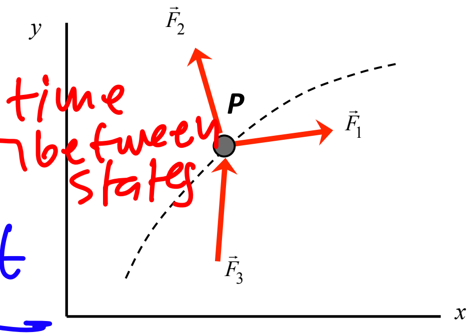
3/6/26

Summary: Linear Impulse-Momentum Equation 1

FUNDAMENTAL equation: the linear impulse-momentum equation:



$$m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \left(\sum \vec{F}\right) dt$$

short time between states

$$\Rightarrow \underbrace{m\vec{v}_1}_{\text{state 1}} + \overbrace{\int_1^2 \left(\Sigma \vec{F}_R\right) dt}^{\text{short time between states}} = \underbrace{m\vec{v}_2}_{\text{state 2}},$$

CONSERVATION: If there is no net force acting of the system in a given direction (say x), $\int_1^2 \left(\sum \vec{F}\right)_x dt = 0$ *, then linear momentum in that direction is conserved.*

SYSTEM CHOICE: Make your choice of system as "large" as reasonable – you want to make as many forces as possible INTERNAL to the system.

W/E - LIM

Some problems require the combined use of work-energy and linear impulse & momentum

As such, both distinctive hints will appear in these types of problem statements:

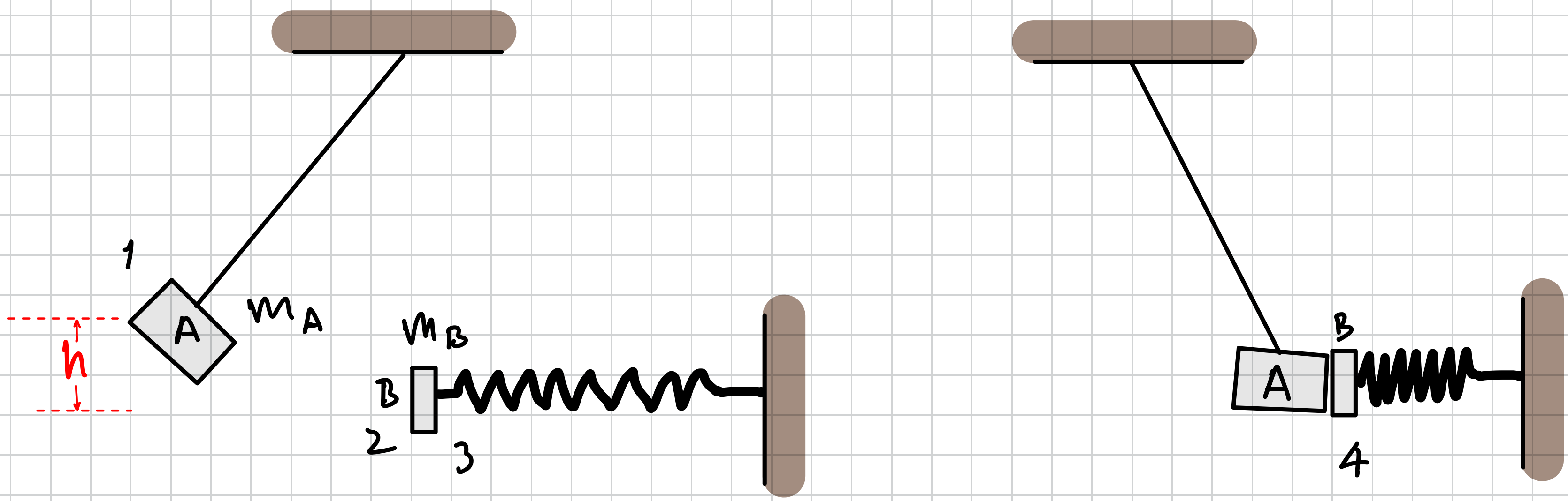
1) Changes in velocity after changes in position

2) Implications of impacts or short time intervals.

The solution procedure may imply more than two states, depending on different intervals stated in problems.

Typical cases include impacts mixed with changes in vertical positions, changes in spring positions and velocities at different states.

For example:



1-2: $T_1 + V_1 + U_{1-2}^{NC} = T_2 + V_2$

$$0 + m_A g h + 0 = \frac{1}{2} m_A v_2^2 + 0 \Rightarrow v_2 = \sqrt{2gh}$$

2-3: $m_A v_2 + \int_1^2 F\,dt = (m_A + m_B) v_3$

$$m_A \sqrt{2gh} + 0 = (m_A + m_B) v_3 \Rightarrow v_3 = \frac{m_A \sqrt{2gh}}{m_A + m_B}$$

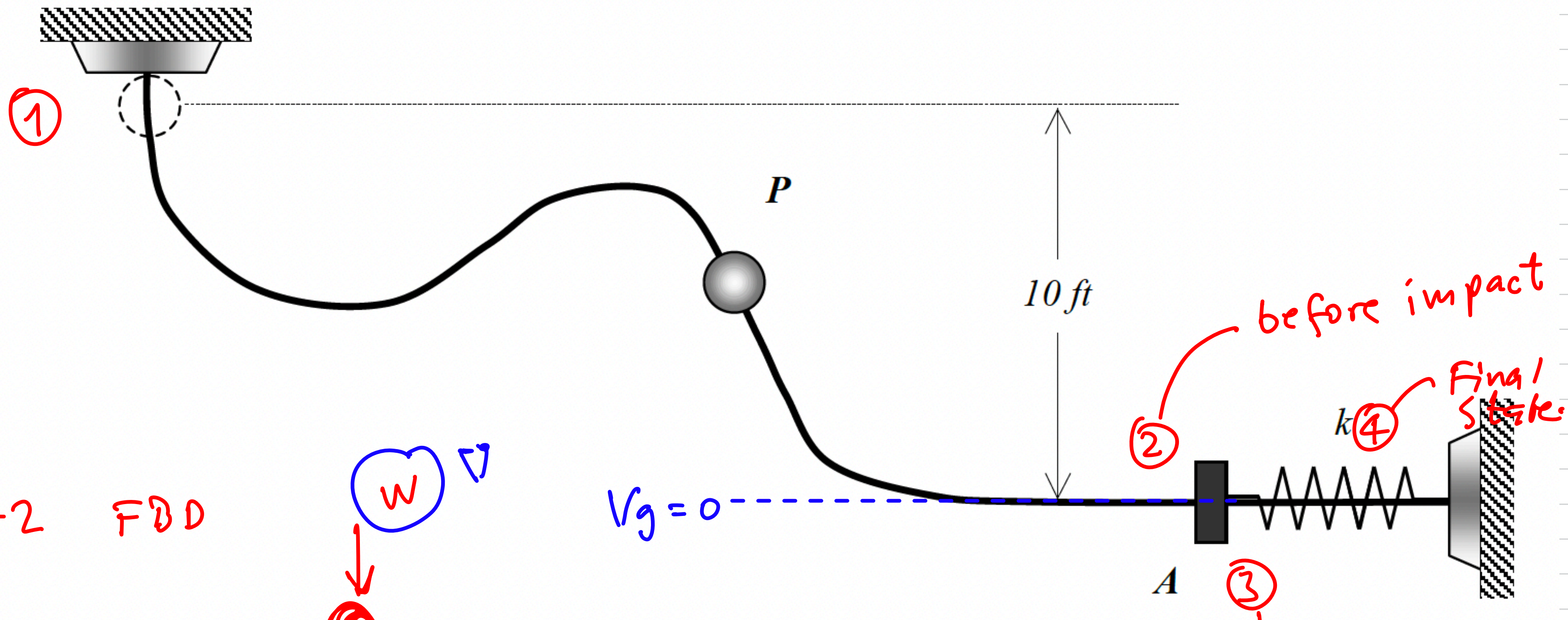
3-4: $T_3 + V_3 + U_{3-4}^{(NC)} = T_4 + V_4$

$$\frac{1}{2}(m_A + m_B) v_3^2 + 0 + 0 = 0 + \frac{1}{2} k \Delta^2 \Rightarrow \Delta =$$

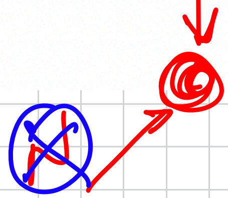
Example 4.C.6

Given: Particle P (weighing 10 lb) is released from rest and slides down a smooth, curved rod and sticks to block A (weighing 5 lb).

Find: Determine the maximum deflection of the spring attached to A, if the spring has a stiffness of $k = 100$ lb/ft.



1-2 FDD

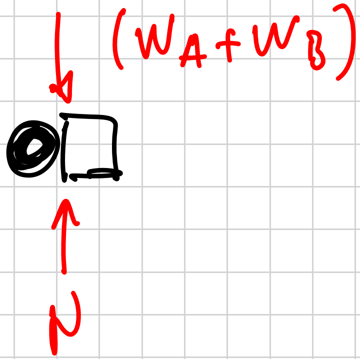


$$\cancel{T_1} + V_1 + U_{1-2}^{NC} = T_2 + V_2$$

$$0 + W_p \cdot h + 0 = \frac{1}{2}\frac{W_p}{g}V_2^2 + 0$$

$$\Rightarrow \quad V_2 = \sqrt{2gh} = 25.3 \text{ ft/s}$$

2-3 L.I.M in x direction, (spring not compressed yet)

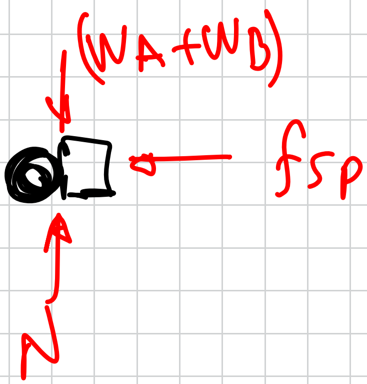


$$m_A V_2 + \int_2^3 R\,dt = (m_A + m_B)V_3$$

$$\Rightarrow \quad \frac{W_p}{g}V_2 + 0 = \frac{(W_A + W_B)}{g}\cdot V_3$$

$$\Rightarrow \quad V_3 = \frac{W_p}{W_p + W_A}V_2 = 8.45 \text{ ft/s}$$

3-4



$$T_3 + V_3 + U^{NC}_{3-4} = T_4 + V_4$$

$$\frac{1}{2}\frac{(W_p + W_A)}{g}V_3^2 + 0 + 0 = 0 + \frac{1}{2}K\Delta^2$$

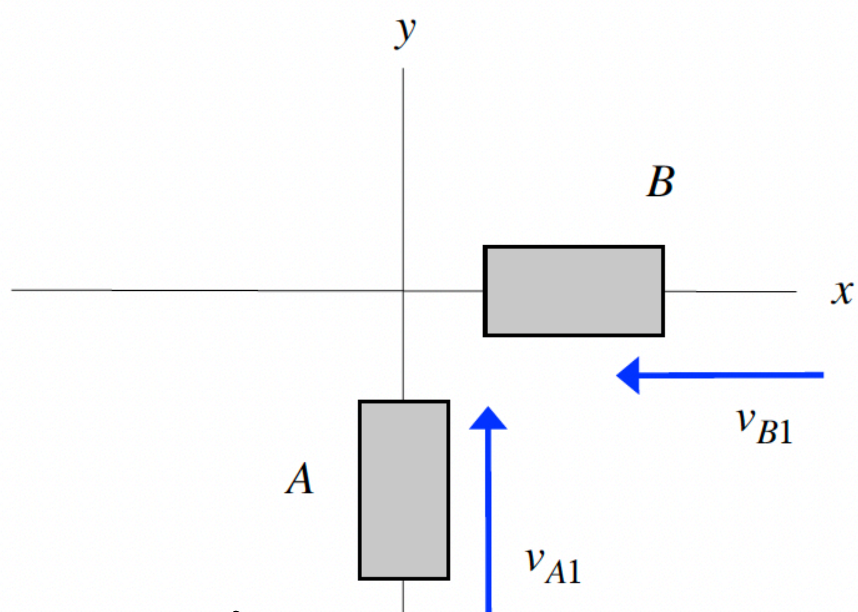
$$\Rightarrow \quad \Delta = \sqrt{\frac{W_p + W_A}{Kg}}\,V_3 = 0.57 \text{ ft}$$

Example 4.C.7

Given: Cars A and B (having weights of W_A and W_B, respectively) strike each other with speeds of v_{A1} and v_{B1}. After a short collision time, the cars stick together.

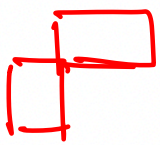
Find: Determine the x and y components of velocity for the cars after the collision.

Use the following parameters in your analysis: $W_A = 3000$ lb, $W_B = 4000$ lb, $v_{A1} = 40$ mph and $v_{B1} = 25$ mph.



Solution

1) FBD

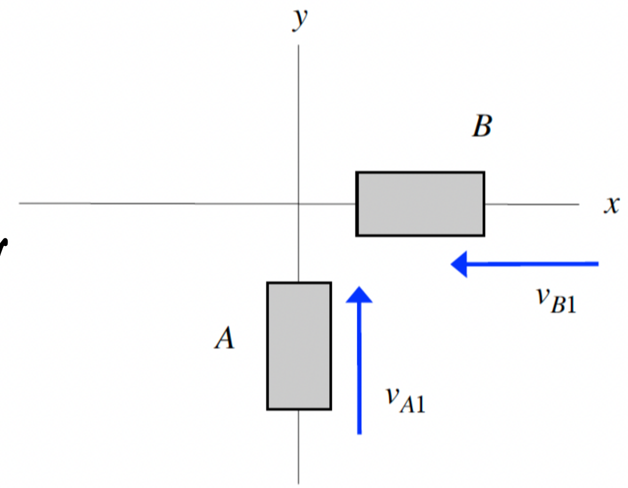


2) LIM in x-dir

$$\frac{W_B}{g}(-v_{B1}) + 0 = \left(\frac{W_A+W_B}{g}\right) V_{2x}$$

$$V_{2x} = -\frac{W_B}{W_A+W_B} V_{B1}$$

LIM in y-dir



$$\frac{W_A}{g} V_{A1} + 0 = \left(\frac{W_A + W_B}{g}\right) V_{2y}$$

$$\Rightarrow V_{2y} = \frac{W_A}{W_A + W_B} V_{A1}$$

So the final velocity

$$\vec{v}_2 = -\frac{W_B}{W_A + W_B} V_{B1}\,\hat{i} + \frac{W_A}{W_A + W_B} V_{A1}\,\hat{j}$$

Example 4.C.8

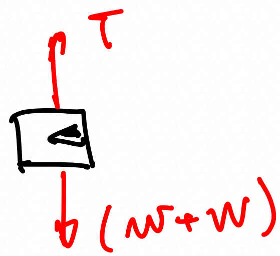
Given: Block A (having a weight of W) is suspended by a cord from fixed point O. Bullet B (having a weight of w) strikes the stationary block A with a speed of v_{B1}. On impact, the bullet sticks to block A.

Find: Determine:
(a) The maximum elevation angle θ of the cord after impact; and
(b) The energy lost during the impact of B with A.

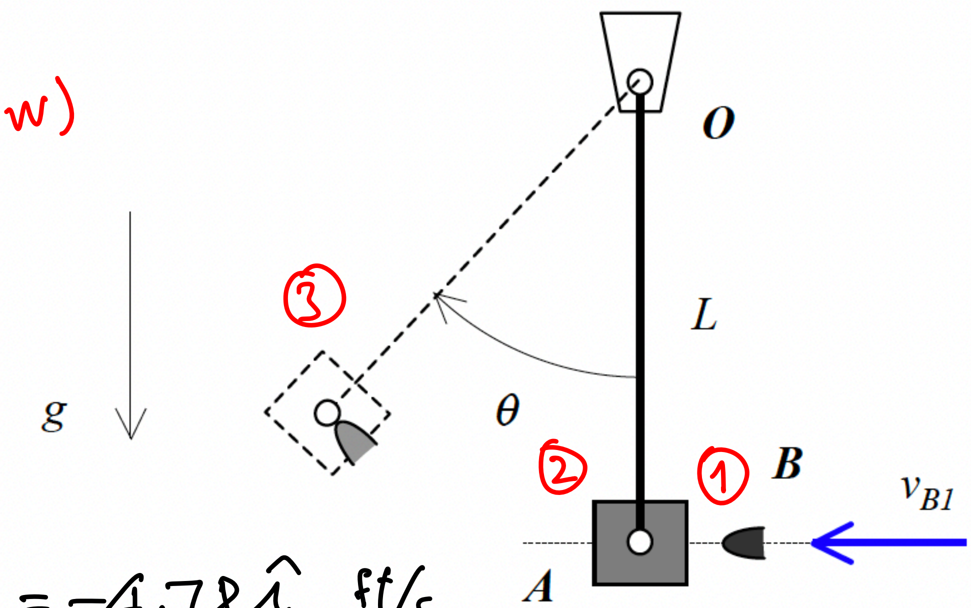
Use the following parameters in your analysis: $w = 0.2$ lb, $W = 75$ lb, $L = 5$ ft and $v_{B1} = 1800$ ft/s.

Solution

1-2 (FBD)



L.M. conserved in x

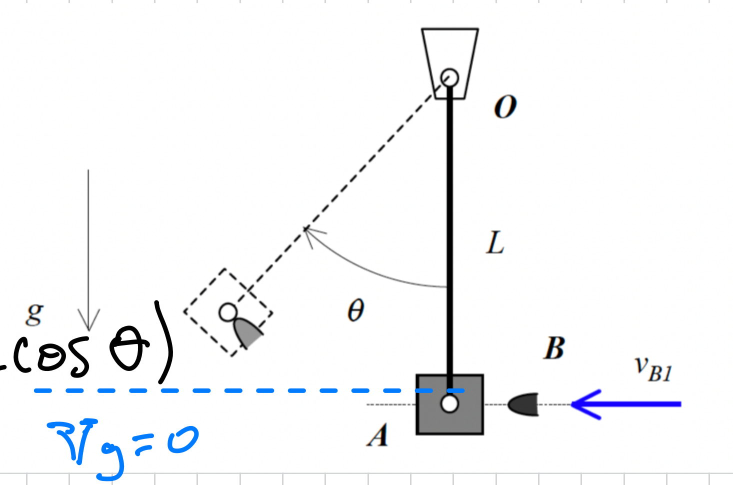


LIM in x-dir

$$-\frac{w}{\cancel{g}} V_{B1} = \frac{w+W}{\cancel{g}} V_2$$

$$V_2 = -\frac{w}{w+W} V_{B1} = -4.78\,\hat{\imath} \text{ ft/s}$$

Segment ② − ③ W/E



$$T_2 + V_2 + U_{2-3}^{NC} = T_3 + V_3$$

$$\frac{1}{2}\frac{(\cancel{w+W})}{g}V_2^2 + 0 + 0 = 0 + (\cancel{w+W})(L - L\cos\theta)$$

$$\theta_{max} = \cos^{-1}\left(\frac{V_2^2}{2Lg} - 1\right) = 21.7^\circ$$

(b) between ① and ② only consider

T1 and T2

$$T_1 + 0 + 0 = T_2 + 0$$

$\Rightarrow$ Energy loss = $\underline{T_2 - T_1}$

$$E_L = \left| \frac{1}{2}\frac{w+W}{g}\cdot V_2^2 - \frac{1}{2}\frac{w}{g}V_{B1}^2 \right| = 10035 \text{ lb-ft}$$