ME 274: Basic Mechanics II

Week 8 – *Monday, March 2*

Work and Energy

Instructor: Manuel Salmerón

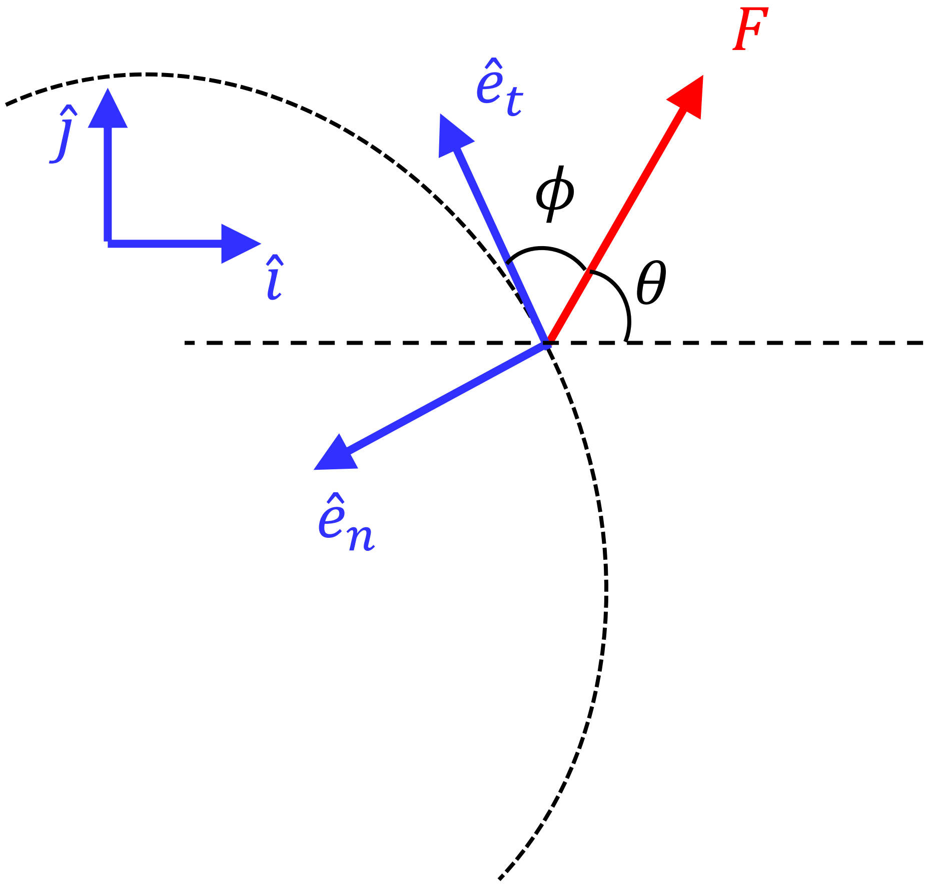
Announcements

- No office hours today (Monday, 03/02)
- Will announce makeup OH later
- Manuel not coming on Wednesday; attendance will be taken!

Today’s Agenda

1. Recap: Work and Energy
2. Conservative Forces and Potential Energy
3. Examples
4. Recommendations for WE Problems

Attendance



Q1. Provide a vector expression for the force F in the Cartesian coordinate system shown.

(a) $\vec{F} = F\sin\theta\,\hat{\imath} - F\cos\theta\,\hat{\jmath}$

(b) $\vec{F} = F\sin\phi\,\hat{\imath} + F\cos\phi\,\hat{\jmath}$

(c) $\vec{F} = F\cos\phi\,\hat{\imath} - F\sin\phi\,\hat{\jmath}$

(d) $\vec{F} = F\cos\theta\,\hat{\imath} + F\sin\theta\,\hat{\jmath}$

Q2. Provide a vector expression for the force F in the path coordinate system shown:

(a) $\vec{F} = F\sin\theta\,\hat{e}_t - F\cos\theta\,\hat{e}_n$

(b) $\vec{F} = F\sin\phi\,\hat{e}_t + F\cos\phi\,\hat{e}_n$

(c) $\vec{F} = F\cos\phi\,\hat{e}_t - F\sin\phi\,\hat{e}_n$

(d) $\vec{F} = F\cos\theta\,\hat{e}_t + F\sin\theta\,\hat{e}_n$

Recap: Work and Energy

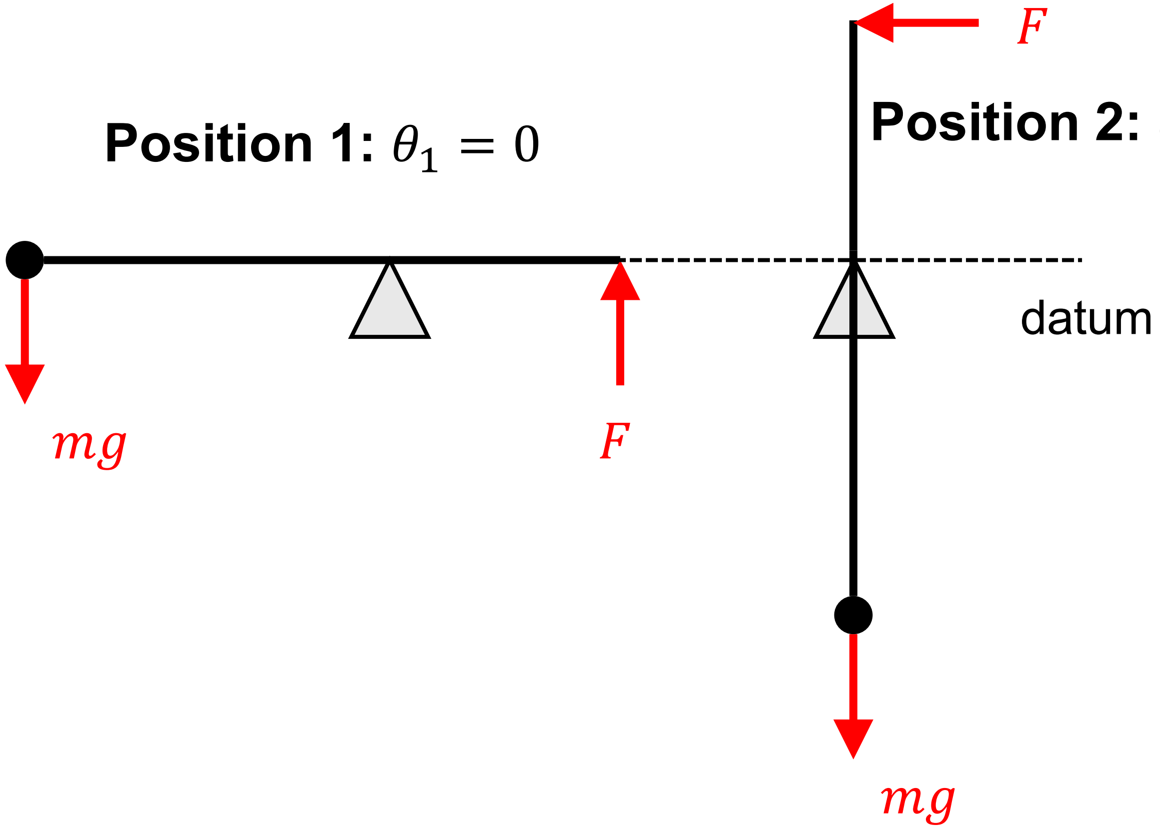
$$\boxed{U_{1\rightarrow 2} = \int_{s_1}^{s_2} \vec{R} \cdot d\vec{s} = T_2 - T_1}$$

Work = Change in Kinetic Energy

Recap: Work and Energy

$$\boxed{U_{1\to2} = \int_{s_1}^{s_2} \vec{R}\cdot d\vec{s} = T_2 - T_1}$$

Work = Change in Kinetic Energy



Work done by the force F:

$$U^{(F)}_{1\to2} = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s} = F\left(\frac{1}{4}\cdot 2\pi r\right)$$

Tangent to the trajectory

Little piece of the trajectory

Work done by the weight:

$$U^{(gr)}_{1\to2} = mg(h_2 - h_1) = -mgR$$

Gravitational force

Height difference between 1 and 2

Work-Energy:

$$U^{(F)}_{1\to2} + U^{(gr)}_{1\to2} = T_2$$

Conservative Forces and Potential Energy

From our example, the work done by the weight was:

$$U_{1\to 2}^{(gr)} = mgh_2 - mgh_1 = -\left(V_2^{(gr)} - V_1^{(gr)}\right)$$

For a spring:

$$U_{1\to 2}^{(sp)} = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = -\left(V_2^{(sp)} - V_1^{(sp)}\right)$$

The work done by springs or the weight of a particle is **<u>independent</u>** of the path of which the forces act.

Such forces are called **<u>conservative</u>**, and their work can be computed using their potential energy functions:

$$V^{(sp)} = \frac{k}{2}(L - L_0)^2 \qquad V^{(gr)} = mgh$$

We can group them in a single function:

$$V = V^{(sp)} + V^{(gr)}$$

Conservative Forces and Potential Energy

The complete work-energy equation is thus:

$U_{1\to 2}^{(nc)} + U_{1\to 2}^{(c)} = T_2 - T_1$ **(work energy equation)**

$U_{1\to 2}^{(nc)} - (V_2 - V_1) = T_2 - T_1$ **(using the definition $U_{1\to 2}^{(c)} = -(V_2 - V_1)$)**

Rearranging:

$$T_2 + V_2 = T_1 + V_1 + U_{1\to 2}^{(nc)}$$

where

$T = \frac{1}{2} m v^2$: kinetic energy of the particle

$V = V^{(gr)} + V^{(sp)} = mgh + \frac{k}{2}(L - L_0)^2$: potential energy done by the conservative forces

$U_{1\to 2}^{(nc)} = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s}$: work done by non-conservative forces, $\vec{F}$

Recommendations for WE Problems

1. Don’t forget the datum line
2. **<u>ALL</u>** bodies go in a **<u>SINGLE</u>** work-energy equation
3. Identify which forces do work, and which don’t (normal is lazy, tangent does all the work)
4. Remember ALL your kinematics
5. Each problem has a “tiny trick”. Your best options are:
   - Get a lot of practice with the most general solution method (line integral)
   - Get familiar with all the possible types of problem (**recommended**)

ME 274: Basic Mechanics II

Week 8 – *Friday, March 6*

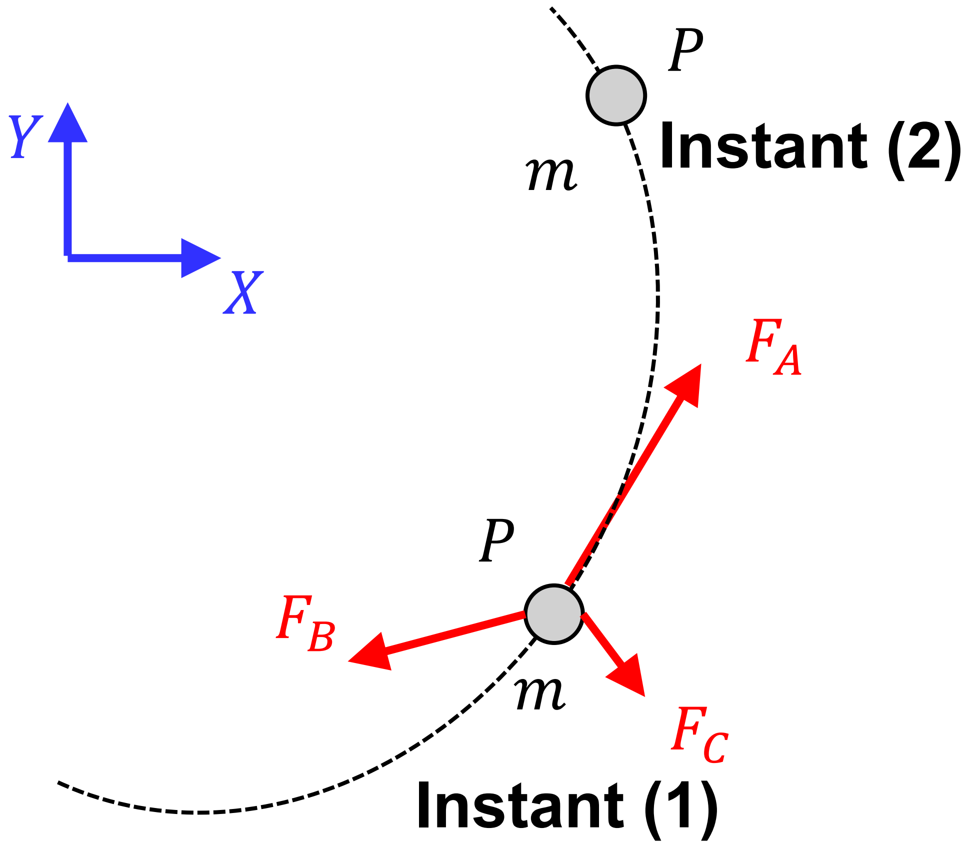
Linear Impulse and Momentum

Instructor: Manuel Salmerón

Today’s Agenda

1. Recap: Linear Impulse and Momentum
2. Example
3. Conservation of linear momentum
4. Example
5. Summary

Linear Impulse and Momentum



$$m\vec{v}_2 = m\vec{v}_1 + \int_{t_1}^{t_2} \left(\sum \vec{F}\right) dt$$

Vector expression!

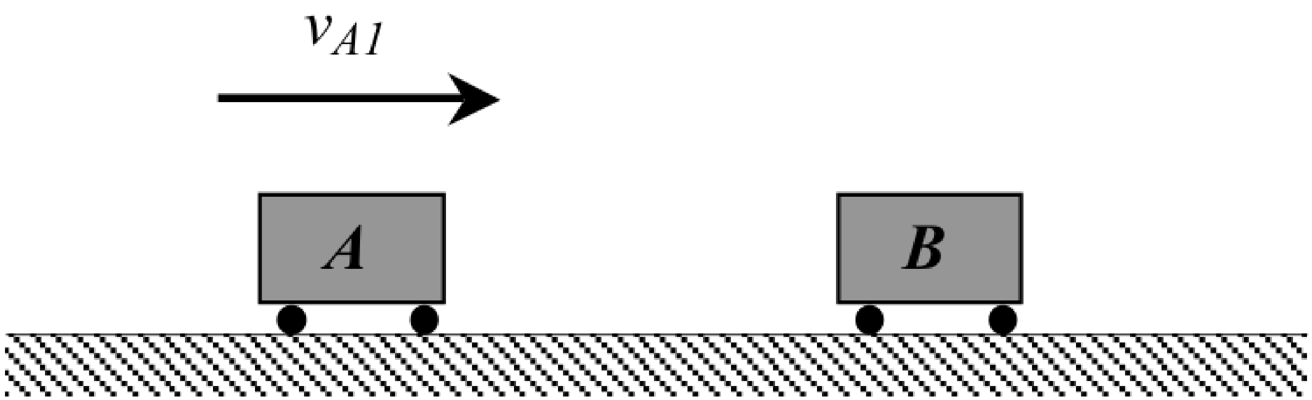
In X: $mv_{2x} = mv_{1x} + \int_{t_1}^{t_2} \left(\sum F_x\right) dt$

In Y: $mv_{2y} = mv_{1y} + \int_{t_1}^{t_2} \left(\sum F_y\right) dt$

Example 4.C.4

Given: Car A (having mass of m_A) travels to the right with an initial speed of v_{A1}. Car A then impacts a stationary car B (having a mass of m_B). After impact, the two cars stick together.

Find: The change in kinetic energy for the system of A and B together after the impact and resulting coupling. What fraction is this change to the initial kinetic energy of the system of A and B?



Two observations

1. It is a good idea to “hide” as many forces as we can
2. “Hidden” forces are called INTERNAL
3. “Visible” forces are called EXTERNAL
4. If we only have internal forces (in the direction we are analyzing), then **the momentum of instant 1 is equal to the momentum of instant 2**. For two bodies A and B:

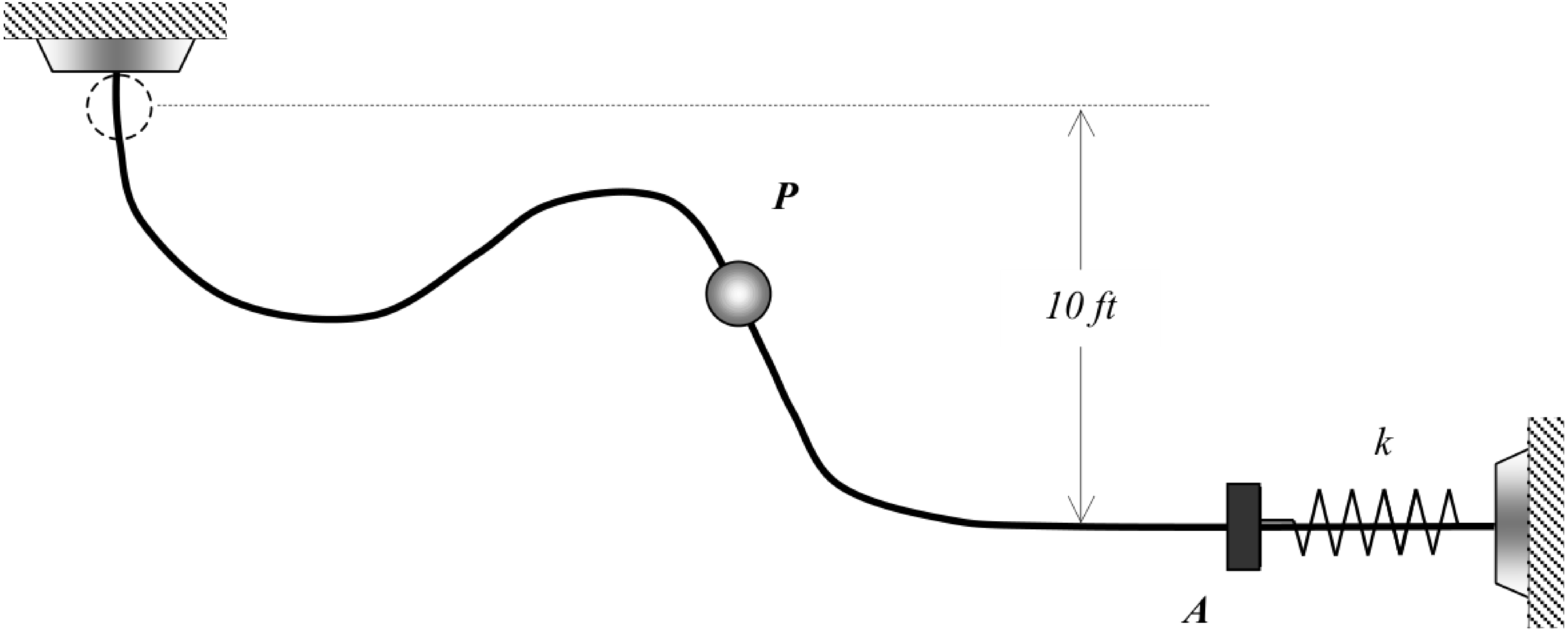
$$m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$$

CONSERVATION OF MOMENTM

Example 4.C.6

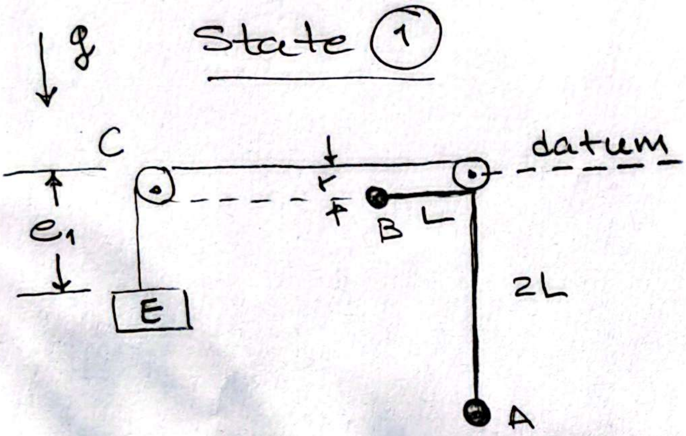
Given: Particle P (weighing 10 lb) is released from rest and slides down a smooth, curved rod and sticks to block A (weighing 5 lb).

Find: Determine the maximum deflection of the spring attached to A, if the spring has a stiffness of $k = 100$ lb/ft.

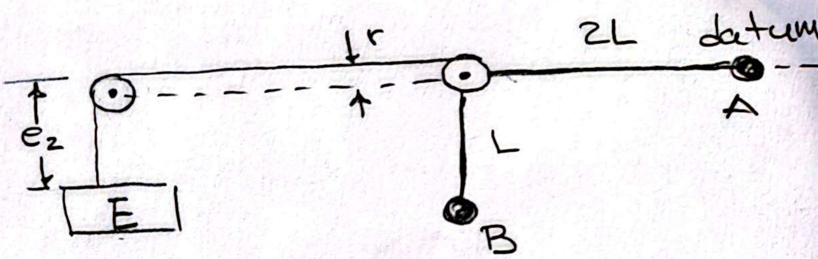


Example 4.B.6.

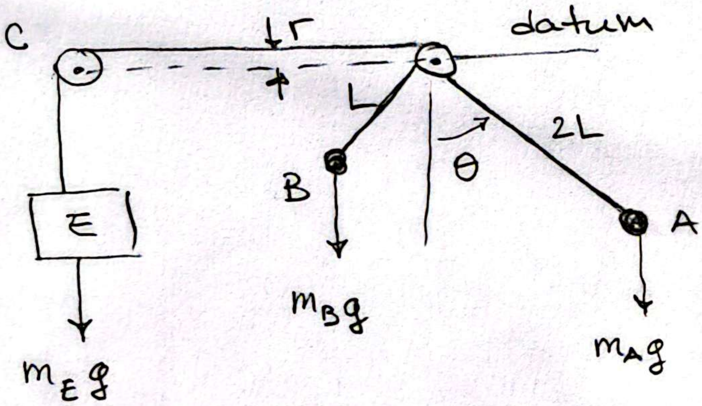
State ①



State ②



1. FBD



2. Kinetics

$$T_1 + V_1 + \mathcal{U}^{(nc)}_{1\to2} = T_2 + V_2$$

$T_1 = 0$: from rest

$V_1 = m_A g(-2L) + m_B g(0) + m_E g(-e_1)$ } ALL BODIES!

$\mathcal{U}^{(nc)}_{1\to2} = 0$: no "random" forces

$T_2 = \frac{1}{2} m_E v_{E_2}^2 + \frac{1}{2} m_B v_{B_2}^2 + \frac{1}{2} m_A v_{A_2}^2$ { ALL BODIES!!

$V_2 = m_B g(-L) + m_E g(-e_2)$

Substitute in the fund. eq:

I'm lazy, want to avoid writing so much!

$$0 - 2m_A gL - m_E g e_1 = T_2 - m_B gL - m_E g e_2$$

Rearranging and substituting T_2:

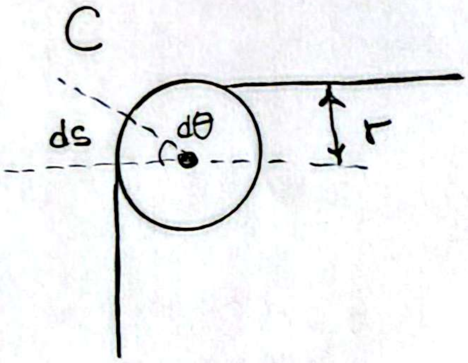
$$m_E\, g\underbrace{(e_2 - e_1)} = \frac{1}{2}m_E v_E^2 + \frac{1}{2}m_B v_{B_2}^2 + \frac{1}{2}m_A v_{A_2}^2 - m_B g L + 2m_A g L \quad \therefore \quad (1)$$

Relabel: ΔE

1 equation, 3 uknowns

3. Kinematics

Zoom in to the pulley at C:



If the pulley rotates a bit, i.e., $d\theta$, the rope segment moves ds. Thus:

$$ds = r\,d\theta \quad \left(\text{arc-length formula}\right)$$

Taking the time derivative:

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow \dot{s} = r\dot{\theta}$$

Thus:

$$\dot{s}_E = r\dot{\theta} = v_E$$

Now, for A and B: $\omega = \dot{\theta} = \dfrac{|v_B|}{|\vec{r}_{B/IC}|} = \dfrac{|v_A|}{|\vec{r}_{A/IC}|}$ (instant centers of rot.)

Hence: $\dot{\theta} = \dfrac{v_B}{L} = \dfrac{v_A}{2L}$

Since $\dot{\theta} = v_E/r$: $v_A = \dfrac{L}{r}v_E \ldots (2)$

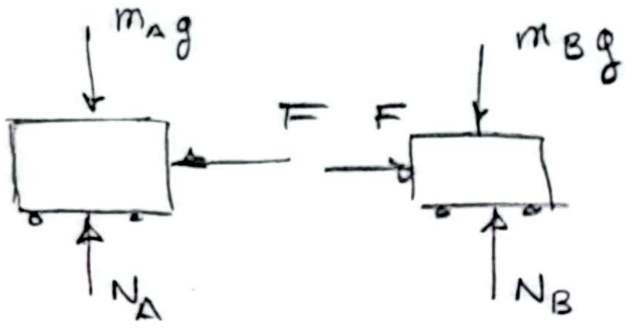
$v_B = \dfrac{2L}{r}v_E \ldots (3)$

4. Solve (1), (2), (3) to get v_A, v_B, and v_C.

If you made it until here, access the following link for one extra attendance point:
https://forms.gle/cjuaWBNqHxHTGeVL9

Example 4.C.4

1. FBD



2. Kinetics

$$m_A v_{A_2} = m_A v_{A_1} + \int_{t_1}^{t_2} (-F)\,dt$$

+

$$m_B v_{B_2} = m_B v_{B_1} + \int_{t_1}^{t_2} F\,dt$$

$$\boxed{m_A v_{A_2} + m_B v_{B_2} = m_A v_{A_1} + m_B v_{B_1}}$$

Conservation of linear momentum

3. Kinematics

$v_{B_1} = 0$, $v_{B_2} = v_{A_2} = v_2$

4. Solve:

$$m_A v_2 + m_B v_2 = m_A v_{A_1}$$

$$(m_A + m_B) v_2 = m_A v_{A_1} \Rightarrow v_2 = \frac{m_A}{m_A + m_B} v_{A_1}$$

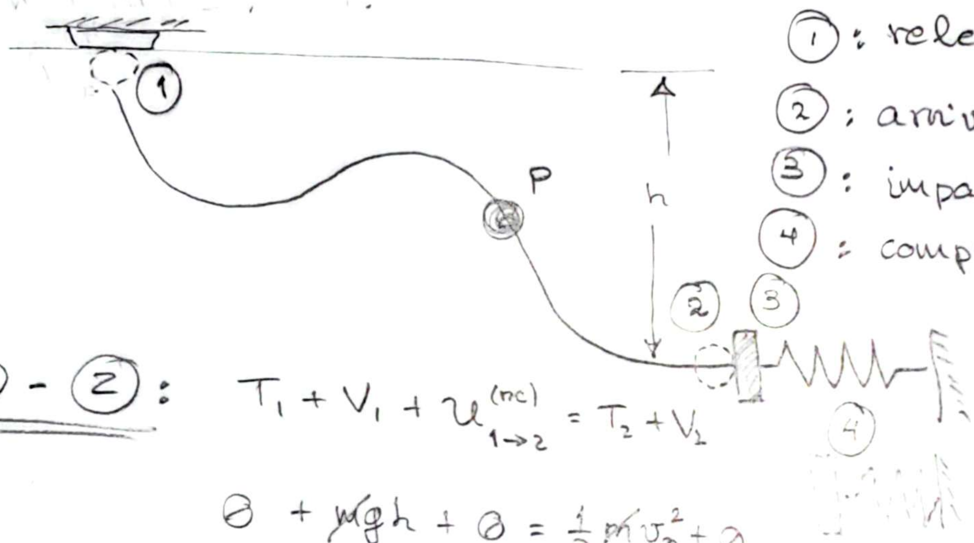
< 1 ! Impact reduces speed !

STOP HERE!

Kinetic energies: $T = \frac{1}{2} m v^2$

Example 4.C.6

- We want the deformation of the spring.
- We recall: $V^{(sp)} = \frac{1}{2}k\underbrace{\left(L-L_0\right)}_{\Delta:\text{ deformation}}^{2} = \frac{1}{2}k\Delta^2$



① : release

② : arrival

③ : impact

④ : compression

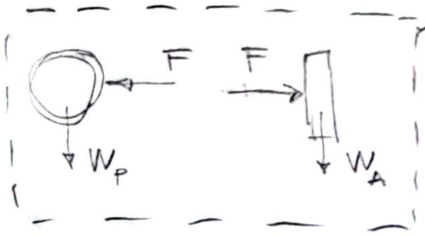
$m = \frac{W}{g}$

①-②: $T_1 + V_1 + \mathcal{U}^{(nc)}_{1\to2} = T_2 + V_2$

$$0 + \cancel{m}gh + 0 = \frac{1}{2}\cancel{m}v_{P_2}^2 + 0$$

$$v_{P_2} = \sqrt{2gh}$$

②-③:



I "hid" the internal forces F & −F!

$$m_P v_{P_2} + m_A \overset{0,\text{ rest}}{\cancel{v_{A_2}}} = m_P v_{P_3} + m_A v_{A_3}$$

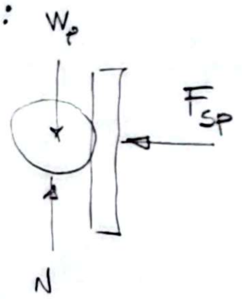
@ impact, $v_{P_3} = v_{A_3}$: $m_P v_{P_2} = (m_P + m_A)\,v_3$

$$v_{P_3} = v_{A_3} = \frac{m_P v_{P_2}}{m_P + m_A}$$

$e = 0$: perfectly plastic impact, $v_{P_3} - v_{A_3} = 0$

assumption

③-④:



$$T_3 + V_3 + \overset{0,\text{ no }\underline{nc}\text{ forces}}{\cancel{\mathcal{U}^{(nc)}_{3\to4}}} = T_4 + V_4$$

$T_3 = \frac{1}{2}(m_P + m_A)\,v_3^2$

$V_3 = 0$: no compression yet!

$T_4 = \frac{1}{2}k\Delta^2$: spring compresses

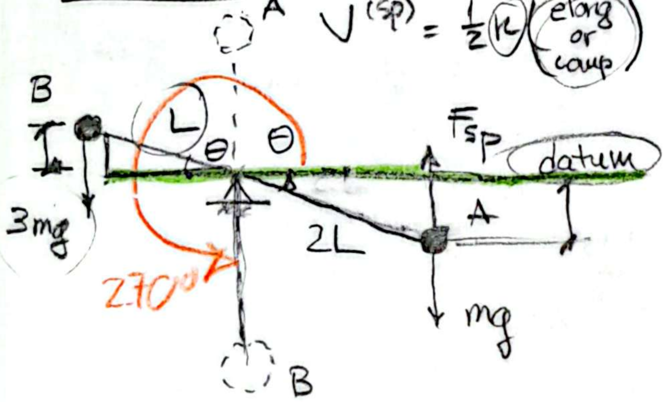
$V_4 = \frac{1}{2}(m_P + m_A)\,v_4^2 = 0$: $v_4 = 0$ because we reach max compression and then change directi

Example 4.B.7

1. FBD

$V^{(gr)} = mgh$

$V^{(sp)} = \frac{1}{2}k\left(\begin{matrix}\text{elong}\\ \text{or}\\ \text{comp}\end{matrix}\right)^2$



2. Kinetics

$$T_1 + V_1 + U^{(nc)}_{1\to 2} = T_2 + V_2$$

$$T_1 = \frac{1}{2}m\cancelto{0}{v_{A_1}^2} + \frac{1}{2}m\cancelto{0}{v_{B_1}^2} = 0 \quad \left(\text{from rest}\right)$$

$$V_1 = (3m)gL\sin\theta + mg(-2L\sin\theta) + \dots$$

$$\dots + \frac{1}{\cancel{2}}(2k)(2L\sin\theta)^2$$

$$U^{(nc)}_{1\to 2} = 0$$

$$T_2 = \underbrace{\frac{1}{2}(3m)v_{B_2}^2}_{\text{Particle B}} + \underbrace{\frac{1}{2}m v_{A_2}^2}_{\text{Particle A}}$$

$$V_2 = (3m)g(-L) + mg(2L) + 0$$

$$0 + 3mgL\sin\theta_1 - 2mgL\sin\theta_1 + k(2L\sin\theta_1)^2 = \dots$$

$$\dots = \frac{3}{2}m\underline{v_{B_2}^2} + \frac{1}{2}m\underline{v_{A_2}^2} \quad \dots (1)$$

$$- 3mgL + 2mgL$$

3. Kinematics

$$\left.\begin{matrix} s_A = 2L\theta \\ s_B = L\theta \end{matrix} \Longrightarrow \begin{matrix} \dot{s}_A = 2L\dot{\theta} = v_A \\ \dot{s}_B = L\dot{\theta} = v_B \end{matrix}\right\} \quad \underline{\underline{v_A = 2v_B}} \quad \dots (2)$$