

ME 274: Basic Mechanics II

Lecture 21: Particle Kinetics – LIM, Central Impact



School of Mechanical Engineering

Applying LIM to Impacts

LIM equation $m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \vec{R} dt$

$m\vec{v}$ - linear momentum of the particle

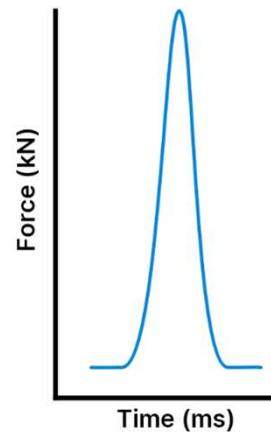
$\int_1^2 \vec{R} dt$ - impulse from the net force acting on the particle

Impacts occur over a very short time interval

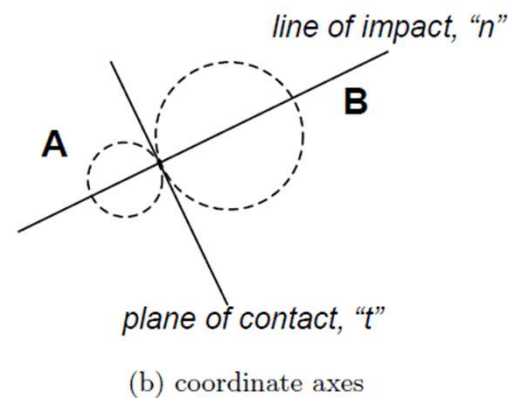
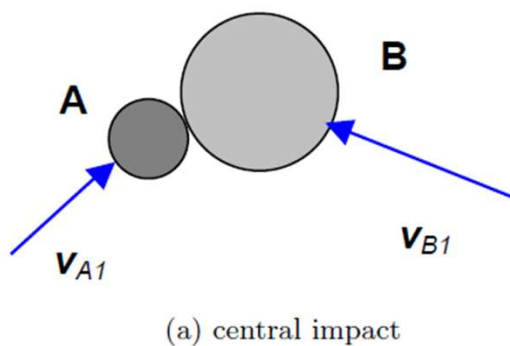
- The impact force between particles is referred to as the **impulsive force** – large magnitude, short duration
- We **neglect** the contributions of **non-impulsive forces** (for example, due to weight or springs during our impact interval Δt)



Impulse on Baseball



Central impact of two particles



Objective: relate the velocities of particles A and B immediately before $(\vec{v}_{A1}, \vec{v}_{B1})$ and immediately after $(\vec{v}_{A2}, \vec{v}_{B2})$ impact

Definitions:

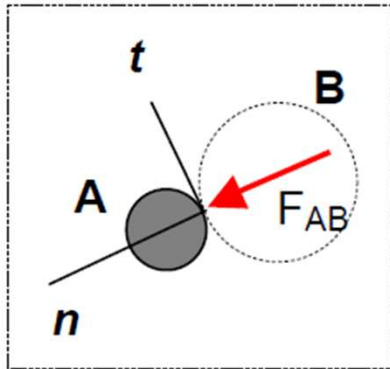
Plane of contact: The plane tangent to the surfaces of A and B

Line of impact: The line perpendicular to the plane of contact for A and B

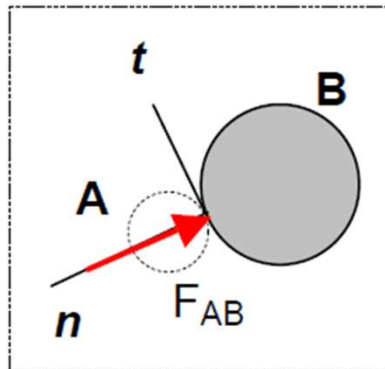
Central impact: an impact in which the line of impact passes through the center of mass of each body

Note: in these problems you will be using the $n - t$ coordinate axis

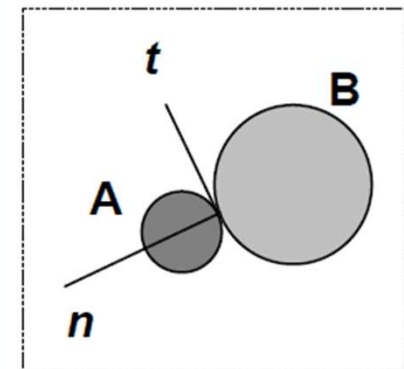
Systems within a contact problem



system A



system B



system AB

Tangent direction:

$$\sum F_t = 0$$

$$m_A v_{At2} = m_A v_{At1} \Rightarrow v_{At2} = v_{At1}$$

Tangent direction:

$$\sum F_t = 0$$

$$m_B v_{Bt2} = m_B v_{Bt1} \Rightarrow v_{Bt2} = v_{Bt1}$$

Tangent direction:

$$\sum F_t = 0$$

$$m_A v_{At2} + m_B v_{Bt2} = m_A v_{At1} + m_B v_{Bt1}$$

Normal direction:

$$\sum F_n = F_{AB} \neq 0$$

Normal direction:

$$\sum F_n = -F_{AB} \neq 0$$

Normal direction:

$$\sum F_n = 0$$

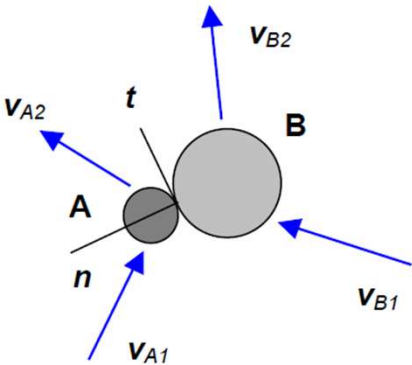
$$m_A v_{An2} + m_B v_{Bn2} = m_A v_{An1} + m_B v_{Bn1}$$

Coefficient of restitution (COR):

$$e = -\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}$$

Central impact problems - Discussion

$$\begin{aligned} (1) \quad & v_{At2} = v_{At1} \\ (2) \quad & v_{Bt2} = v_{Bt1} \\ (3) \quad & m_A v_{An2} + m_B v_{Bn2} = m_A v_{An1} + m_B v_{Bn1} \\ (4) \quad & e = -\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}} \end{aligned}$$



- Equations 1-4 can be used in cases where the impact force is the **only impulse force** between A and B
- The COR equation is only valid in the **normal direction**.
- Mechanical energy in system AB is **not conserved** during impact - you should **never** use work-energy in impact analysis

What is the coefficient of restitution?

The COR is a measure of the elasticity of a collision

- COR = 1 – perfectly elastic collision – energy is conserved (non-physical)
- COR = 0 – perfectly inelastic collision – particles stick together

$$e = -\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}$$

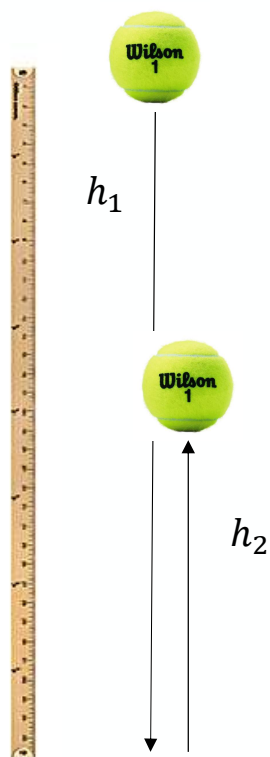
Demo: Calculating the coefficient of restitution (COR) of two balls

We drop a tennis and ping-pong ball from the same height and measure their max height after bouncing. How do we determine the COR?

Weight tennis ball: – 0.125 lbf

Weight ping-pong ball: – 0.006 lbf

Drop height, $h_1 = 3'$

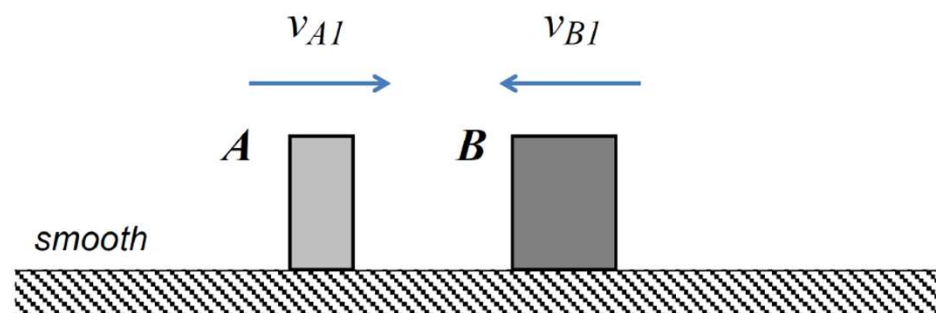


Example 4.C.10

Given: Blocks A and B (having masses of m_A and m_B , respectively) are initially moving to the right and left, respectively, on a smooth horizontal surface with speeds of v_{A1} and v_{B1} , respectively. At some instant in time, A strikes B. The coefficient of restitution of this impact is e . As a result of the impact, block B becomes stationary.

Find: Determine the initial speed v_{B1} of block B.

Use the following parameters in your analysis: $m_A = 2$ kg, $m_B = 3$ kg, $v_{A1} = 1.5$ m/s and $e = 0.4$.

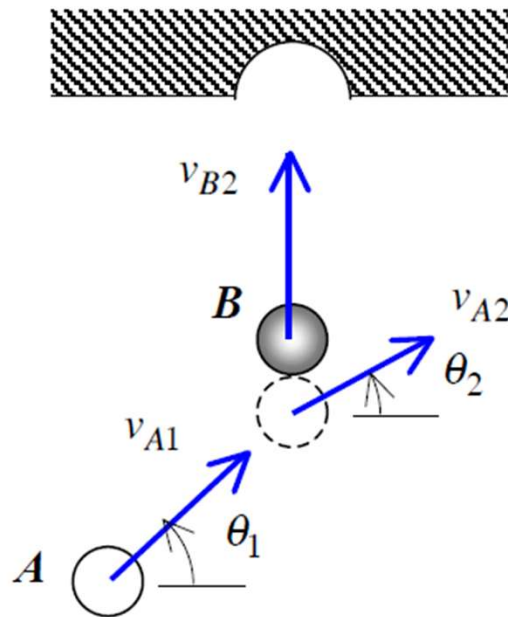


Example 4.C.12

Given: Cue ball A strikes a stationary object ball B, with a speed v_{A1} as shown in the figure below. The coefficient of restitution for this impact is e . After impact, A moves along a line defined by the angle θ_2 , and B moves directly to the side pocket.

Find: Determine the numerical value of the rebound angle θ_2 of A, assuming the masses of A and B are the same.

Use the following parameters in your analysis: $\theta_1 = 45^\circ$ and $e = 0.9$.

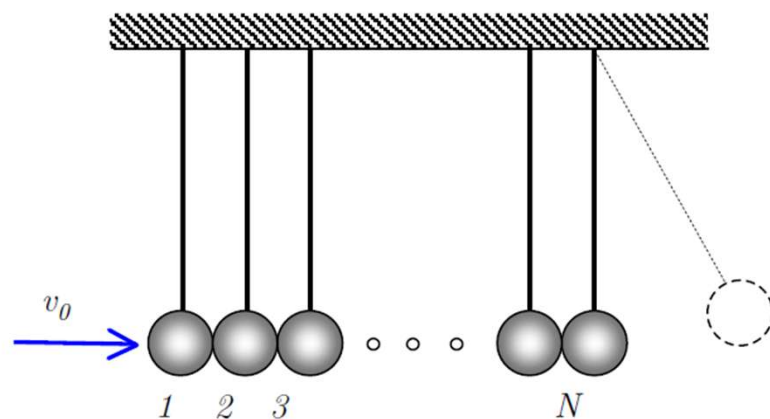


Example 4.C.13

Given: A “Newton’s Cradle” is made up of N identical pendulums. The particle for pendulum 1 has a horizontal velocity of v_0 when it strikes the particle for pendulum 2 (all pendulums except 1 are at rest prior to this impact). The coefficient of restitution for the impacts of all particles is e .

Find: Determine:

- (a) The speed of particle N immediately after the final impact; and
- (b) The maximum height h reached by particle N .



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Lecture 21: Particle Kinetics – Angular Impulse Momentum



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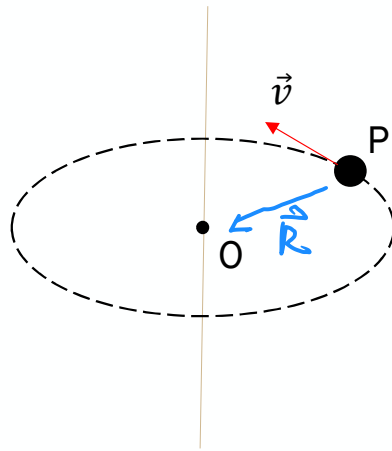
Requirements: 10 credit hours

- 1-credit **foundational course** in dynamical systems
- **Two advanced courses** from a designated list focusing on dynamics, vibrations, or approved research, and choose
- **One additional course** from an extended list covering control systems, signal processing, numerical methods, machine learning, nonlinear systems, aeroelasticity, research, or related topics



Talk to your ME [undergraduate advisor](#) to determine if this concentration is right for you!

Consider a particle P of mass m constrained to rotate about a fixed point O with a velocity of $\vec{v}$.



Remember from LIM:

$$m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \vec{R} dt$$

If $\vec{R} = 0$, $m\vec{v}_2 = m\vec{v}_1 \rightarrow$ Linear momentum is conserved

In this scenario, the speed v of the particle is constant but the direction is changing.

- $\rightarrow$ a net force $\vec{R}$ must be acting on the particle. $\Sigma F \neq 0$
- $\rightarrow$ Linear momentum is not conserved. $m\vec{v}_1 \neq m\vec{v}_2$
- $\rightarrow$ LIM does not simplify the problem.

need a rotation specific analysis method!

Remember from ME 270 that changes in rotational motion are caused by moments (the effect of a force about a point).

here, $v = \text{constant} \rightarrow \Sigma M = 0$

$\rightarrow$ How do we adapt this knowledge to find a rotational analog of the LIM equation?

Deriving the Angular Impulse Momentum Equation

Starting with Newton's 2nd Law:

$$\vec{R} = \sum \vec{F} = m\vec{a} = m \frac{d\vec{v}_P}{dt}, \quad \vec{v}_P = \frac{d\vec{r}_{P/O}}{dt}$$

The moment of the force $\vec{R}$ about point O is found as:

$$\vec{M}_O = \vec{r}_{P/O} \times \vec{R}$$

Substituting in Newton's 2nd Law:

$$\vec{M}_O = \vec{r}_{P/O} \times \left(m \frac{d\vec{v}_P}{dt} \right)$$

Recall the time derivative of the cross product of two vectors is:

$$\frac{d}{dt}(\vec{a} \times \vec{b}) = \frac{d\vec{a}}{dt} \times \vec{b} + \vec{a} \times \frac{d\vec{b}}{dt} \Rightarrow \vec{a} \times \frac{d\vec{b}}{dt} = \frac{d}{dt}(\vec{a} \times \vec{b}) - \frac{d\vec{a}}{dt} \times \vec{b}$$

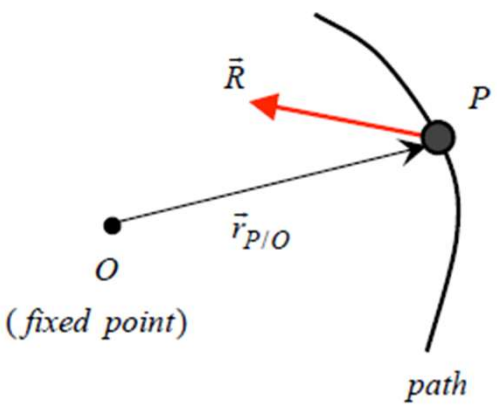
Substituting into our moment equation:

$$\begin{aligned} \vec{M}_O &= \frac{d}{dt} [\vec{r}_{P/O} \times m\vec{v}] - \frac{d\vec{r}_{P/O}}{dt} \times m\vec{v} \\ &= \frac{d}{dt} [\vec{r}_{P/O} \times m\vec{v}] - \cancel{\vec{v} \times m\vec{v}}^0 = \frac{d}{dt} [\vec{r}_{P/O} \times m\vec{v}] = \frac{d\vec{H}_O}{dt} \end{aligned}$$

Integrating with respect to time:

$$\int_1^2 \vec{M}_O dt = \int_1^2 d\vec{H}_O = \vec{H}_{O2} - \vec{H}_{O1}$$

Angular impulse momentum



Key Takeaways

→ $\vec{H}_O$ is the angular momentum about point O , $\vec{H}_O = \vec{r}_{P/O} \times m\vec{v}_P$.

→ Note $\vec{H}_O$ is a vector. For 2D rotation, it will always be pointed in or out of the page ($\hat{k}$).

→ AIM is the rotational equivalent of LIM.

→ AIM relates the time integral of the moment to the change in angular momentum of a particle rotation about a point.

LIM: relates Δv to $\Delta t, F(t)$
 AIM: relates $\Delta \omega$ to $\Delta t, M(t)$

Calculating the Angular Momentum of a Particle

For our particle P rotating about the fixed point O , angular momentum is calculated as $\vec{H}_O = \vec{r}_{P/O} \times m\vec{v}_P$. *← general form, all coord. descriptions*

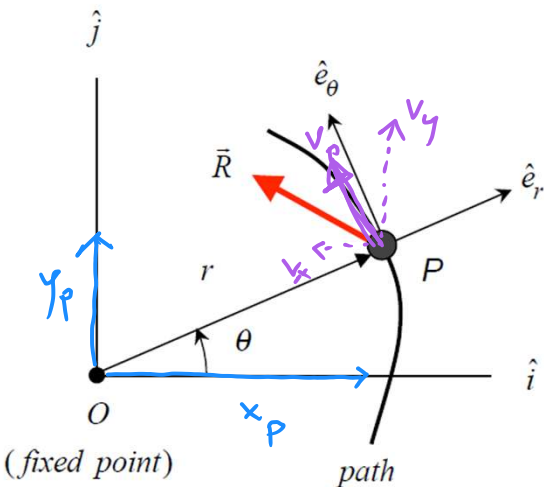
To calculate in the cartesian description:

$$\begin{aligned} \vec{H}_O &= \vec{r}_{P/O} \times m\vec{v}_P \\ &= m[(x_P \hat{i} + y_P \hat{j}) \times (v_{Px} \hat{i} + v_{Py} \hat{j})] \\ &= m(x_P v_{Py} - y_P v_{Px}) \hat{k} \end{aligned}$$

To calculate in the polar description:

$$\begin{aligned} \vec{H}_O &= \vec{r}_{P/O} \times m\vec{v}_P \\ &= m[(r\hat{e}_r) \times (r\dot{\theta}\hat{e}_\theta)] \\ &= m r^2 \dot{\theta} \hat{k} \end{aligned}$$

$\vec{H}_O$ always in $\hat{k}$ for 2D.

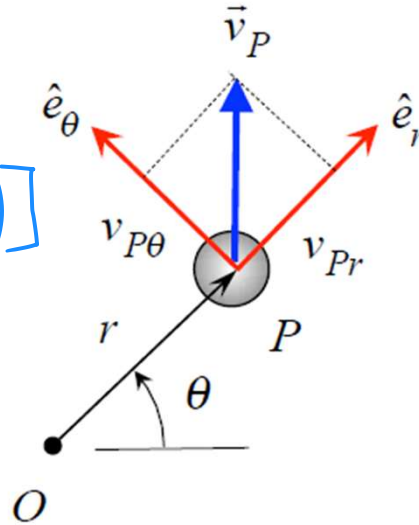


Note: the polar description is frequently the most useful form for angular momentum.

CHALLENGE QUESTION: The radial component of velocity, v_{Pr} , for a particle P cannot make a contribution to the angular momentum of the particle about point O. Why is that?

$$\begin{aligned}\vec{H}_O &= \vec{r}_{P/O} \times m \vec{v}_{P/O} \\ &= m \left[(r \hat{e}_r) \times (v_{Pr} \hat{e}_r + v_{P\theta} \hat{e}_\theta) \right] \\ &\quad \underbrace{\hat{e}_r \times \hat{e}_r = 0}\end{aligned}$$

$$\begin{aligned}\vec{H}_O &= m \left[(r \hat{e}_r) \times (v_{P\theta} \hat{e}_\theta) \right] \\ &= m r v_{P\theta} \hat{k}\end{aligned}$$

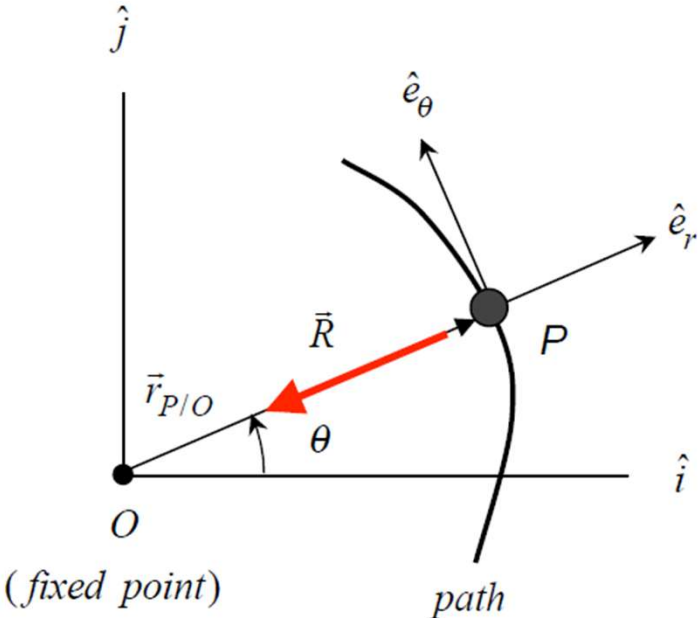


Key Takeaways

→ Angular momentum only gives info about the rotational component $r\dot{\theta}$ of velocity $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$.

→ if the total velocity is needed, you will need to also use WE to find $\dot{r}$.

Central Force Problems and Conservation of Angular Momentum



A “central force” problem is a special case where our resultant force $\vec{R}$ is aligned with our $\hat{e}_r$ direction $\rightarrow \vec{R}$ point directly towards/away from the fixed point O .

The moment of this central force will always be zero!

$$\vec{M}_O = \vec{r}_{P/O} \times \vec{R} = r_{P/O} \hat{e}_r \times R \hat{e}_r = 0$$

From our angular impulse-momentum equation:

$$\int_1^2 \vec{M}_O dt = \int_1^2 d\vec{H}_O = \vec{H}_{O2} - \vec{H}_{O1}$$

This means the angular momentum of the particle is constant!

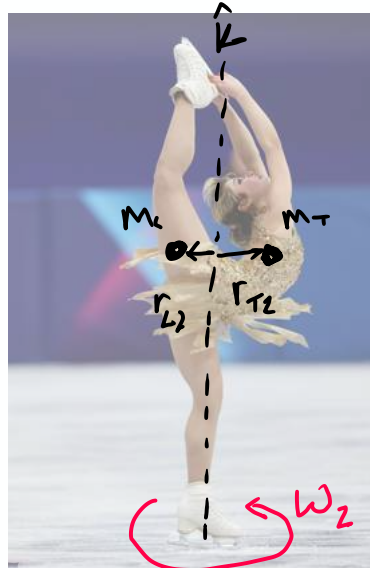
$$\int_1^2 \vec{M}_O dt = 0 = \vec{H}_{O2} - \vec{H}_{O1}$$

$$\vec{H}_{O2} = \vec{H}_{O1}$$

Key Takeaways

- $\rightarrow$ In central force problems $\vec{H}_{O2} = \vec{H}_{O1}$
- $\rightarrow$ Angular momentum is conserved!

Olympic gold medalist Alyssa Liu enters a spin rotating slowly. As the spin progresses her rate of rotation increases. How does Alyssa speed up during her spin?



$$r_{L1} > r_{L2} \quad , \quad r_{T1} > r_{T2} \quad \Rightarrow \quad \omega_1 < \omega_2$$

By using conservation of angular momentum!

(and years of flexibility and balance training...but mostly angular momentum.)

$$\vec{H}_O = \vec{r}_{P/O} \times m\vec{v}$$

In polar form:

$$\vec{H}_O = m(r^2\dot{\theta})\vec{k}$$

When angular momentum is conserved:

$$\vec{H}_{O2} = \vec{H}_{O1}$$

$$m(r_1^2\dot{\theta}_1) = m(r_2^2\dot{\theta}_2)$$

$$\dot{\theta}_2 = \dot{\theta}_1 \frac{r_1^2}{r_2^2}$$

Key Takeaways

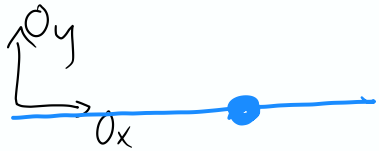
→ Angular velocity $\vec{\omega}$ must decrease (or increase) as the radial distance r increases (or decreases) when angular momentum is conserved.

$$r \uparrow \omega \downarrow \quad r \downarrow \omega \uparrow$$

Example 4.D.1

Given: Particle P (weighing 2 lb) is able to slide on a smooth, lightweight horizontal arm that is rotating about a vertical axis. Initially, P is stationary relative to the arm when the arm is rotating at a rate of $\omega_1 = 20 \text{ rad/s}$ and P is $R = 3 \text{ in}$ from the rotation axis of the arm.

Find: Determine the angular speed of the arm when the P has moved outward to a position of $R = 24 \text{ in}$ from the axis of the shaft.



$$\sum M_O = 0$$

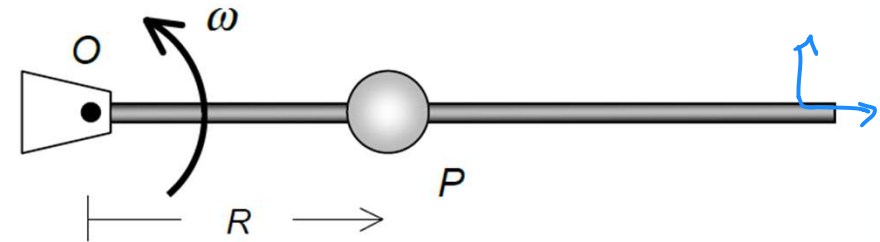
$$(\vec{H}_O)_1 = (\vec{H}_O)_2$$

$$\vec{H}_{O1} = \vec{r}_1 \times m\vec{v}_1 = r_1 \hat{e}_r \times \frac{W}{g} (\cancel{\dot{r}_1} \hat{e}_r + r_1 \omega_1 \hat{e}_\theta)$$

$$= \frac{W}{g} r_1^2 \omega_1 \hat{k}$$

$$\vec{H}_{O2} = \vec{r}_2 \times m\vec{v}_2 = r_2 \hat{e}_r \times \frac{W}{g} (\dot{r}_2 \hat{e}_r + r_2 \omega_2 \hat{e}_\theta)$$

$$= \frac{W}{g} r_2^2 \omega_2 \hat{k}$$



HORIZONTAL PLANE

$$r_1^2 \omega_1 = r_2^2 \omega_2$$

$$\omega_2 = \frac{r_1^2}{r_2^2} \omega_1$$

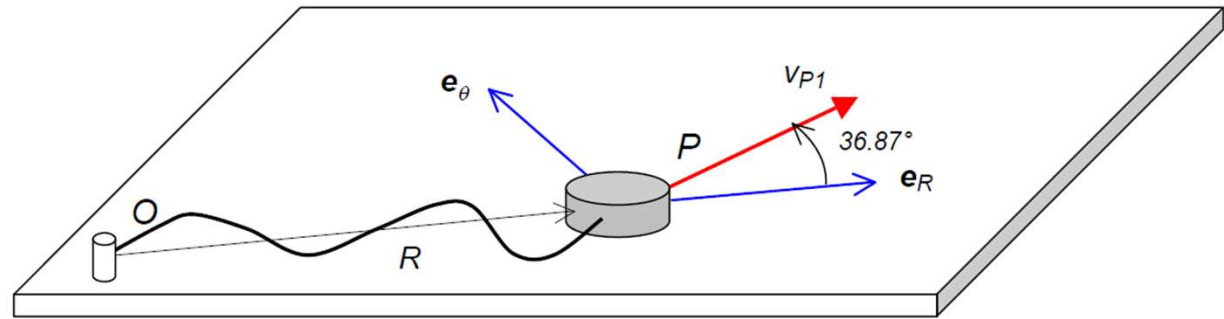
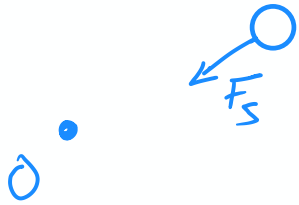
$$= \frac{3^2}{24^2} 20 = 0.3 \text{ rad/s}$$

Example 4.D.3

Given: Disk P having a mass of 0.2 kg is able to slide on a smooth, horizontal surface. A rubber band (having a stiffness of $k = 10 \text{ N/m}$ and unstretched length of 0.6 m) attaches B to a fixed peg at O. Disk P is set into motion with a speed of $v_{P1} = 15 \text{ m/s}$ in the direction shown with $R = 0.4 \text{ m}$.

Find: Determine the polar coordinates of the velocity of P when P is a distance 1.5 m from O.

FBD



Kinetics

$$\sum M_O = 0 \Rightarrow \vec{H}_{O1} = \vec{H}_{O2}$$

$$\begin{aligned} \vec{H}_{O1} &= \vec{r}_1 \times m \vec{v}_1 = r_1 \hat{e}_r \times m (v_{P1} \cos \theta \hat{e}_r + v_{P1} \sin \theta \hat{e}_\theta) \\ &= m r_1 v_{P1} \sin \theta \hat{k} \end{aligned}$$

$$\begin{aligned} \vec{H}_{O2} &= \vec{r}_2 \times m \vec{v}_2 = r_2 \hat{e}_r \times m (v_{2r} \hat{e}_r + v_{2\theta} \hat{e}_\theta) \\ &= m r_2 v_{2\theta} \hat{k} \end{aligned}$$

angular component: $m r_1 v_{P1} \sin \theta = m r_2 v_{2\theta} \rightarrow v_{2\theta} = \frac{r_1}{r_2} v_{P1} \sin \theta$

To find radial component, use WE!

$$T_1 + V_1 + U_{1 \rightarrow 2}^{Nc} = T_2 + V_2$$

$$T_1 = \frac{1}{2} m v_{p1}^2$$

$$V_1 = 0 \quad (\text{elastic band cannot be in compression})$$

$$U_{1 \rightarrow 2}^{Nc} = 0$$

$$T_2 = \frac{1}{2} m (v_{2r}^2 + v_{2\theta}^2)$$

$$V_2 = \frac{1}{2} k (R_2 - R_0)^2$$

$$\Rightarrow \frac{1}{2} m v_{p1}^2 = \frac{1}{2} m v_{2r}^2 + \frac{1}{2} m v_{2\theta}^2 + \frac{1}{2} k (R_2 - R_0)^2$$

Solve for v_{2r}

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Lecture 21: Particle Kinetics – Angular Impulse Momentum



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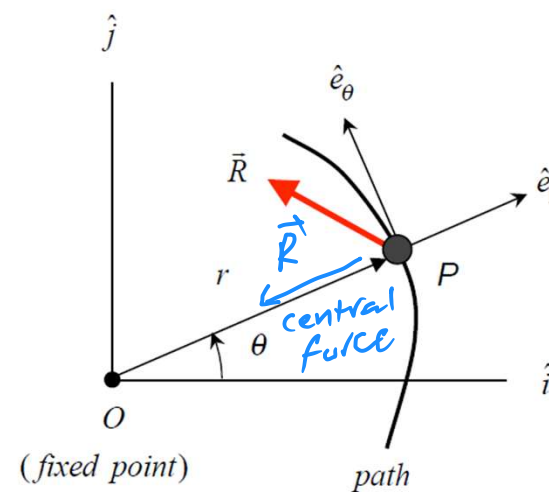
Overview: Angular Impulse Momentum

AIM equation: $\int_1^2 \vec{M}_O = \vec{H}_{O2} - \vec{H}_{O1}$,

Moment about O
 $\vec{M}_O = \vec{r}_{P/O} \times \vec{R}$,

general polar
 $\vec{H}_O = \vec{r}_{P/O} \times m\vec{v}_P$ or $\vec{H}_O = m(r^2\dot{\theta})\hat{k}$

- The AIM equation is the rotational analog to the LIM equation. It relates a change in angular velocity to a moment applied over time.
- Angular momentum, $\vec{H}_O$ is a vector. For 2D motion, it will always be pointed in or out of the page ($\hat{k}$).
- The AIM equation only gives info about the rotational component $r\dot{\theta}$ of velocity $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$. If you also need the radial component, use WE.
- In central force problems, $\vec{R}$ points directly towards/away from O . The net moment in these problems = 0.
- When $\sum \vec{M}_O = 0$, angular momentum is conserved! $\Rightarrow \vec{H}_{O2} = \vec{H}_{O1}$
- Angular velocity $\vec{\omega}$ must decrease (or increase) as the radial distance r increases (or decreases) **when angular momentum is conserved**.
 $(r \uparrow, \omega \downarrow \text{ or } r \downarrow, \omega \uparrow)$

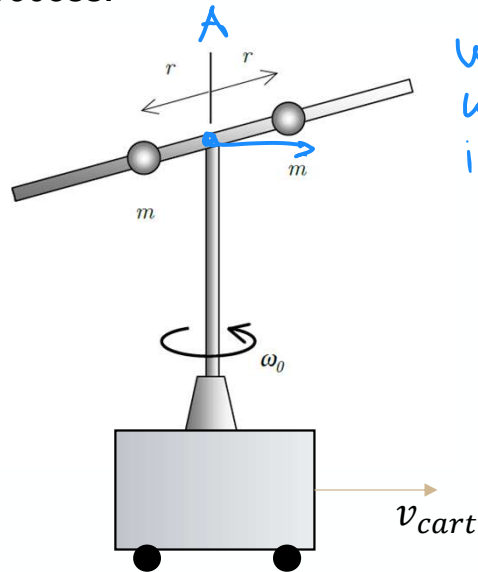


AIM for a system of multiple particles

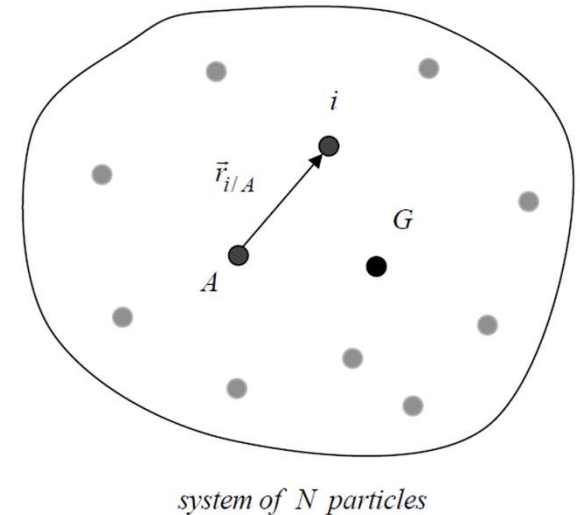
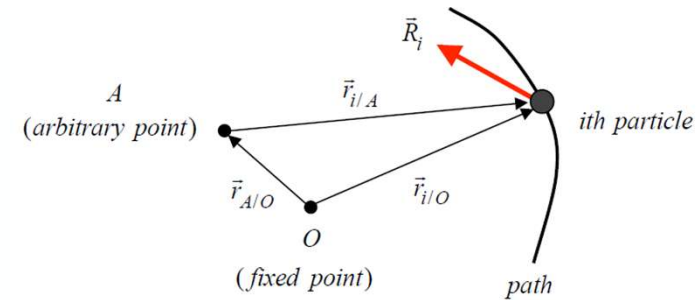
Note: these equations will be useful in our next chapter on angular momentum of rigid bodies. *discrete sum vs. integral*

For a system of particles, we sum the contribution of the angular momentum of each point.

Here we use an arbitrary reference point A (can be a moving point) to make these equations more general – later we will see how our selection of this point can simplify our solution process.



useful in cases where our system is rotating & translating



AIM for a system of multiple particles

If we have a system of N particles, looking at the particle i

Kinematics:

$$\vec{r}_{i/O} = \vec{r}_{A/O} + \vec{r}_{i/A}, \quad \vec{v}_i = \vec{v}_A + \vec{v}_{i/A}$$

if O is origin

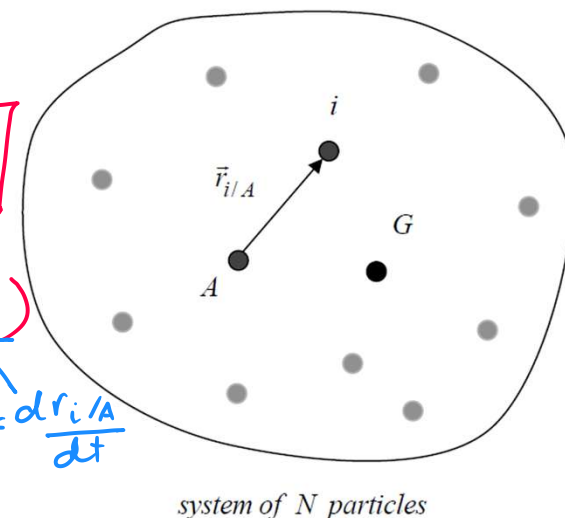
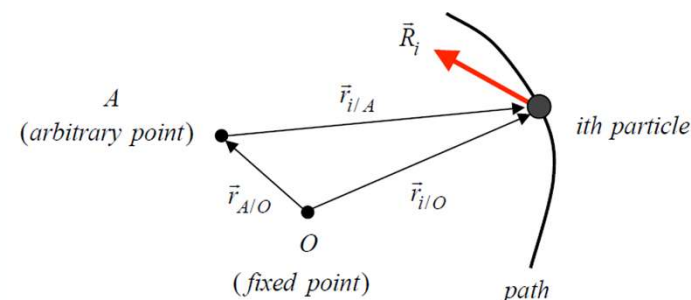
$$\vec{r}_i = \vec{r}_A + \vec{r}_{i/A}$$

Newton's 2nd law:

$$\vec{R}_i = \sum \vec{F}_i = m_i \vec{a}_i = m_i \frac{d\vec{v}_i}{dt}$$

The moment of $\vec{R}_i$ about our reference point A:

$$\begin{aligned} \vec{M}_{A,i} &= \vec{r}_{i/A} \times \vec{R}_i = \vec{r}_{i/A} \times m_i \frac{d\vec{v}_i}{dt} \\ &= \vec{r}_{i/A} \times m_i \frac{d}{dt} (\vec{v}_A + \vec{v}_{i/A}) = m_i \left[\vec{r}_{i/A} \times \frac{d\vec{v}_A}{dt} + \left(\vec{r}_{i/A} \times \frac{d\vec{v}_{i/A}}{dt} \right) \right] \\ &= (\vec{r}_{i/A} \times m_i \vec{a}_i) + \frac{d}{dt} (\vec{r}_{i/A} \times m_i \vec{v}_{i/A}) - \underbrace{\frac{d\vec{r}_{i/A}}{dt} \times (m_i \vec{v}_{i/A})}_0 \\ &= (\vec{r}_{i/A} \times m_i \vec{a}_i) + \frac{d}{dt} (\vec{r}_{i/A} \times m_i \vec{v}_{i/A}) \\ &= \vec{H}_{i/A} \end{aligned}$$



*

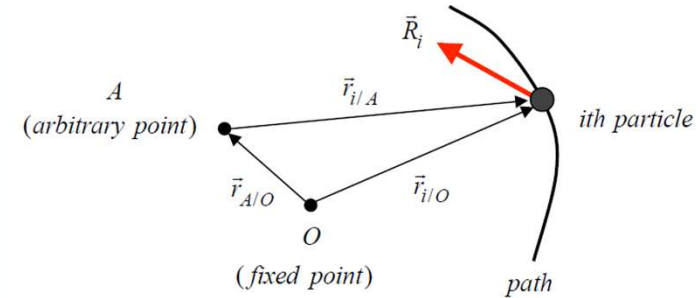
Remember from last lecture: $\frac{d}{dt} (\vec{a} \times \vec{b}) = \frac{d\vec{a}}{dt} \times \vec{b} + \vec{a} \times \frac{d\vec{b}}{dt} \Rightarrow \vec{a} \times \frac{d\vec{b}}{dt} = \frac{d}{dt} (\vec{a} \times \vec{b}) - \frac{d\vec{a}}{dt} \times \vec{b}$

AIM for a system of multiple particles

Now looking at the total for all N particles (i.e. for $i = 1, 2, 3, \dots, N$)

Remember the definition of the center of mass for multiple particles:

$$\ast \vec{r}_G = \frac{1}{m} \sum_{i=1}^N m_i \vec{r}_i, \quad \ast m = \sum_{i=1}^N m_i$$

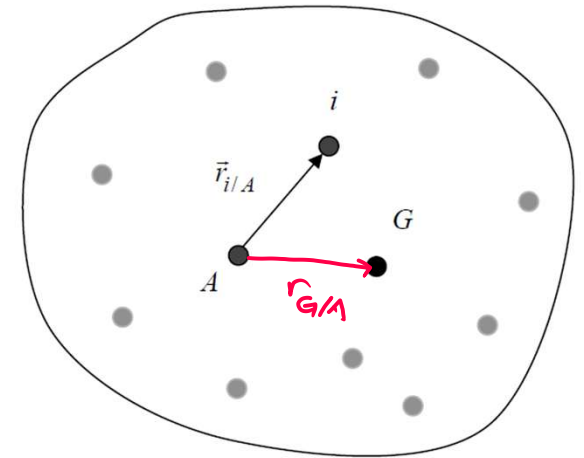


We sum the angular momentum equation of all particles:

$$\vec{M}_{A,i} = (\vec{r}_{i/A} \times m_i \vec{a}_A) + \frac{d}{dt} (\vec{r}_{i/A} \times m_i \vec{v}_{i/A}) \leftarrow \text{for 1 particle}$$

$$\sum_{i=1}^N \vec{M}_{A,i} = \sum_{i=1}^N (m_i \vec{r}_{i/A} \times \vec{a}_A) + \sum_{i=1}^N \frac{d}{dt} (\vec{r}_{i/A} \times m_i \vec{v}_{i/A})$$

$$= \underbrace{m}_{\text{total mass}} \vec{r}_{G/A} \times \vec{a}_A + \sum_{i=1}^N \frac{d}{dt} (\vec{r}_{i/A} \times m_i \vec{v}_{i/A})$$

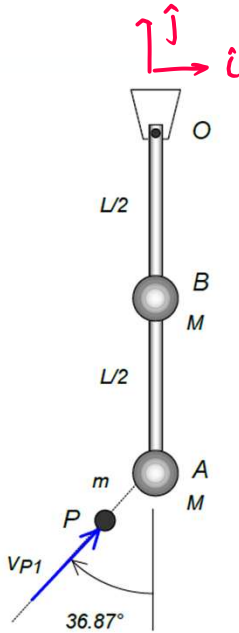


system of N particles

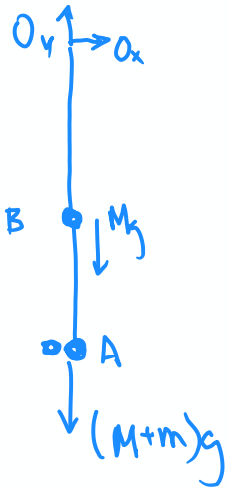
Example 4.D.5

Given: Particles A and B (each having a mass of M) are attached to rigid bar OA (this bar has negligible mass). Bar OA is pinned to ground at end O. This system is at rest when A is struck by a bullet P (having a mass of m) with the bullet traveling in the direction shown with a speed of v_{P1} . Immediately upon impact, the bullet becomes embedded in particle A.

Find: Determine the angular velocity of bar OA immediately after the collision is completed.



remember for AIM, make system as big as possible



$$\sum M_O = 0 \rightarrow \text{ang. Mo. cons.}$$

$$\vec{H}_{O1} = \vec{H}_{O2}$$

$$\vec{H}_{O1} \rightarrow \text{ang. mo. of bullet before impact}$$

$$= \vec{r}_{P/O} \times m_P \vec{v}_{P1}$$

$$= -L \hat{j} \times m(v_{P1} \sin \theta \hat{i} + v_{P1} \cos \theta \hat{j}) = mL v_{P1} \sin \theta \hat{k}$$

$$\vec{H}_{O2} \rightarrow \text{ang. mo. of bullet, A + B after impact}$$

$$\vec{H}_{O2} = \sum_i \vec{r}_{2i} \times m_i \vec{v}_{2i}$$

$\underbrace{O}_{\text{is IC}} \rightarrow v_A = r\omega$

$$= -L \hat{j} \times (m+M)(L\omega_2 \hat{i}) - \frac{L}{2} \hat{j} \times M \frac{L}{2} \omega_2 \hat{i}$$

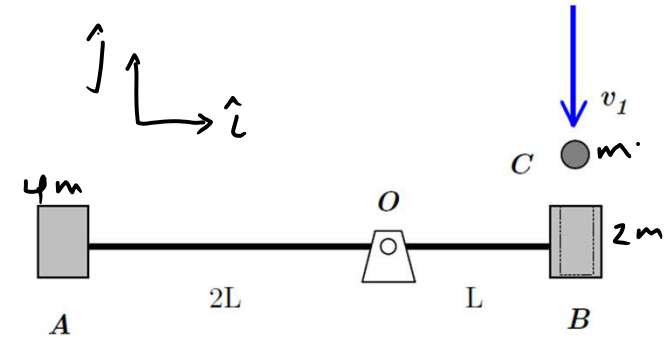
$$= (m+M)L^2 \omega_2 \hat{k} + M \frac{L^2}{4} \omega_2 \hat{k}$$

$$mL v_{P1} \sin \theta \hat{k} = (m+M)L^2 \omega_2 \hat{k} + M \frac{L^2}{4} \omega_2 \hat{k} \rightarrow \text{solve for } \omega_2$$

Example 4.D.6

Given: Particles A and B, of masses $4m$ and $2m$, respectively, are attached to the ends of a stationary rigid rod of negligible mass. The rod is pinned to ground at O. A third particle C, of mass m , strikes particle B with a speed of v_1 . On impact, C sticks to B. The system lies in the horizontal plane.

Find: Determine the angular speed of the bar immediately after the impact occurs.



$$\sum \vec{M} = 0 \rightarrow \vec{H}_{20} = \vec{H}_{10}$$

$\vec{H}_{01} \rightarrow$ just bullet moving

$$\vec{H}_{01} = \sum \vec{r}_{i1} \times m_i \vec{v}_{i1} = L \hat{i} \times m (-v_1 \hat{j}) = -mv_1 L \hat{k}$$

$$\begin{aligned} \vec{H}_{02} &= \sum \vec{r}_{i2} \times m_i \vec{v}_{i2} = L \hat{i} \times 3m (L \omega_2 \hat{j}) + (-2L \hat{i}) \times 4m (-2L \omega_2 \hat{j}) \\ &= 3m \omega_2 L^2 \hat{k} + 16m \omega_2 L^2 \hat{k} = 19m \omega_2 L^2 \hat{k} \end{aligned}$$

$$-mv_1 L = 19m \omega_2 L^2 \rightarrow \omega_2 = \frac{-v_1}{19L}$$

ME 274: Basic Mechanics II

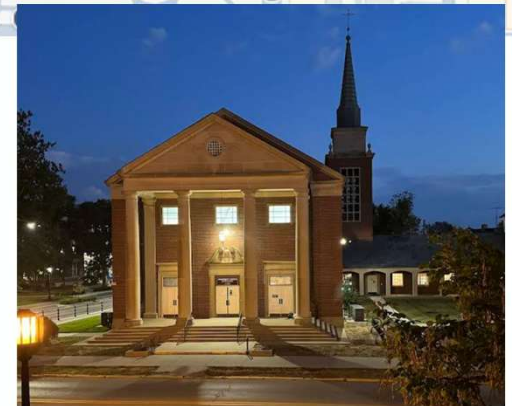
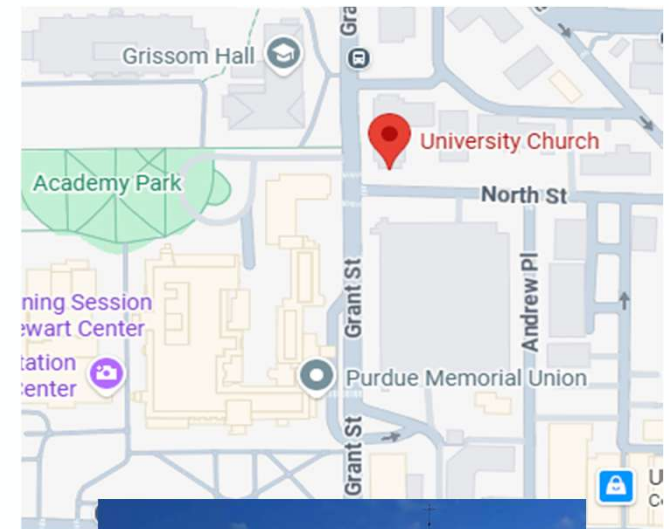
Lecture 26: Particle Kinetics – Review



School of Mechanical Engineering

Announcements

- HW 25 (the one released Friday before Spring Break) DUE TONIGHT!
- Exam 2, Thursday, 4/2, 8:00 – 9:30pm
 - Topics covered: Lectures 11-26 (Moving reference frames – Angular impulse momentum)
 - Location: UC 114
 - DRC students – expect an email from Dr. Krousgrill
 - Let me know of any conflicts ASAP
- Review sessions:
 - Pi Tau Sigma: Tuesday, March 31, 6:30-7:30 PM, WTHR 104
 - Wednesday, April 1, 7:00 PM over Zoom
 - Recordings of both sessions will be posted



Kinetics: Four-Step Problem Solving Method

1) FBD

- Draw appropriate FBD (note- you may have to draw more than 1)
- Choose coordinate system (cartesian, path, polar)

2) Kinetics

- Choose appropriate method to solve the problem – practice determining when to use each!
 - Newton/Euler
 - Work/Energy
 - Linear impulse/Momentum
 - Angular impulse/Momentum

3) Kinematics

- Perform a kinematic analysis of the system using techniques we have developed in the previous chapters.
- Use your kinetics equations from step 2 to determine what additional information you need to solve the problem

4) Solve

- Count the number of equations and unknowns. Do they match?
- If not:
 - Draw more FBDs
 - Do additional kinematic analysis

Newton's 2nd Law

When to use: relating forces to accelerations

Fundamental equations:

$$\Sigma F = m\vec{a}$$

Remember:

Break into scalar equations by component!

$$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

Work-Energy

When to use: relating change in **speed** to a change in **position**

Fundamental equations:

$$T_1 + V_1 + U_{1 \rightarrow 2}^{NC} = T_2 + V_2$$

Kinetic energy: $T = \frac{1}{2}mv^2$

Potential energy: gravity $\rightarrow V = mgh$

$$\text{spring} \rightarrow V = \frac{1}{2}k(\Delta L)^2$$

Non-conservative work: $U_{1 \rightarrow 2}^{NC} = \int_1^2 (\vec{R} \cdot \hat{e}_t) ds$

Remember:

- If $U_{1 \rightarrow 2}^{NC} = 0$, mechanical energy is conserved!
- Only force components tangential to the path contribute to $U_{1 \rightarrow 2}^{NC}$

Linear Impulse Momentum

When to use: relating change in **velocity** to a change in **time**

Fundamental equations:

$$\int_{t_1}^{t_2} \vec{R} dt = m\vec{v}_2 - m\vec{v}_1$$

Remember:

- If $\int_{t_1}^{t_2} \vec{R} dt = 0$, linear momentum is conserved!
- In multiparticle problems, make the system as large as possible so internal forces cancel.

Angular Impulse Momentum

When to use: relating change in **angular velocity** to a change in **time**

Fundamental equations:

$$\int_{t_1}^{t_2} \Sigma \vec{M}_O dt = \vec{H}_{O2} - \vec{H}_{O1}$$

Angular momentum: $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$

Remember:

- If $\int_{t_1}^{t_2} \Sigma \vec{M}_O dt = 0$, linear momentum is conserved! (central force problems)

“Rapid fire” questions from Chapter 4 – Kinetics of Particles
ME 274 – Spring 2025
cmk

Attached is a set of problems from past exams in this course that are related to the material on particle kinetics found in Chapter 4. I would like for us to use these problems to sharpen our skills on two issues: 1) deciding which method(s) to use in solving the problem (Newton's 2nd law, the work/energy equation, the linear impulse momentum equations and/or the angular impulse momentum equations), and 2) drawing the appropriate free body diagram(s) for solving the problems. I am calling these “rapid fire” questions in that these are two issues that you with which you need to be extremely comfortable, and that you can handle in a relatively short time at the beginning of the problem solution. Also attached here is page 352 of the course lecture book which can be helpful in guiding you on your choice of method(s) to use in solving.

With this, I challenge you to do these two steps on the following problems. You may continue on with the solution beyond these two steps of course; however, focus on developing skills in answering these two steps first before solving.

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler (relating forces to accelerations)	particle	$\sum \vec{F} = m\vec{a}$
	rigid body (G = c.m. and A = any point on body)	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy (relating change in speed to change in position)	particle	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body (G = c.m. and A = any point on body)	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$
Linear impulse-momentum (relating change in velocity to change in time)	particle	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body (G = c.m.)	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum (relating change in angular velocity to change in time)	particle (O = fixed point)	$\int_{t_1}^{t_2} \sum \vec{M}_O dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body (A = fixed point or c.m.)	$\int_{t_1}^{t_2} \sum \vec{M}_A dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A \vec{\omega}$

ME 274 – Summer 2009
Examination No. 2
PROBLEM NO. 1

Name _____

Given: Pellet P having a mass of m is pulled through a barrel (having negligible mass) by means of radial force $F = 60R$, where F is in Newtons and R is in meters. The barrel is constrained to move in a HORIZONTAL plane by rotating about shaft passing through point O. The system is released with $R = R_1$, $\dot{R} = \dot{R}_1$ and $\dot{\theta} = \dot{\theta}_1$.

Find: For the instant when $R = R_2$:

- determine the rotation rate of the barrel, $\dot{\theta}_2$.
- determine the value of $\dot{R}_2$.

Use the following parameters in your analysis: $m = 20\text{kg}$, $R_1 = 1.5\text{ meters}$, $\dot{R}_1 = 4\text{ m/sec}$, $\dot{\theta}_1 = 8\text{ rad/sec (CCW)}$ and $R_2 = 3\text{ meters}$.

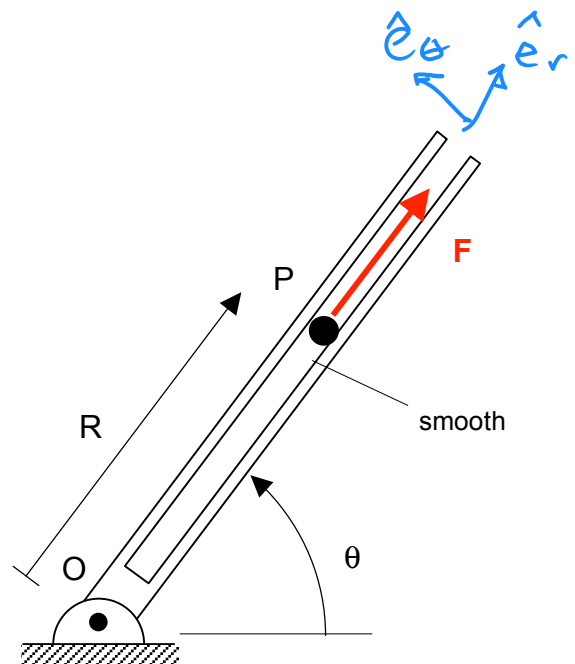
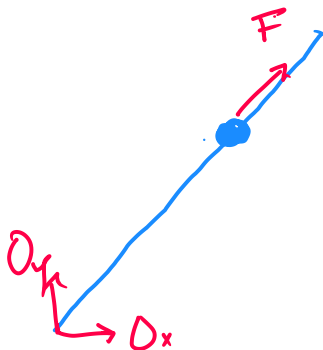
using our 4 step problem solving method:

Step 1: FBD & coord sys

From our kinematics section, we should be able to see that the radial positioning of the particle along the slot & the angle theta indicate POLAR cords

We will make our system the combined slot & particle

FBD:



HORIZONTAL PLANE

From this FBD, we can see that this is a Central Force problem with no net moment.

$$\sum \vec{M}_O = 0$$

Step 2: Kinetics

To decide what solution method(s) we should use, let's look at the information we are given as well as what we're asked to solve for

in part (a) we are asked to find a change in angular velocity. Also, from our FBD, we know this is a central force problem. This is an indication that we should use AIM.

(Note: AIM generally is used to find a change in angular momentum over a change in time; however, if our net moment = 0, we can use AIM without an explicitly given time frame)

Remember that AIM only gives the velocity in the transverse θ direction, so we know we will have to use Work-Energy to solve for $\dot{r}$ in part (b).

part a

$$\text{AIM: } \sum \vec{M}_O = 0 \rightarrow \vec{H}_{O2} = \vec{H}_{O1}$$

$$m \vec{r}_2 \times \vec{v}_2 = m \vec{r}_1 \times \vec{v}_1$$

recast in polar coords: $\vec{r} = r \hat{e}_r$
 $\vec{v} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta$

$$m(R_2 \hat{e}_r) \times (\dot{R}_2 \hat{e}_r + R_2 \dot{\theta}_2 \hat{e}_\theta) = m(R_1 \hat{e}_r) \times (\dot{R}_1 \hat{e}_r + R_1 \dot{\theta}_1 \hat{e}_\theta)$$

$$m R_2^2 \dot{\theta}_2 \hat{k} = m R_1^2 \dot{\theta}_1 \hat{k}$$

$$\Rightarrow \dot{\theta}_2 = \left(\frac{R_1}{R_2} \right)^2 \dot{\theta}_1$$

part b

In part b we apply WE

$$T_1 + \cancel{V_1} + U_{1 \rightarrow 2}^{NC} = T_2 + \cancel{V_2} \quad \text{v}^2 \text{ in polar}$$

$$T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} m (\dot{R}_1^2 + (R_1 \dot{\theta}_1)^2)$$

$$U_{1 \rightarrow 2}^{NC} = \int_{R_1}^{R_2} F dR = \int_{R_1}^{R_2} 60R dR = 30(R_2^2 - R_1^2)$$

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} m (\dot{R}_2^2 + (R_2 \dot{\theta}_2)^2)$$

$$\frac{1}{2} m (\dot{R}_1^2 + (R_1 \dot{\theta}_1)^2) + 30(R_2^2 - R_1^2) = \frac{1}{2} m (\dot{R}_2^2 + (R_2 \dot{\theta}_2)^2) \quad \text{from part A}$$

Solve for $\dot{R}_2$!

ME 274 – Summer 2009
Examination No. 2
PROBLEM NO. 3

Name _____

Given: Blocks A and B are connected by the cable-pulley shown. The system is released from rest. Consider all surfaces to be smooth and that the masses of the pulleys are small compared to the masses of A and B.

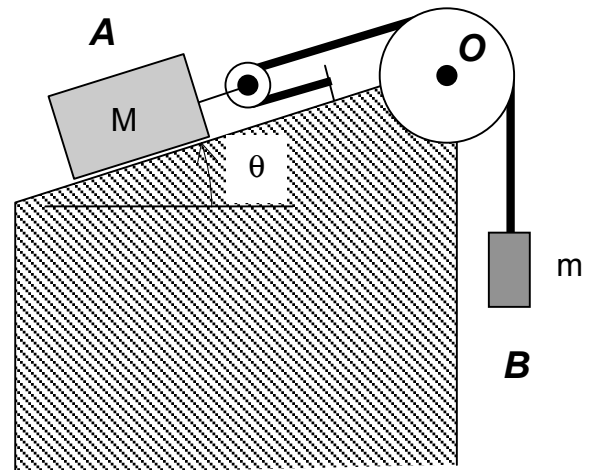
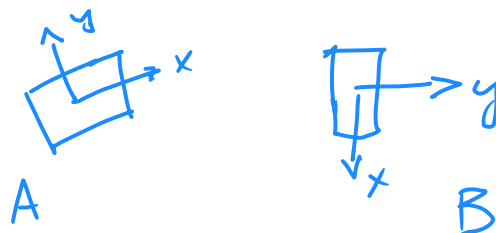
Find: Upon release,
 a) determine the acceleration of block B. Write your answer as a vector.
 b) determine the tension in the cable.

Use the following parameters in your analysis: $m = 5\text{kg}$, $M = 20\text{kg}$ and $\theta = 36.87^\circ$.

For pulley problems like this be careful
 In defining your axis. You want
 your axis to be continuous along
 The length of the cable, so it should
 "bend" around the pulley
 So if for block A, your X
 axis is aligned with the cable, it
 should still be aligned with the cable at b.
 You can draw that like with one axis like:

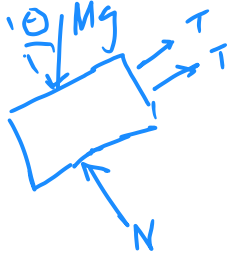


or with separate axes for each block



FBD

Block A



Block B



Kinetics $\rightarrow$ we are asked for the acceleration $\rightarrow$ Newton's 2nd

Block A

$$\sum F_x = 2T - Mg \sin \theta = M a_{Ax} \quad (1)$$

$$\sum F_y = N - Mg \cos \theta = 0 \quad \leftarrow \text{does not provide new info}$$

Block B

$$\sum F_x = mg - T = m a_{Bx} \quad (2)$$

from our kinetics, we have 2 eq^{ns},

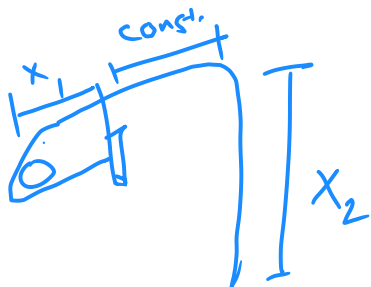
unknowns: a_{Ax} , a_{Bx} , T

use cable constraints:

$$L = 2x_1 + x_2 + \text{const.}$$

$$\frac{d^2 L}{dt^2} = 0 = 2\ddot{x}_1 + \ddot{x}_2$$

$$\Rightarrow a_B = 2a_A$$



Solve!

Examination No. 2

PROBLEM NO. 4

Given: Block B, having a mass of m , is pressed against a spring (of stiffness k) that is attached to cart A. Cart A (having a mass of M) rests on a horizontal surface. The system is released from rest with the spring compressed by an amount of Δ . After release, block B impacts A, with this impact having a coefficient of restitution of e . Assume all surfaces to be smooth. (Note that since B is simply pressed against the spring, the spring *can push but not pull* on B.)

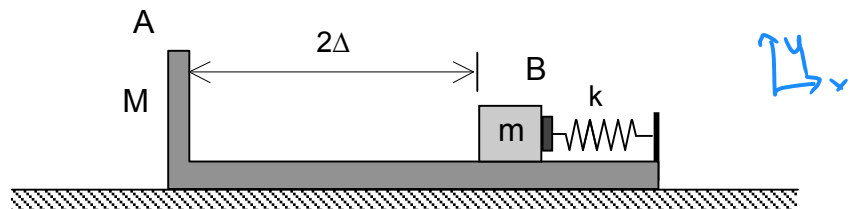
Find: For this problem,

- determine the velocities of A and B immediately BEFORE impact.
- determine the velocities of A and B immediately AFTER impact

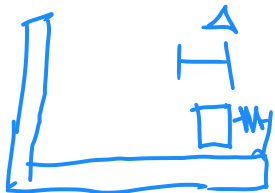
Write your answers as vectors.

Use the following parameters in your analysis: $m = 20\text{kg}$, $M = 40\text{kg}$, $k = 3000\text{ N/m}$, $\Delta = 0.2\text{meters}$ and $e = 0.5$.

This problem asks for velocity at two points in time, but you really need to evaluate the transition between 3 states

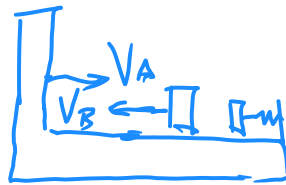


State 0



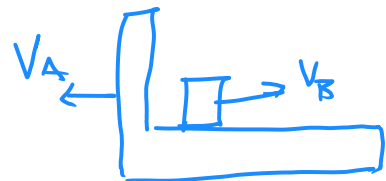
A & B stationary,
spring compressed

State 1



Spring released
A & B moving
before impact

State 2



post impact

from ① → ② use W-E

$$T_0 + V_0 + U_{0 \rightarrow 1}^{NC} = T_1 + V_1$$

our system is A+B

$$T_0 = 0$$

$$T_1 = \frac{1}{2} m V_{B1}^2 + \frac{1}{2} M V_{A1}^2$$

$$V_0 = \frac{1}{2} k \Delta^2$$

$$V_1 = 0$$

$$U_{0 \rightarrow 1}^{NC} = 0$$

$$\frac{1}{2} k \Delta^2 = \frac{1}{2} m V_{B1}^2 + \frac{1}{2} M V_{A1}^2$$

← from W-E
we have 1 eqn,
2 unknowns

Now use LIM!

no external impulse → momentum conserved!

$$M \vec{V}_{A0} + m \vec{V}_{B0} = M \vec{V}_{A1} + m \vec{V}_{B1}$$

$$M \vec{V}_{A1} = -m \vec{V}_{B1} \Rightarrow V_{A1} = -\frac{m}{M} V_{B1}$$

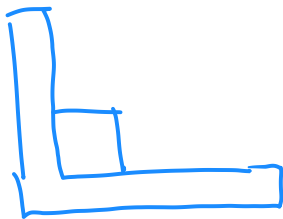
plug into WE

$$\frac{1}{2} k \Delta^2 = \frac{1}{2} m V_{B1}^2 + \frac{1}{2} M \left(-\frac{m}{M} V_{B1} \right)^2$$

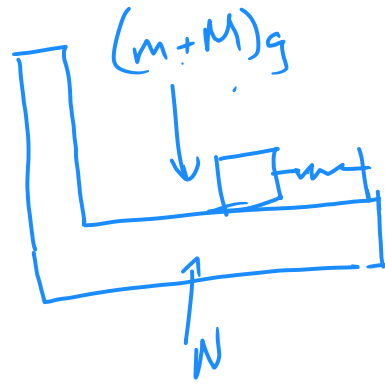
Solve for V_{B1} , V_{A1}

from ① → ② use LIM + COR

FBD



→ no external forces on the
system → momentum conserved



$$M v_{A1} + m v_{B1} = M v_{A2} + m v_{B2}$$

$$e = - \frac{v_{B2} - v_{A2}}{v_{B1} - v_{A1}}$$

2 eqn, 2 unknowns $\rightarrow$ Solve for v_{A2}, v_{B2} !

ME 274 – Summer 2009
Final Examination
PROBLEM NO. 1

Name _____

Given: A small object of mass m is placed on the inner surface of a conical dish that is rotating at a constant rate of ω . The coefficients of static and kinetic friction between the object and the dish are known to be μ_s and μ_k , respectively.

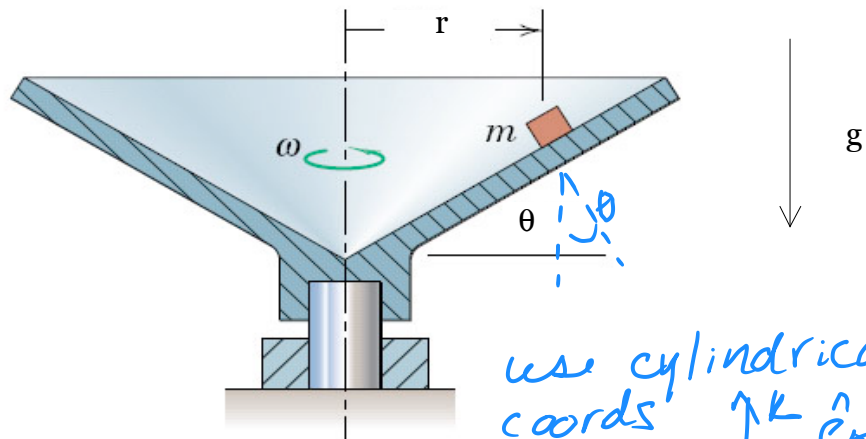
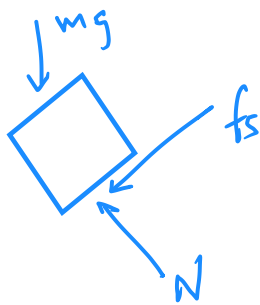
Find: Determine the maximum rotation rate ω for which the object does not slip on the dish.

implies if we increase rotation, the box will fly up out of the dish

Use the following parameters in your analysis: $m = 5\text{ kg}$, $r = 0.92\text{ meters}$, $\mu_s = 0.4$, $\mu_k = 0.1$ and $\theta = 36.87^\circ$.

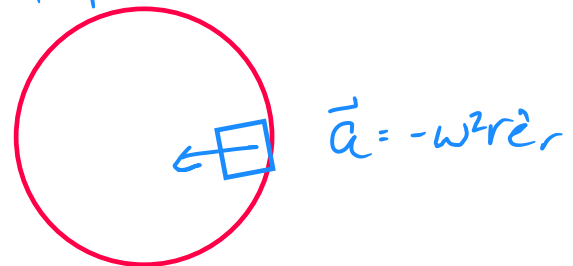
*Solution Method:
Newton's 2nd!*

FBD



use cylindrical coords
 $\hat{e}_\theta$
 $\hat{e}_r$

Top view



$$\sum F_r = -f_s \cos \theta - N \sin \theta = m a_r$$

$$\sum F_k = -mg + N \cos \theta = 0$$

$$f_s = \mu_s N \quad a_r = \omega^2 r$$

solve for ω !

Final Examination

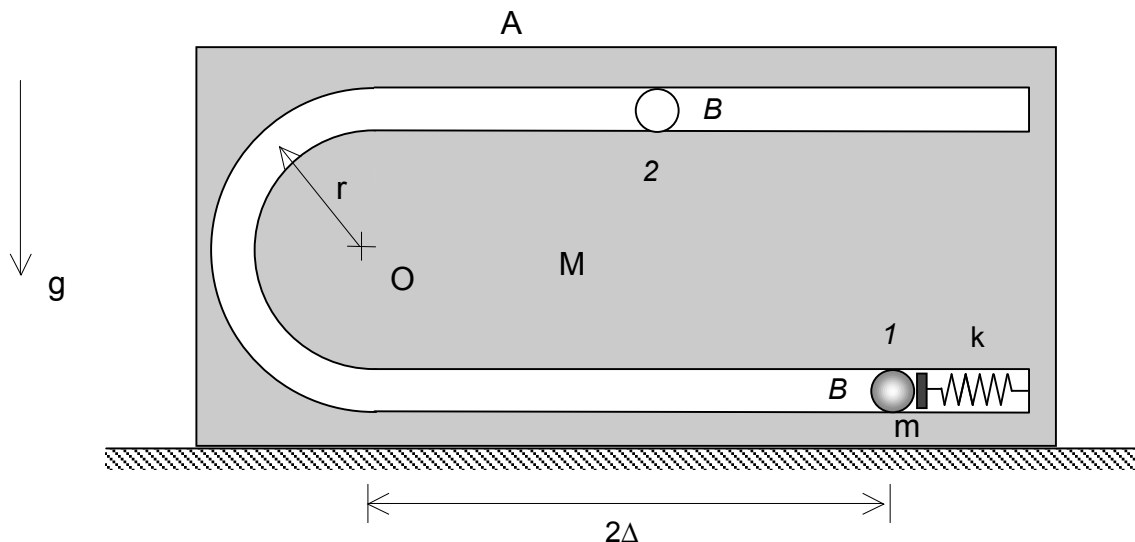
PROBLEM NO. 3

Given: Particle B, having a mass of m , is pressed against a spring (of stiffness k) that is attached to cart A. Cart A (having a mass of M) rests on a horizontal surface. The system is *released from rest* when B is at Position 1 with the spring compressed by an amount of Δ . After release, B travels within a slot cut into cart A, with the slot having straight horizontal and circular sections (the circular section has a radius of r and center at O). A position 2, B has reached the upper horizontal slot but has not yet impacted the cart at the right end of this slot.

Find: Determine the velocities of A and B when B is at Position 2. Write your answers as vectors.

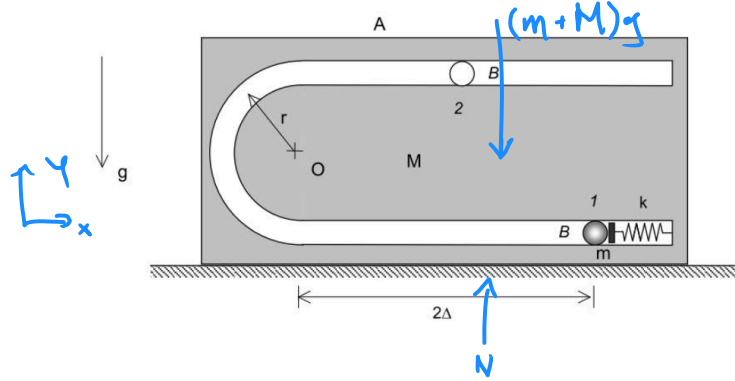
Note that since B is simply pressed against the spring, the spring *can push but not pull* on B. Assume all surfaces to be smooth.

Use the following parameters in your analysis: $m = 30\text{ kg}$, $M = 60\text{ kg}$, $k = 3000\text{ N/m}$, $\Delta = 0.5\text{ meters}$ and $r = 0.2\text{ meters}$.



PLEASE START YOUR WORK ON THE NEXT PAGE.

Using a system of cart A & particle B, there are no net external forces



(the spring force & any reaction forces between A+B are internal)
 → Linear momentum conserved

Soln Method → WE & LIM

WE: we are told the slot is smooth, so there are no NC forces → mechanical energy conserved.

$$\cancel{T} + V_1 + \cancel{U_{1 \rightarrow 2}^{NC}} = T_2 + V_2$$

$$\frac{1}{2} k \Delta^2 = \frac{1}{2} m V_{B2}^2 + \frac{1}{2} M V_{A2}^2 + mg(2r)$$

$$\text{LIM: } m \vec{V}_{B1} + M \vec{V}_{A1} = m \vec{V}_{B2} + M \vec{V}_{A2}$$

2 eqn, 2 unknowns → solve for V_{B2}, V_{A2}

Examination No. 2

PROBLEM NO. 4d (4 points max)

Given: Particle A (of mass m) is traveling with a speed of v_{A1} in the direction shown below when it strikes a stationary particle B (of mass $2m$). The coefficient of restitution for the impact of A with B is known to be $e = 0.5$.

Find: If $\theta_1 = 25^\circ$, what is the direction of travel of particle A *AFTER* impacting B? Provide a mathematical justification for your answer.

Solution method:

LIM $\rightarrow$ COR

FBD System A+B



$\leftarrow$ In an impact, all reaction forces are internal & cancel

No net external forces $\rightarrow$ momentum is conserved

Kinetics $\rightarrow$ central impact

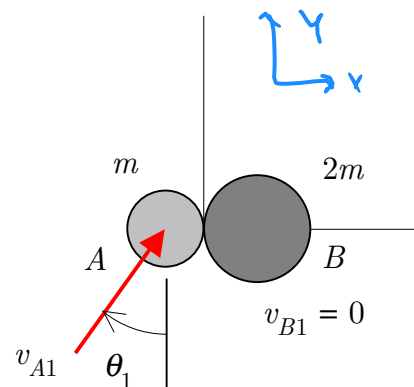
in this problem x is the normal direction & y is the tangent direction

$$X: m v_{A1x} + 2m v_{B1x} = m v_{A2x} + 2m v_{B2x}$$

$$v_{A1} \sin \theta = v_{A2x} + 2v_{B2x} \quad (1)$$

in central impact problems, the velocity in the tangent direction is constant

$$\Rightarrow v_{A1y} = v_{A1} \cos \theta = v_{A2y}, \quad v_{B1y} = 0 = v_{B2y}$$



COR

$$e = - \frac{V_{B2N} - V_{A2N}}{V_{B1N} - V_{A1N}}$$

remember this can only be used in the
Normal direction $\rightarrow (x)$

We now have 2 eqn, 2 unknowns,
Solve for V_{B2x} & V_{A2x}

Examination No. 2

PROBLEM NO. 2

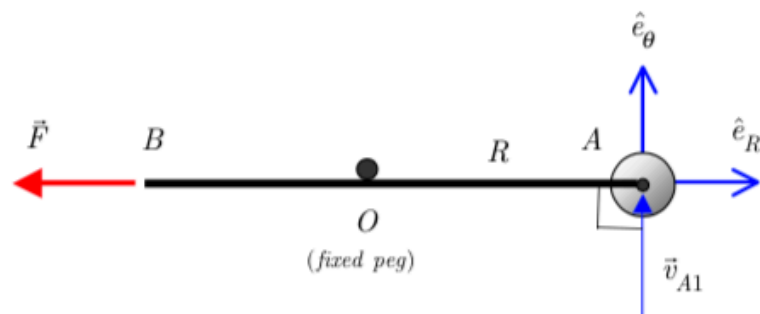
Given: Particle A (of mass m) slides upon a *smooth* HORIZONTAL surface. A flexible, inextensible cord is connected to A at one end and has a constant force $\vec{F}$ acting to the left on the other end. Initially, when A is at a radial distance of $R = R_1$ from O, the cord is in contact with a small, smooth peg at O. At this instant, A is moving perpendicular of line OA with a speed of v_{A1} , as shown in the figure.

Find: When A is at a radial distance of $R = R_2$ from O, determine the speeds of ends A and B of the cord. Use the following parameter values in your work:

$$m = 10 \text{ kg}, R_1 = 2 \text{ meters}, R_2 = 3 \text{ meters}, |\vec{F}| = 280 \text{ N and}$$

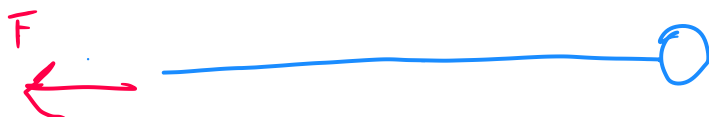
$$v_{A1} = 15 \text{ m / sec.}$$

Please clearly indicate the four steps in a neat and orderly presentation of your work.



HORIZONTAL SURFACE

the system is the cord, particle, & peg



F is the only external force $\rightarrow$ central force problem $\vec{F}_{O1} = \vec{F}_{O2}$

We will use AIM to find the speed in the $\hat{e}_\theta$ direction & WE to find speed in $\hat{e}_r$

AIM

$$r_1 \times m v_{A1} = r_2 \times m v_{A2\theta}$$

$$v_{A2\theta} = \frac{r_1}{r_2} v_{A1}$$

WE

$$T_1 + \cancel{V_1} + U_{1 \rightarrow 2}^{NC} = T_2 + \cancel{V_2}$$

remember in polar coords $\vec{v} = \dot{r}\hat{e}_r + \underbrace{r\dot{\theta}\hat{e}_\theta}_{\substack{\text{we found this} \\ \text{with AIM}}} = v_{A2\theta}$

$$T_1 = \frac{1}{2} m v_{A1}^2$$

$$U_{1 \rightarrow 2}^{NC} = \int_{R_1}^{R_2} F dr = F(R_2 - R_1)$$

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} m (\dot{R}^2 + v_{A2R}^2)$$

$$\frac{1}{2} m v_{A1}^2 + F(R_2 - R_1) = \frac{1}{2} m (\dot{R}^2 + v_{A2R}^2)$$

Solve for $\dot{R}$

$$v_A = \sqrt{\dot{R}^2 + v_{A2\theta}^2}$$

$$v_B = \dot{R}$$

} remember we are asked for speed

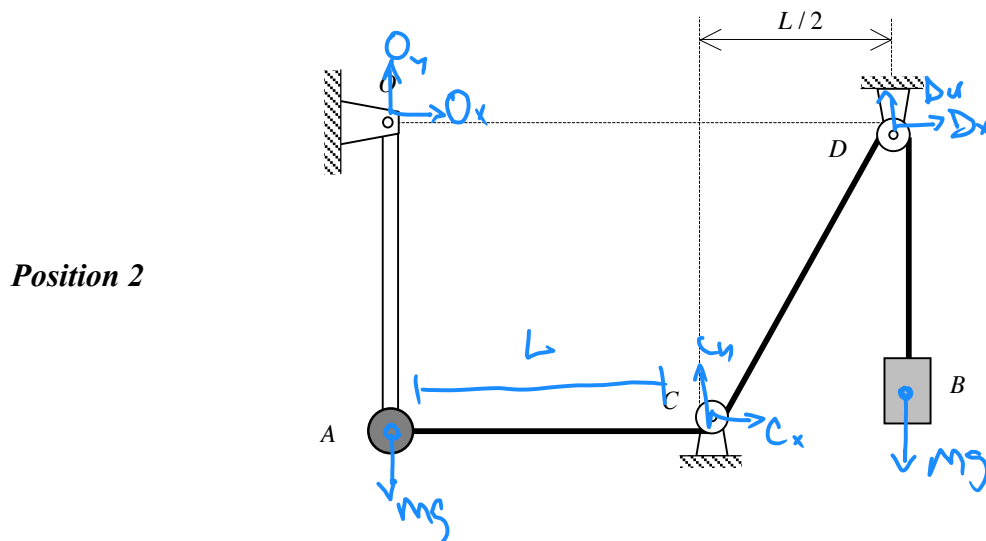
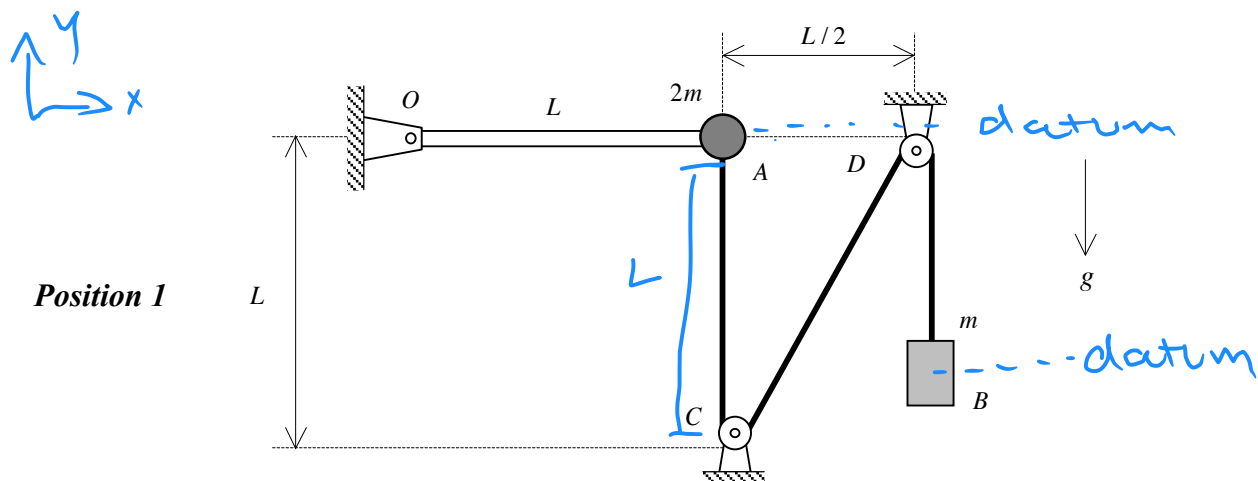
Examination No. 2

PROBLEM NO. 3

Given: Particle A (of mass $2m$) is attached to a rigid bar of negligible mass. Particle A is also connected to a cable that is wrapped around two pulleys and connected to particle B on its other end. The system is released from rest with OA being horizontal and with section AC of the cable being vertical. Assume that the radii of the pulleys to be negligible.

Find: Determine the *angular velocity* of the bar at Position 2 where it has rotated 90° CW to a vertical orientation. (At Position 2, section AC of the cable is horizontal.) Use the following parameter values in your work: $m = 10\text{ kg}$ and $L = 4\text{ meters}$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.



PLEASE START YOUR ANALYSIS ON THE NEXT PAGE.

System: bar + cable + mass

$$T_1 + \cancel{V_1} + \cancel{U_{1 \rightarrow 2}^{NC}} = T_2 + V_2$$

$$T_1 = 0, \text{ RFR}$$

$$\dot{V}_1 = 0$$

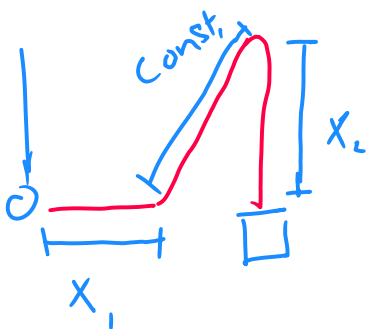
$$T_2 = \frac{1}{2} (2M) V_A^2 + \frac{1}{2} m V_B^2$$

$$V_2 = -2mgL \quad \leftarrow \text{note the block is at the same position at position 1 \& 2}$$

$$0 = M V_A^2 + \frac{1}{2} m V_B^2 - 2mgL$$

$$V_B = \sqrt{2 V_A^2 - 4 g L}$$

2 unknowns, 1 eqn $\rightarrow$ use kinematics
 $\rightarrow$ inextensible cable



$$L_{\text{tot}} = x_1 + x_2 + \text{const.}$$

$$\frac{dL_{\text{tot}}}{dt} = \dot{x}_1 + \dot{x}_2 = 0$$

$$V_A = -V_B$$

$\rightarrow$ sub in & solve!

Examination No. 2

PROBLEM NO. 3

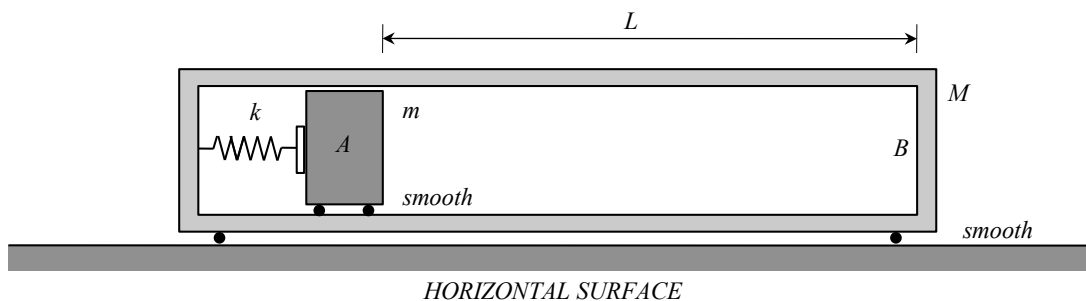
Given: A box having a mass of M is constrained to move along a smooth, horizontal surface. Block A (having a mass of m) is able to slide along smooth, horizontal surface inside the box, as shown in the figure below. Block A is pressed against a spring having a stiffness of k . Initially, the system is at rest and the spring is compressed by an amount Δ_1 . The coefficient of restitution between A and the box at end B is e .

Find: After the spring is released:

- Determine the speed of the box and the speed of block A immediately BEFORE A impacts the box at B. Write your answers as vectors.
- Determine the speed of the box and the speed of block A immediately AFTER A impacts the box at B. Write your answers as vectors.

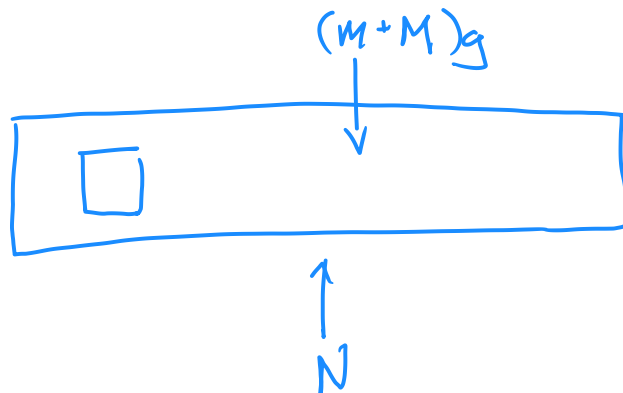
Use the following: $m = 3\text{ kg}$, $M = 5\text{ kg}$, $k = 12,000\text{ N/m}$, $e = 0.5$, $\Delta_1 = 0.3\text{ m}$ and $L = 2\text{ m}$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.



Work appearing above this line will NOT be graded.

Solution method LIM & WE



System
Block A + car + B

No net external forces
→ lin. mo. conserved

We are looking at 3 configurations of our system

State 1 $\rightarrow$ system at rest, spring compressed

State 2 $\rightarrow$ carts A & B both moving after spring released, before impact

State 3 $\rightarrow$ carts A & B both moving after impact

① $\rightarrow$ ②

WE: $\frac{1}{2} K \Delta_1^2 = \frac{1}{2} m v_{A2}^2 + \frac{1}{2} M v_{B2}^2$

LIM: $m \cancel{v_{A1}}^0 + M \cancel{v_{B1}}^0 = m v_{A2} + M v_{B1} \Rightarrow v_{B1} = - \frac{m v_{A2}}{M}$

Solve for v_{B2} & v_{A2}

② $\rightarrow$ ③ no external forces during impact $\rightarrow$ Lin. mo. conserved

LIM: $m v_{A2} + M v_{B2} = m v_{A3} + M v_{B3}$

COR:
$$e = - \frac{v_{B3} - v_{A3}}{v_{B2} - v_{A2}}$$

Solve for v_{B3} & v_{A3}

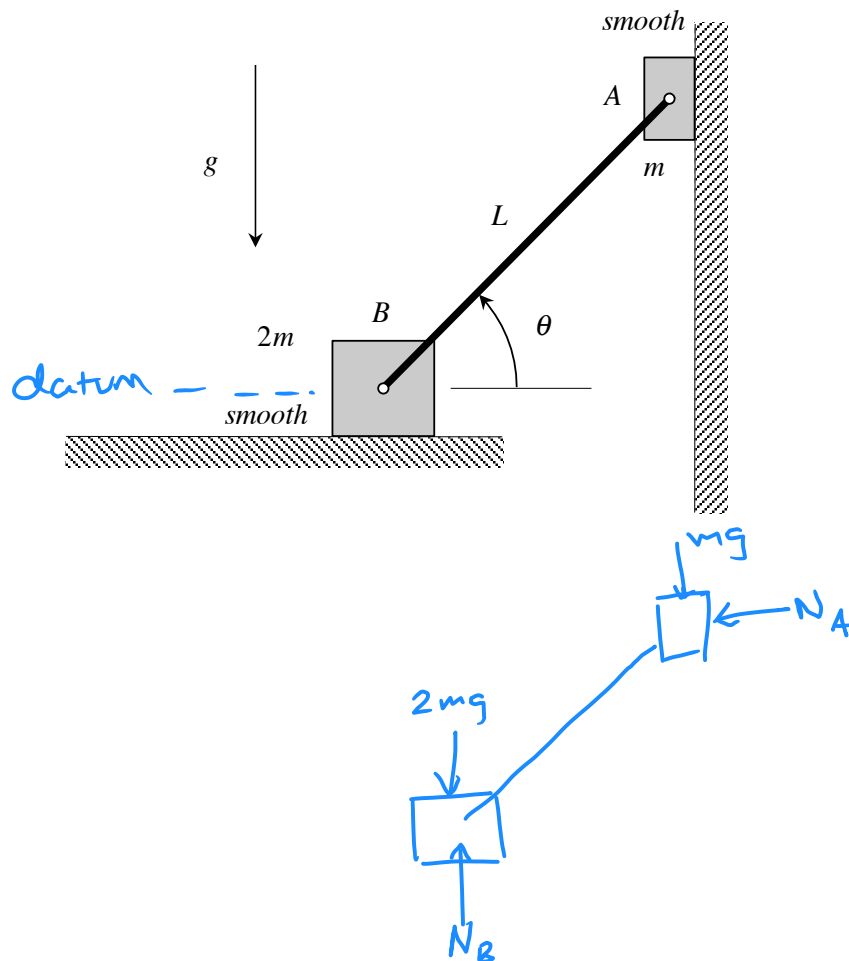
Examination No. 2

PROBLEM NO. 2 – 20 points

Given: Blocks A and B (having masses of m and $2m$, respectively) are connected by rigid bar AB, with bar AB having negligible mass. Block A is constrained to move along a smooth, vertical wall, whereas block B moves along a smooth horizontal surface. The system is released from rest with $\theta = 53.13^\circ$.

Find: Determine the speeds of blocks A and B at the position where $\theta = 0$.

Use the following parameter values: $L = 2\text{ m}$ and $m = 10\text{ kg}$.



Solution method: WE

System: Blocks A + B

Note reaction forces are internal & cancel

Kinetics

$$\text{WE: } T_1 + V_1 + U_{1 \rightarrow 2}^{\text{nc}} = T_2 + V_2$$

$$T_1 = 0 \text{ (RFR)}$$

$$T_2 = \frac{1}{2}(2m)V_B^2 + \frac{1}{2}mV_A^2$$

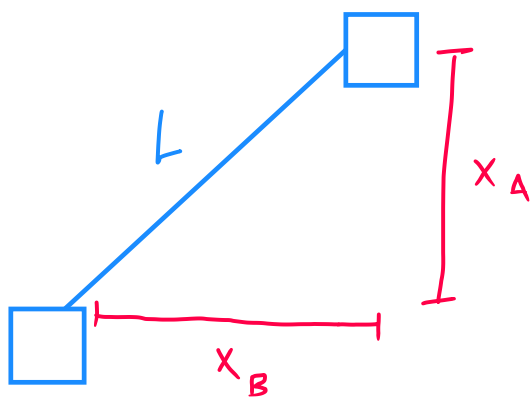
$$V_1 = mgL \sin \theta$$

$$V_2 = 0$$

$$U_{1 \rightarrow 2}^{\text{nc}} = 0$$

$$mgL \sin \theta = \cancel{m}V_B^2 + \frac{1}{2}\cancel{m}V_A^2 \quad (1)$$

kinetics:



$$L^2 = x_A^2 + x_B^2$$

$$\frac{d(L^2)}{dt} = 0 = 2x_A \dot{x}_A + 2x_B \dot{x}_B$$

$$x_A = L \cos \theta$$

$$x_B = L \sin \theta$$

$$0 = 2L \cos \theta V_A + 2L \sin \theta V_B$$

$$V_A = -\tan \theta V_B \quad (2)$$

2 eqn, 2 unknown, solve