ME 274: Basic Mechanics II

Week 10 – *Monday, March 23*

Particle Kinetics: Summary

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Today’s Agenda

1. Recap: Particle Kinetics Summary
2. Examples

Particle Kinetics Summary

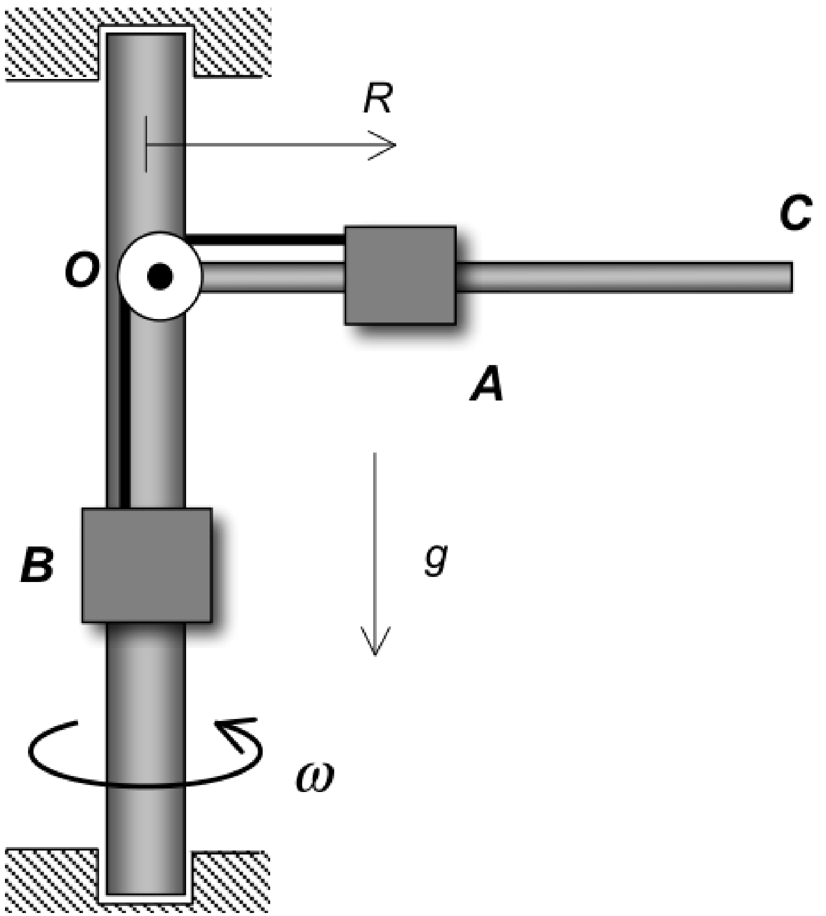
Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $where\ \ T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $where\ \ T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \bullet (\vec{\omega} \times \vec{r}_{G/A})$

Method	Body model	Fundamental equations
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_{O2} - \vec{H}_{O1}$ $where\ \ \vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\, dt = \vec{H}_{A2} - \vec{H}_{A1}$ $where\ \ \vec{H}_A = I_A\vec{\omega}$

Example 4.D.4

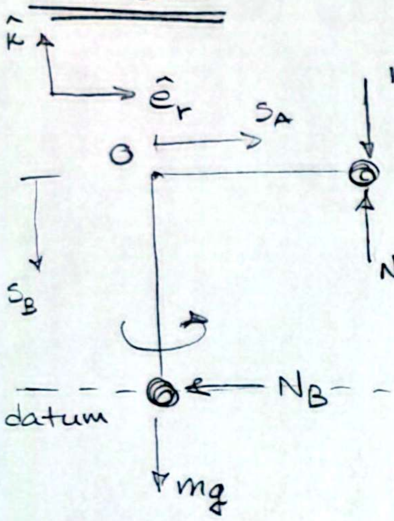
Given: Particles A and B, whose masses are 0.5 kg each, are NOT sliding over their smooth guides at a position of $R = 0.8$ m with $\omega = 6$ rad/s. Assume the mass of the guides and pulley to the negligible and that the influences of friction are negligible everywhere.

Find: For the position where $R = 1.2$ m, find:
(a) The angular speed of arm OC; and
(b) The speed of particle B.



Example 4.D.4

1. FBD



2. Kinetics : $(\vec{H}_O)_2 = (\vec{H}_O)_1 + \int_{t_1}^{t_2} (\Sigma M_O)$

$+\circlearrowleft \quad \Sigma M_O = -mgR + N_A R - N_B R_B$

$\Sigma F_{k_A} = -mg + N_A = m a_z = m(0) = 0$

$\boxed{N_A = mg}$

$\Sigma F_{r_B} = -N_B = m a_r = m(0) = 0$

$\boxed{N_B = 0}$

$\Sigma M_O = -mgR + mgR - 0(R_B) = 0$

$(\vec{H}_O)_1 = \left(\vec{r}_1 \times m\vec{v}_1\right)_A = (R_1 \hat{e}_r) \times m\vec{v}_{A/O} = (R_1 \hat{e}_r) \times m\omega R_1 \hat{e}_\theta = mR_1^2 \omega \hat{k}$

$+ (\cdots)_B \to$ none!

Recall:

$$\vec{v}_A = \vec{v}_O + (\omega \hat{k}) \times \vec{r}_{A/O} = (\omega \hat{k}) \times (R_1 \hat{e}_r) = \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{k} \\ 0 & 0 & \omega \\ R_1 & 0 & 0 \end{vmatrix} = +\omega R_1 \hat{e}_\theta$$

$(\vec{H}_O)_2 = \left(\vec{r}_2 \times m\vec{v}_2\right)_A = R_2 \hat{e}_r \times m\left(\dot{R}_2 \hat{e}_r + R_2 \omega_2 \hat{e}_\theta\right) = mR_2^2 \omega_2 \hat{k}$

Recall:

$$\vec{v}_A = \vec{v}_O + (\vec{v}_{A/O})_{rel} + \vec{\omega} \times \vec{r}_{A/O}$$

$$= \vec{0} + \dot{R}_2 \hat{e}_r + (\omega \hat{k}) \times (R_2 \hat{e}_r) = \dot{R}_2 \hat{e}_r + \omega R_2 \hat{e}_\theta$$

$(\vec{H}_O)_1 = (\vec{H}_O)_2 \; : \; mR_1^2 \omega \hat{k} = mR_2^2 \omega_2 \hat{k} \; : \; \omega_2 = \dfrac{R_1^2}{R_2^2} \omega$

v_B? : $T_1 + V_1 + \mathcal{U}_{1\to2}^{(nc)} = T_2 + V_2$

$T_1 = \frac{1}{2} m (\omega R_1)^2 \; ; \; V_1 = mg(0) = 0 \; ; \; \mathcal{U}_{1\to2}^{(nc)} = 0$

$T_2 = \frac{1}{2} m (v_A)^2 + \frac{1}{2} m (v_B)^2$

$v_A = \sqrt{\dot{R}_2^2 + \omega^2 R_2^2}$

Recall: the speed in polar coordinates is the sum of the squares of the components:
$\vec{v} = \dot{R}\hat{e}_r + R\dot{\theta}\hat{e}_\theta \Rightarrow |\vec{v}| = \sqrt{\dot{R}^2 + R^2\dot{\theta}^2}$

for v_B :

3. Kinematics

$$L = S_A + S_B$$

$$0 = \dot{S}_A + \dot{S}_B = \dot{R}_{2A} + \dot{R}_{2B} \Rightarrow \dot{R}_{2B} = -\dot{R}_{2A} = -\dot{R}_2$$

So: $$T_2 = \frac{1}{2} m \left(\dot{R}_2^{\,2} + \omega^2 R_2^2 \right) + \frac{1}{2} m \dot{R}_2$$

$$V_2 = mg \left(R_2 - R_1 \right)$$

Substitute and solve for $\dot{R}_2$.