\frac{dv}{F(v)}$$

velocity changing with time

$$(t_2 - t_1) = m\int_{v_1}^{v_2} F(v)$$

or

$$F(v) = mv\frac{dv}{ds}$$

$$\int_{s_1}^{s_2} ds = m\int_{v_1}^{v_2}\frac{v}{F(s)}dv$$

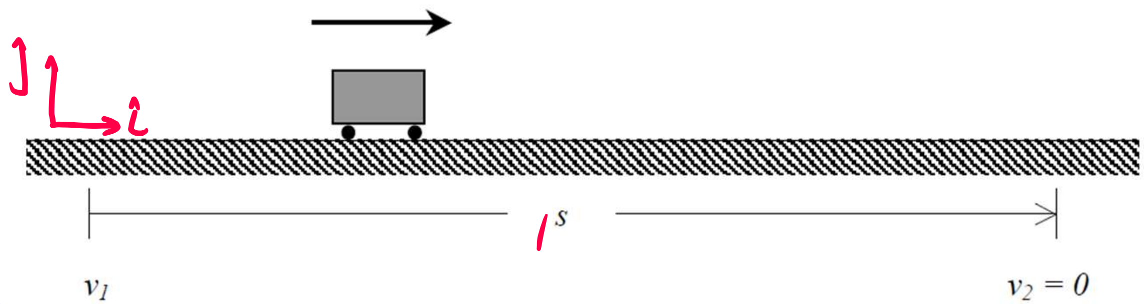
$$\underbrace{(s_2 - s_1) = m\int_{v_1}^{v_2}\frac{v}{F(s)}dv}_{\text{Velocity changing with position}}$$

Example 4.A.1

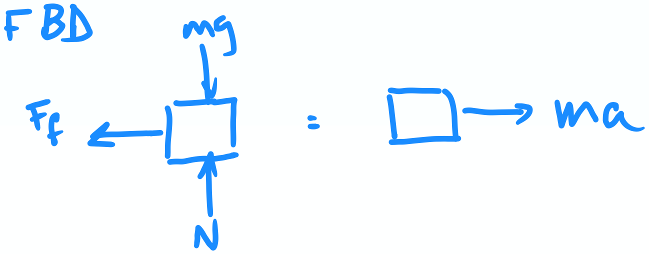
Given: As a car of mass m brakes with a constant braking force F_f, the speed of the car drops from v_1 to a speed of $v_2 = 0$ in a distance of s and in a time t.

Find: Determine:

(a) The distance s; and
(b) The time t.



1) FBD



2) kinetics

$\Sigma F_x = -F_f = ma$

3) kinematics $a = \frac{dv}{dt} = v\frac{dv}{ds}$

4) solve

b) time t to stop

$-F_f = ma = m\frac{dv}{dt}$

$\int_0^{t_2} -F_f\, dt = \int_{v_1}^{0} m\, dv$

$-F_f t\Big|_0^{t_2} = mv\Big|_{v_1}^{0}$

$-F_f t_2 = -mv_1$

$t_2 = \frac{mv_1}{F}$

a) distance s to stop

$-F_f = mv\frac{dv}{ds}$

$\int_0^{s_2} -F_f\, ds = \int_{v_1}^{0} mv\, dv$

$-F_f s\Big|_0^{s_2} = \frac{mv^2}{2}\Big|_{v_1}^{0}$

$-F_f s_2 = -\frac{mv_1^2}{2}$

$s_2 = F_f \frac{mv_1^2}{2}$

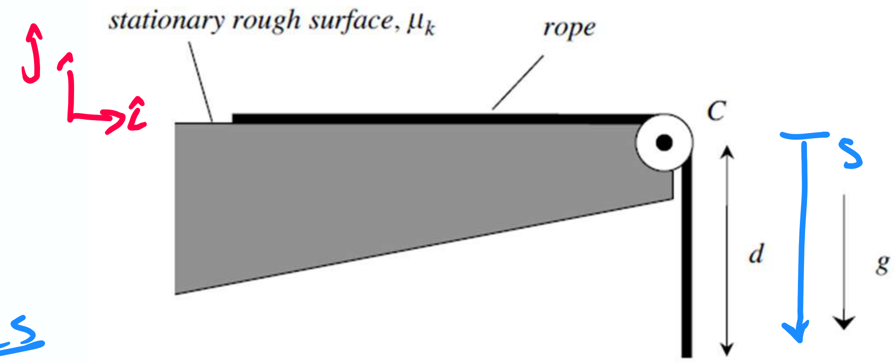
Example 4.A.2

Given: A rope of length L and mass per length ρ is initially at rest on a rough, horizontal surface (with coefficient of kinetic friction μ_k) and with a portion of its length d hanging over a pulley on the right end of the resting surface. Ignore the mass of the pulley.

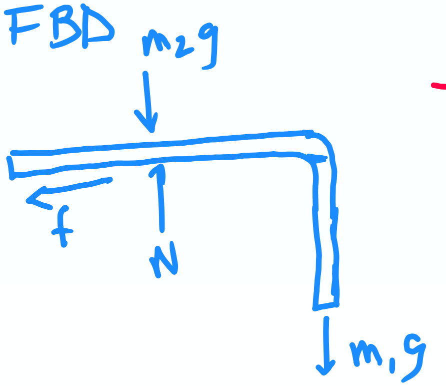
Find: Determine the speed of the rope when the left end of the rope reaches the pulley. Assume that the rope remains taut.

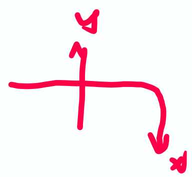
↳ v ↳ $s = L$

Use the following relationship in your analysis: $d = 0.2L$.



1) FBD





2) Kinetics

$$\Sigma F_x = -f + m_1 g = ma \quad (1)$$

$$\Sigma F_y = N - m_2 g = 0 \quad (2)$$

Kinematics

$m_1 = \rho s$

$m_2 = \rho (L-s)$

$m = \rho L$

$f = \mu_k N$

→

$N = \rho (L-s) g$

$f = \mu_k \rho (L-s) g$

$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$

Solve:

$$-\mu_k \cancel{\rho}(L-s)g + \cancel{\rho} s g = \cancel{\rho} L v \frac{dv}{ds}$$

$$g \int_{0.2L}^{L} \left[s(1+\mu_k) - \mu_k L\right] ds = L \int_0^{v_1} v \, dv$$

$$g \left[\frac{s^2}{2}(1+\mu_k) - \mu_k L s \right]_{0.2L}^{L} = L \frac{v^2}{2} \Big|_0^{v_1}$$

$$g \left[\frac{(L^2 - 0.04L^2)}{2}(1+\mu_k) - \mu_k (L^2 - 0.2L^2) \right] = L \frac{v_1^2}{2}$$

$$v_1 = \sqrt{2g \left[\frac{0.96L}{2}(1+\mu_k) - 0.8\mu_k L \right]}$$

simplify further