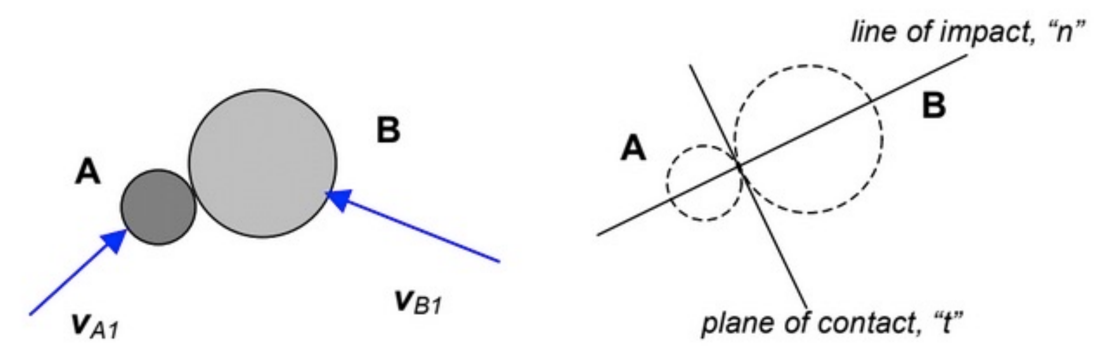
CENTRAL IMPACT

3/9/2026

Particles A & B approach each other @ $\vec{v}_{A1}$ and $\vec{v}_{B1}$ and impact each other. What would the velocities immediately after the impact $\vec{v}_{A2}$ and $\vec{v}_{B2}$ be?



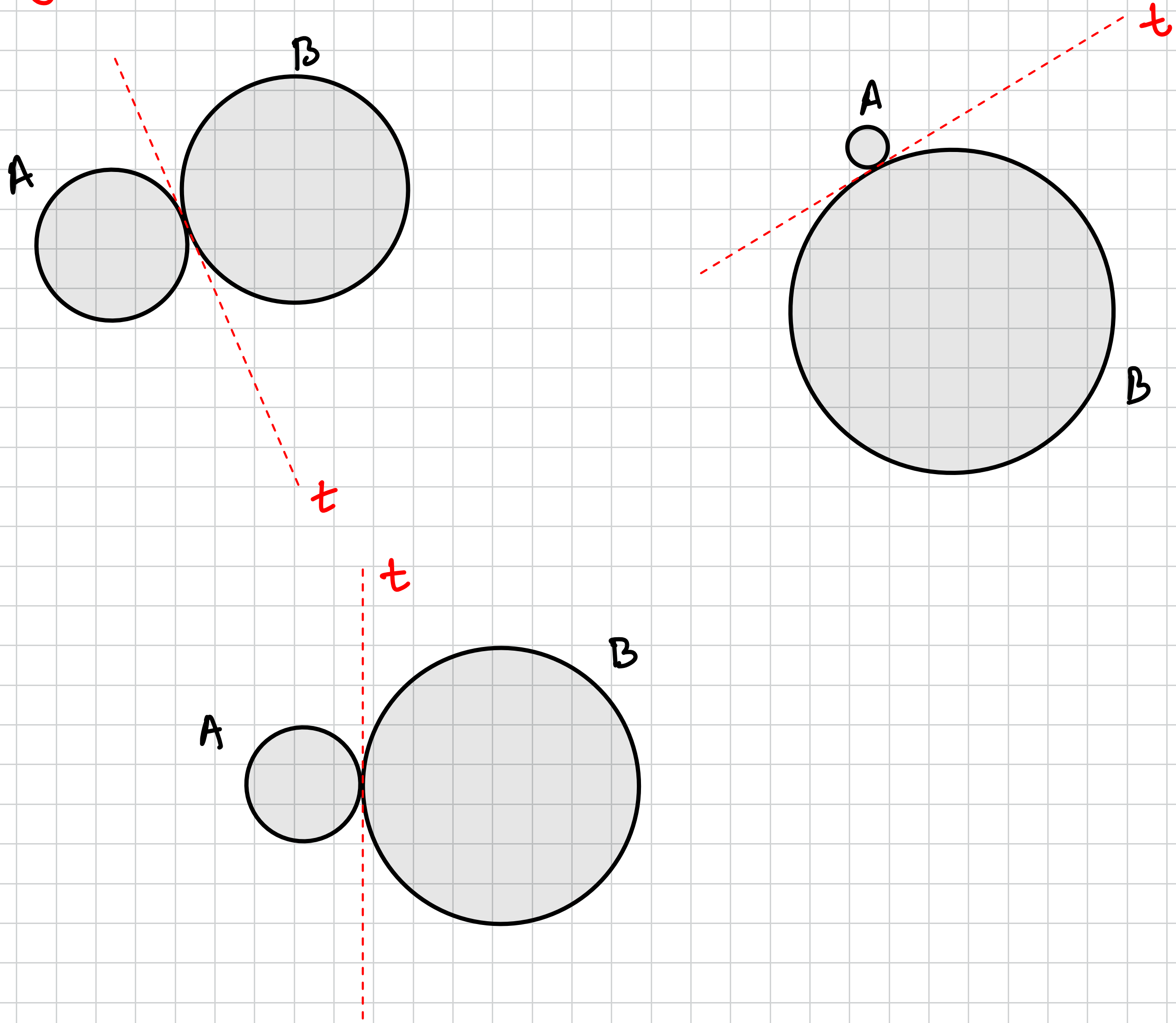
(a) central impact (b) coordinate axes

Impact refers to the collision between two bodies and is characterized by large contact forces acting over a very short interval of time. In this section, we will introduce direct and oblique central impact.

In setting up the central impact problem, let's consider the following definitions:

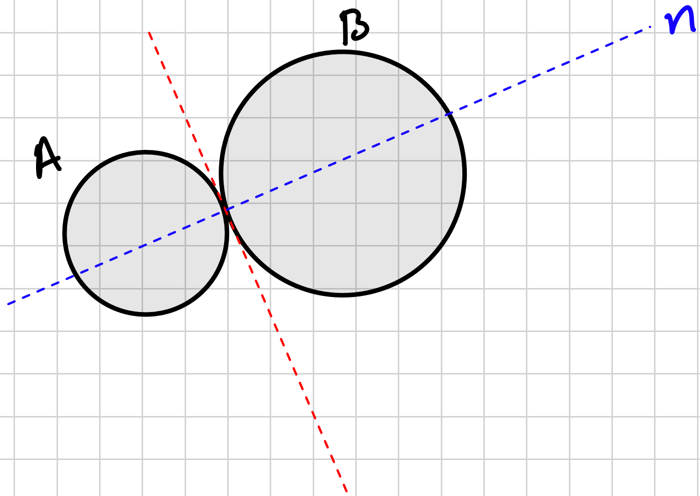
Plane of contact: (tangent, line in 2D)

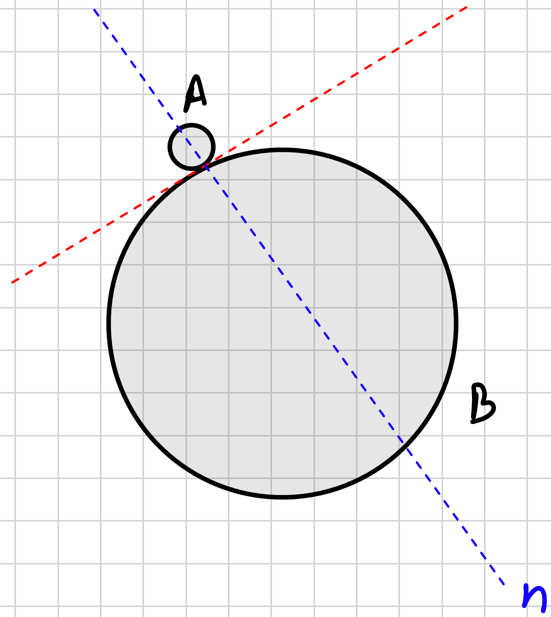
Plane tangent to the contact surfaces of A & B

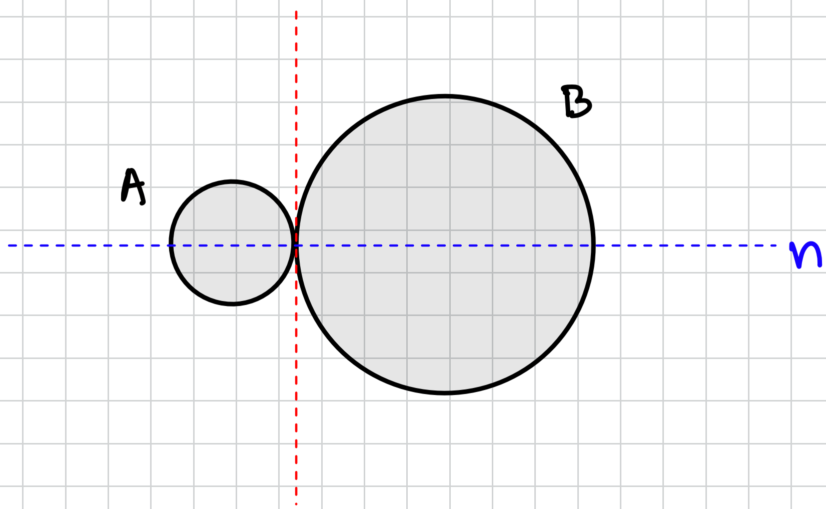


Line of impact : (normal)

Line normal to plane of contact (can have any direction)





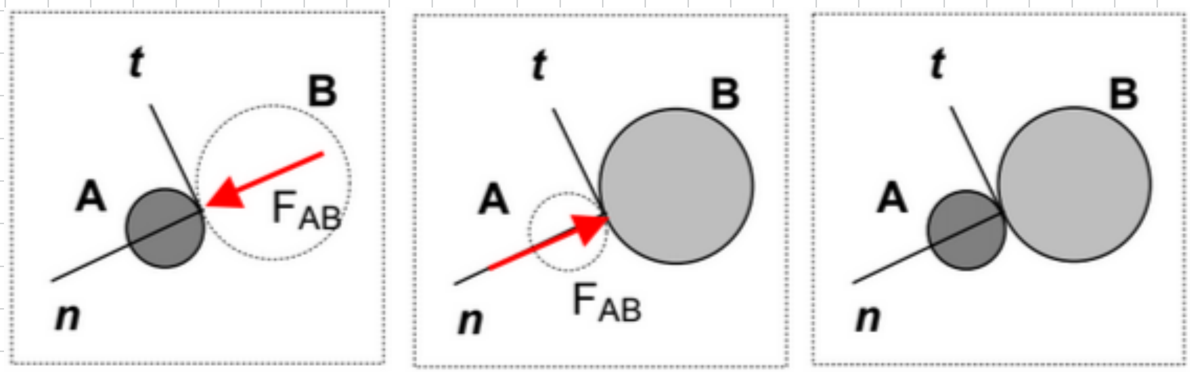


Central impact:

Impact in which the line of impact passes through the centers of mass of both particles.

In this analysis, we consider three systems:

Body A alone	Body B alone	Bodies A&B together



system A　　system B　　system AB

1: BEFORE

2: AFTER

n: $V_{An_2} \neq V_{An_1}$

t: $V_{At_2} = V_{At_1}$

$F_{AB} \neq 0$

n: $V_{Bn_1} \neq V_{Bn_2}$

t: $V_{Bt_1} = V_{Bt_2}$

$F_{AB} \neq 0$

n: $m_A V_{An_1} + m_B V_{Bn_1}$

$= m_B V_{Bn_2} + m_B V_{Bn_2}$

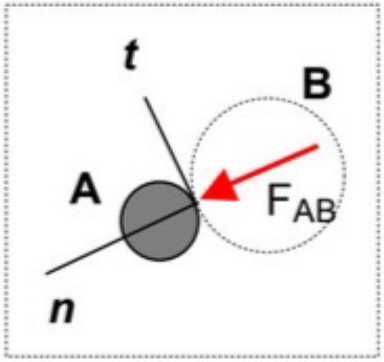
t: no new information

System A:

$$\Sigma F_t = 0 \Rightarrow \cancel{m_A} V_{At2} = \cancel{m_A} V_{A1t}$$

$$\Rightarrow V_{At2} = V_{At1} \quad (1)$$

$\Sigma F_n = F_{AB} \neq 0 \Rightarrow$ momentum NOT conserved in the "n" direction for A



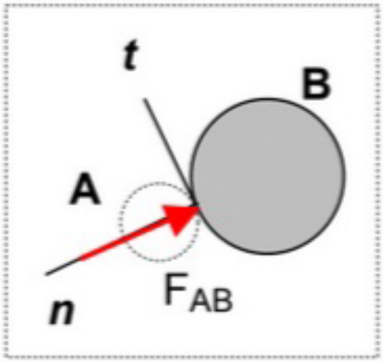
system A

System B

$$\Sigma F_t = 0 \Rightarrow \cancel{m_B} V_{Bt2} = \cancel{m_B} V_{Bt1}$$

$$\Rightarrow V_{Bt2} = V_{Bt1} \quad (2)$$

$\Sigma F_n = -F_{AB} \neq 0 \Rightarrow$ momentum NOT conserved in the "n" direction for B



system B

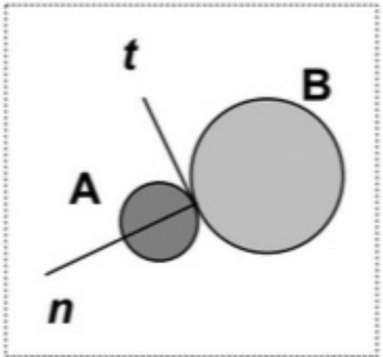
System AB

$$\Sigma F_t = 0$$

$$\Rightarrow m_A V_{At2} + m_B V_{Bt2} = m_A V_{At1} + m_B V_{Bt1}$$

$$\Sigma F_n = 0$$

$$\Rightarrow m_A V_{An2} + m_B V_{Bn2} = m_A V_{An1} + m_B V_{Bn1} \quad (3)$$



system AB

Coefficient of restitution

Another equation is required to find the final velocities as the momentum equation contains two unknowns: v_{A2} & v_{B2}. Let's look at the capacity of A and B to recover from the impact, i.e., the ratio e of the magnitude of the restoration impulse to the magnitude of the deformation impulse $\Rightarrow$ coef. of restitution:

$$(4)\quad e = -\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}} \quad \frac{\leftarrow \text{rel. velocity of separation}}{\leftarrow \text{rel. velocity of approach}}$$

Only the normal components of the velocities are used in the e equation.

SUMMARY – CENTRAL IMPACT

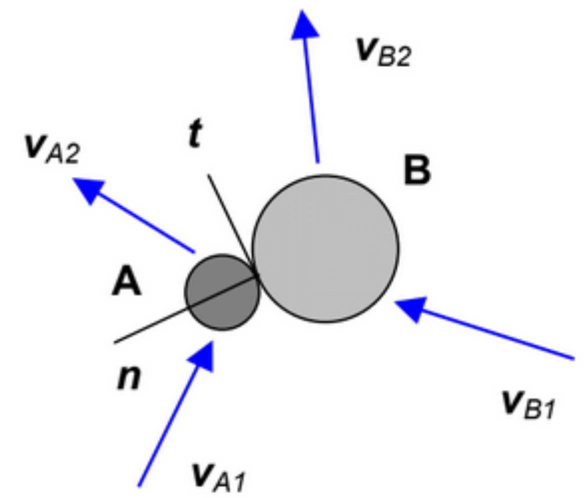
(1) $v_{At2} = v_{At1}$

(2) $v_{Bt2} = v_{Bt1}$

(3) $m_A v_{An2} + m_B v_{Bn2}$

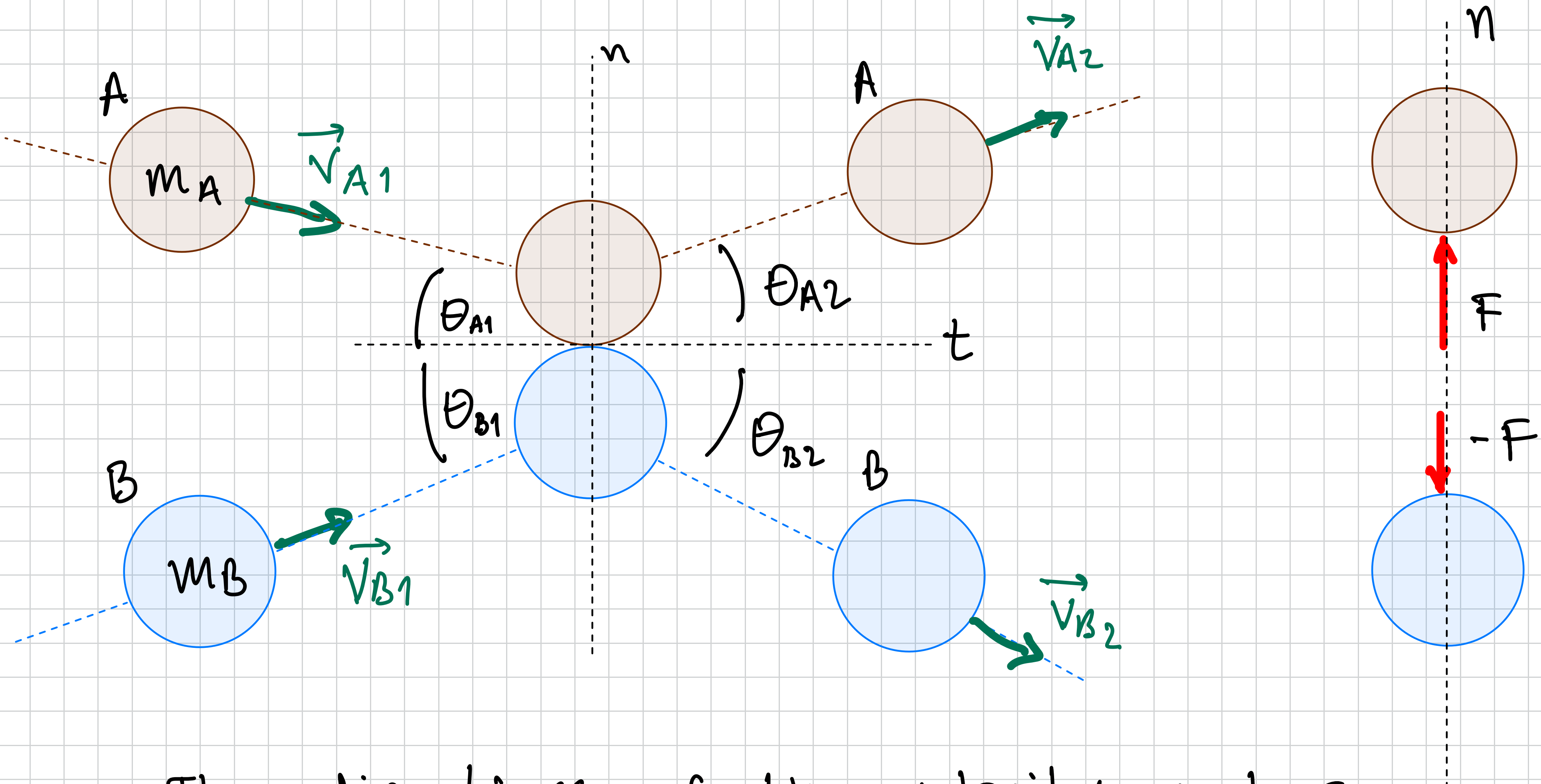
$= m_A v_{An1} + m_B v_{Bn1}$

(4) $$e = -\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}$$



OBLIQUE CENTRAL IMPACT

Let us extend our analysis to cases where V_1 is not parallel to V_2



The directions of the velocity vectors are measured from the tangent plane of contact.

$$\Rightarrow \quad V_{A1n} = -V_{A1}\sin\theta_{A1} \qquad V_{A1t} = V_{A1}\cos\theta_{A1}$$

$$V_{B1n} = V_{B1}\sin\theta_{B1} \qquad V_{B1t} = V_{B1}\cos\theta_{B1}$$

This produces four unknowns: V_{A2n}, V_{A2t}, V_{B2t}, V_{B2n}

therefore, we use:

$$m_A V_{A1n} + m_B V_{B1n} = m_A V_{A2n} + m_B V_{B2n}$$

And, since momentum is conserved in the t direction:

$$m_A V_{A1t} = m_A V_{A2t}$$

$$m_B V_{B1t} = m_B V_{B2t}$$

Along with

$$e = \frac{V_{B2n} - V_{A2n}}{V_{A1n} - V_{B1n}}$$

Finally, θ_{A2} and θ_{B2} can be obtained.

→ Impulsive forces are large in magnitude and short in duration.

→ Reaction forces can be impulsive. Spring and gravitational forces are NOT impulsive

→ In problems, make sure to clearly define t-n coordinates @ beginning of solution

→ The e equation is only valid for velocity components along the line of impact.

→ Mechanical energy is not conserved during impact. Do NOT use the work-energy equation for impact problems.

Example 4.C.11

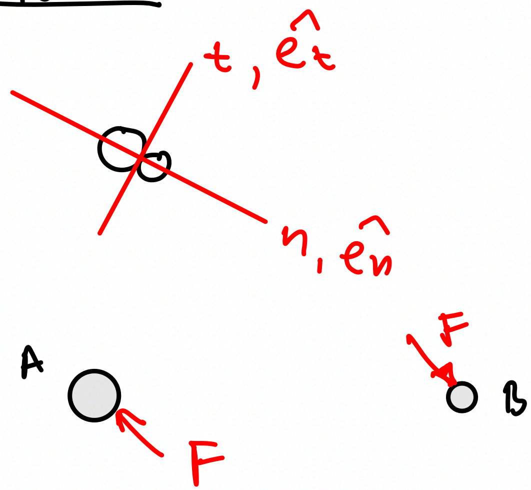
Given: Sphere A has a mass of 20 kg and a radius of 75 mm, while B has a mass of 5 kg and a radius of 48 mm. The coefficient of restitution for the impact of A and B is known to be $e = 0.6$.

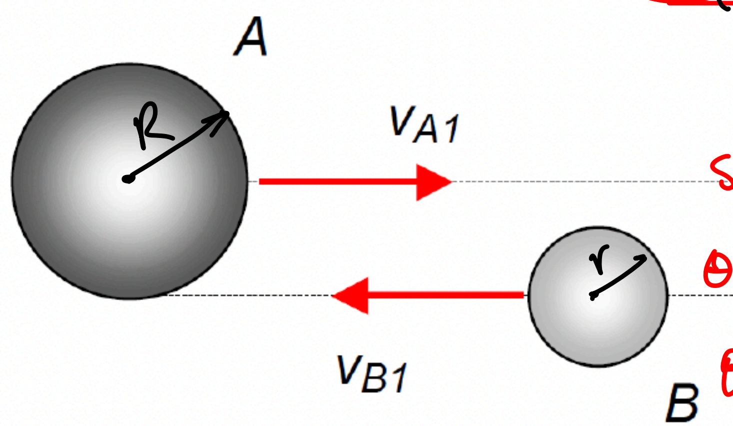
Find: Determine the velocities of the spheres immediately after impact.

Use the following parameters in your analysis: $v_A = 5$ m/s and $v_B = 15$ m/s.

Solution

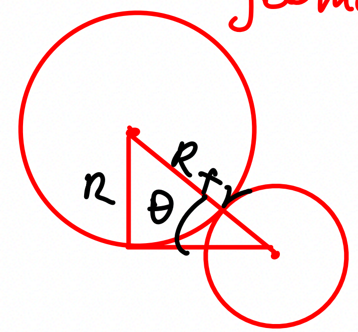
1) FBD





HORIZONTAL PLANE

3) Kinematics/geometry



$$\sin\theta = \frac{R}{R+r}$$

$$\theta = \sin^{-1}\left(\frac{R}{R+r}\right)$$

$$\theta = \sin^{-1}\left(\frac{75}{75+48}\right)$$

$$\theta = 37.6^\circ$$

2) Kinetics → Linear momentum in t-dir is conserved For each particle

$$\cancel{m_A} V_{A1t} = \cancel{m_A} V_{A2t}$$

$$V_{A1} \sin\theta = V_{A2t} \implies V_{A2t} = 5 \sin 37.6^\circ$$

$$(1) \quad V_{A2t} = 3.05 \text{ m/s}$$

$\cancel{m_B}\,(-V_{B1t}) = \cancel{m_B}\,V_{B2t}$

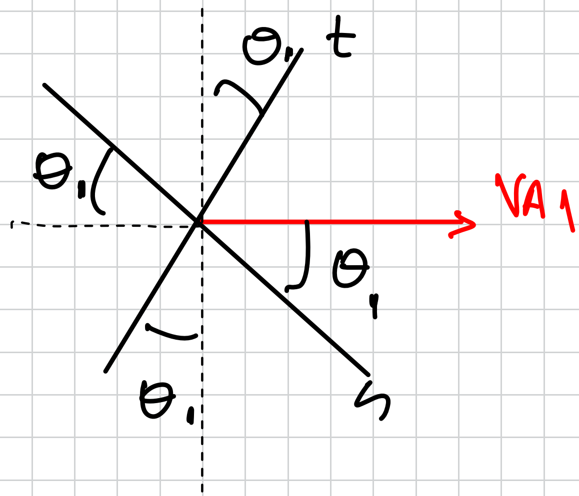
$-V_{B1}\sin\theta = V_{B2t}$

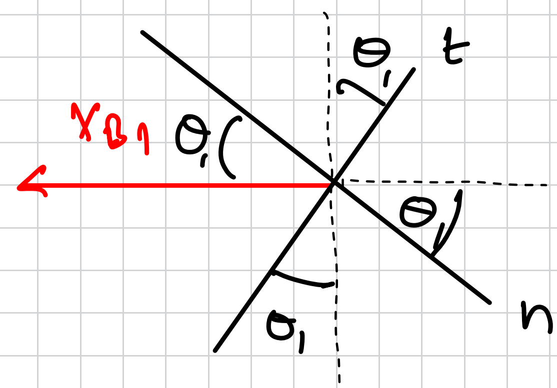
$\Rightarrow V_{B2t} = -15\sin 37.6°$

(2) $V_{B2t} = -9.15$ m/s

→ Linear momentum in n-dir conserved for entire sys.

$m_A V_{A1n} + m_B V_{B1n} = m_A V_{A2n} + m_B V_{B2n}$





$m_A V_{A1}\cos\theta_1 - m_B V_{B1}\cos\theta_1 = m_A V_{A2n} + m_B V_{B2n}$

$\Rightarrow 20(5\cos 37.6°) - 5(15\cos 37.6°) = 20V_{A2n} + 5V_{B2n}$

$79.2 - 59.5 = 20V_{A2n} + 5V_{B2n}$

$\Rightarrow 20V_{A2n} + 5V_{B2n} = 19.8$ (3)

$\rightarrow$ Impact rule (coef. rest)

$$e = \frac{V_{B2n} - V_{A2n}}{V_{A1n} - V_{B1n}} = \frac{V_{B2n} - V_{A2n}}{V_{A1} \cos\theta_1 - (-V_{B1} \cos\theta_1)}$$

$$e(V_{A1} \cos\theta_1 + V_B \cos\theta_1) = V_{B2n} - V_{A2n}$$

$$0.6(5 \cos 37.6^\circ + 15 \cos 37.6^\circ) = V_{B2n} - V_{A2n}$$

$$\Rightarrow V_{B2n} - V_{A2n} = 9.5 \text{ m/s} \quad (4)$$

A) Solve From (4)

$$V_{B2n} = 9.5 + V_{A2n}$$

plug in (3)

$$20 V_{A2n} + 5(9.5 + V_{A2n}) = 19.8$$

$$V_{A2n}(20 + 5) = 19.8 - 5(9.5)$$

$V_{A2n} = -1.11$ m/s $\longrightarrow$ in (4) $\Rightarrow V_{B2n} = 8.4$ m/s

$$V_{A2} = +3.05\,\hat{e}_t - 1.11\,\hat{e}_n$$

$$V_{B2} = -9.15\,\hat{e}_t + 8.4\,\hat{e}_n$$

ANSWER.

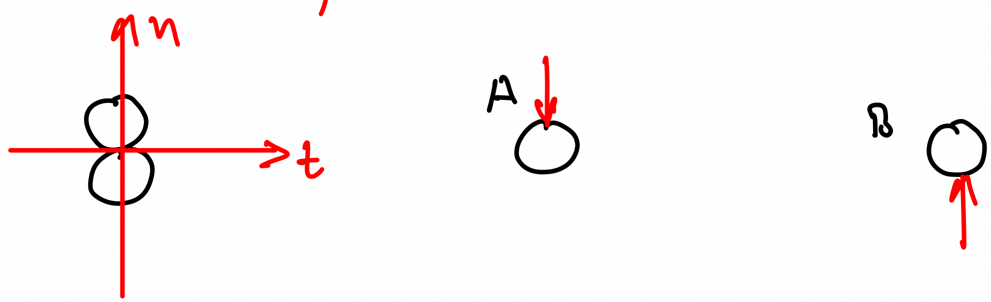
Example 4.C.12

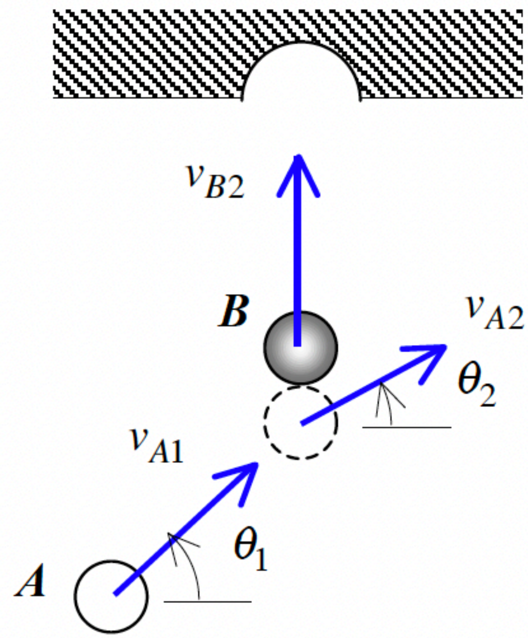
Given: Cue ball A strikes a stationary object ball B, with a speed v_{A1} as shown in the figure below. The coefficient of restitution for this impact is e. After impact, A moves along a line defined by the angle θ_2, and B moves directly to the side pocket.

Find: Determine the numerical value of the rebound angle θ_2 of A, assuming the masses of A and B are the same.

Use the following parameters in your analysis: $\theta_1 = 45°$ and $e = 0.9$.

Solution 1) FBD





2) Kinetics – Linear momentum in n-dir conserved for entire system:

$$m v_{A1} + m \cancel{v_{B1}}^{\,0} = m v_{A2} + m v_{B2}$$

$$\cancel{m} v_{A1} \sin\theta_1 = \cancel{m} v_{A2} \sin\theta_2 + \cancel{m} v_{B2}$$

$$\frac{\sqrt{2}}{2} v_{A1} = v_{A2} \sin\theta_2 + v_{B2} \qquad (1)$$

- Conservation of Linear momentum in t-dir:

A: $\cancel{m} V_{A1} \cos\theta_1 = \cancel{m} V_{A2} \cos\theta_2$

$$\frac{\sqrt{2}}{2} V_{A1} = \underbrace{V_{A2} \cos\theta_2}_{V_{A2t}} \Rightarrow V_{A2t} = \frac{\sqrt{2}}{2} V_{A1} \quad (2)$$

B: $0 = 0$

- Impact rule

$$e = \frac{V_{B2n} - V_{A2n}}{V_{A1n} - V_{B1n}} = \frac{V_{B2} - V_{A2}\sin\theta_2}{V_{A1}\sin\theta_1 - 0}$$

$$e V_A \sin\theta_1 = V_{B2} - V_{A2}\sin\theta_2$$

$$\frac{\sqrt{2}}{2} e V_{A1} = -V_{A2}\sin\theta_2 + V_{B2} \quad (3)$$

3) Solve

Add (1) + (3)

$$\frac{\sqrt{2}}{2}V_{A1} = \cancel{V_{A2}\sin\theta_2} + V_{B2}$$

$$+\ \frac{\sqrt{2}}{2}eV_{A1} = \cancel{-V_{A2}\sin\theta_2} + V_{B2}$$

$$\frac{\sqrt{2}}{2}(1+e)V_{A1} = 2V_{B2}$$

$$V_{B2} = \frac{\sqrt{2}}{4}(1+e)V_{A1} \quad (4)$$

Subtract (1) − (3)

$$\frac{\sqrt{2}}{2}V_{A1} = V_{A2}\sin\theta_2 \cancel{+V_{B2}}$$

$$-\ \frac{\sqrt{2}}{2}eV_{A1} = V_{A2}\sin\theta_2 \cancel{-V_{B2}}$$

$$\frac{\sqrt{2}}{2}(1-e)V_{A1} = 2\underbrace{V_{A2}\sin\theta_2}_{V_{A2n}}$$

$$\frac{\sqrt{2}}{4}(1-e)V_{A1} = V_{A2n} \quad (5)$$

Combine (2) and (5)

$$\Rightarrow \vec{V}_{A2} = V_{A2t}\,\hat{e}_t + V_{A2n}\,\hat{e}_n$$

$$\vec{V}_{A2} = \frac{\sqrt{2}}{2}V_{A1}\,\hat{e}_t + \frac{\sqrt{2}}{4}(1-e)V_{A1}\,\hat{e}_n \quad m/s$$

Angle θ_2 is defined by the direction of $\vec{V}_{A2}$

$$\Rightarrow \theta_2 = \tan^{-1}\left(\frac{V_{A2n}}{V_{A2t}}\right) = \tan^{-1}\left[\frac{\frac{\cancel{\sqrt{2}}}{\cancel{4}\,2}(1-e)\cancel{V_{A1}}}{\frac{\cancel{\sqrt{2}}}{\cancel{2}}\cancel{V_{A1}}}\right] \Rightarrow \boxed{\theta_2 = \tan^{-1}(0.05) \cong 2.86^\circ}$$

ANSWER