LINEAR IMPULSE - MOMENTUM

3/4/2026

what happened when we integrated $\Sigma F = ma$ over a path of a particle?

$\rightarrow$ we got $T_1 + V_1 + \int_1^2 R_t\,ds = T_2 + V_2$

This is useful when we need to relate changes in speed wit changes in position, and also to acknowledge that energy is conserved.

What if we now need to relate changes in speed with changes in time? In this case, we need a different strategy. Let us introduce the concept of conservation of linear momentum.

$$\int_1^2 \vec{R} = m\vec{a} = \int_1^2 m\,\frac{d\vec{v}}{dt}$$

$$\int_1^2 \vec{R}\,dt = m\vec{v}_2 - m\vec{v}_1$$

The quantity $m\vec{v}$ is defined as the linear momentum of particle P, and the integral, $\int^2 \vec{R}\,dt$ is the linear impulse of the net force acting over P.

So, the linear impulse-momentum equation (LIM)

$$m\vec{v}_1 + \int_1^2 \vec{R}\,dt = m\vec{v}_2$$

THIS IS A VECTOR EXPRESSION, so it can be decomposed into components:

$$m v_{1x} + \int R_x\,dt = m v_{2x}$$
$$m v_{1y} + \int R_y\,dt = m v_{2x}$$

If the resulting force in a given direction is zero, then, linear momentum in that direction is conserved in that direction.

If $R_x = 0 \Rightarrow$

$$m v_{1x} + \cancelto{0}{\int_1^2 R_x \, dt} = v_{2x}$$

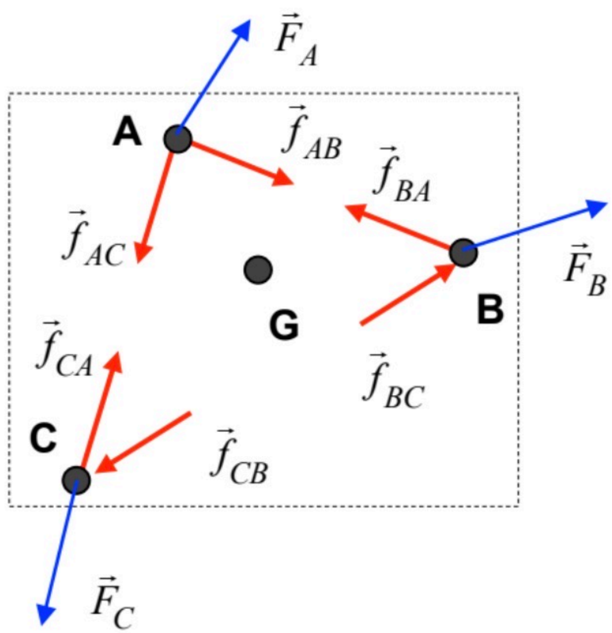
$$\Rightarrow \quad m v_{1x} = m v_{2x}$$

Note that this conclusion does not come from conservation of energy. Momentum and energy are two different quantities. Therefore there may be cases where momentum is conserved, but energy is not, and viceversa.

Linear Impulse-Momentum → System of particles

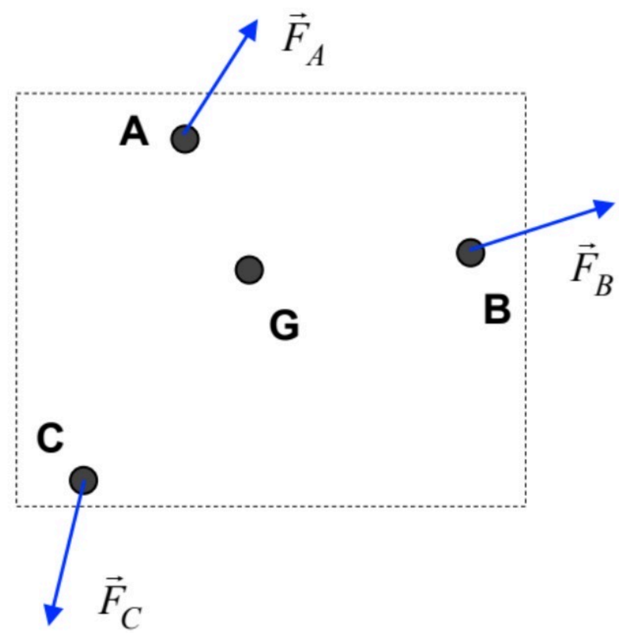
When looking at a system of particles, some forces may result from interactions with other particles of the system, while other forces may indeed represent external influences

Internal forces



all forces acting on system

External forces



"external" forces acting on system

From the FBD above:

$$m_A \vec{v}_{A1} + \int_1^2 (\vec{F}_A + \vec{f}_{AB} + \vec{f}_{AC})\,dt = m_A \vec{v}_{A2}$$

$$m_B \vec{v}_{B1} + \int_1^2 (\vec{F}_B - \vec{f}_{AB} + \vec{f}_{BC})\,dt = m_B \vec{v}_{B2}$$

$$m_C \vec{v}_{C1} + \int_1^2 (\vec{F}_C - \vec{f}_{AC} - \vec{f}_{BC})\,dt = m_C \vec{v}_{C2}$$

$$m_A \vec{v}_{A1} + m_B \vec{v}_{B1} + m_C \vec{v}_{C1} + \int_1^2 (\vec{F}_A + \vec{F}_B + \vec{F}_C)\,dt = m_A \vec{v}_{A2} + m_B \vec{v}_{B2} + m_C \vec{v}_{C2}$$

$$\Rightarrow \sum m_j \vec{v}_{j1} + \int_1^2 \left(\sum \vec{F}_{ext}\right) dt = \sum m_j \vec{v}_{j2}$$

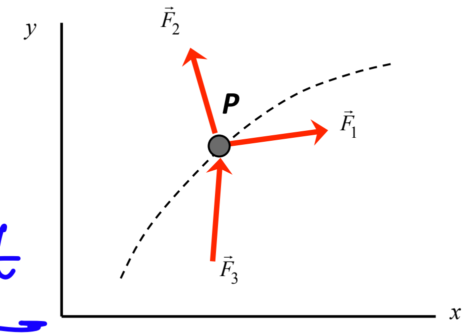
In conclusion, if one component of the net external force is zero, the total linear momentum in that direction is conserved (constant) $\Rightarrow$ Make system as inclusive as possible to not include internal forces in the analysis.

Linear Impulse-Momentum Equation

$$\boxed{\sum m_j \vec{v}_{j1} + \int_1^2 \left(\sum F_j^{(ext)} dt \right) = \sum m_j \vec{v}_{j2}}$$

Summary: Linear Impulse-Momentum Equation 1

FUNDAMENTAL equation: the linear impulse-momentum equation:



$$m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \left(\sum \vec{F}\right) dt \Rightarrow m\vec{v}_1 + \int_1^2 \left(\sum \vec{F}\right) dt = m\vec{v}_2$$

CONSERVATION: If there is no net force acting of the system in a given direction (say x), $\int_1^2 \left(\sum \vec{F}\right)_x dt = 0$ *, then linear momentum in that direction is conserved.*

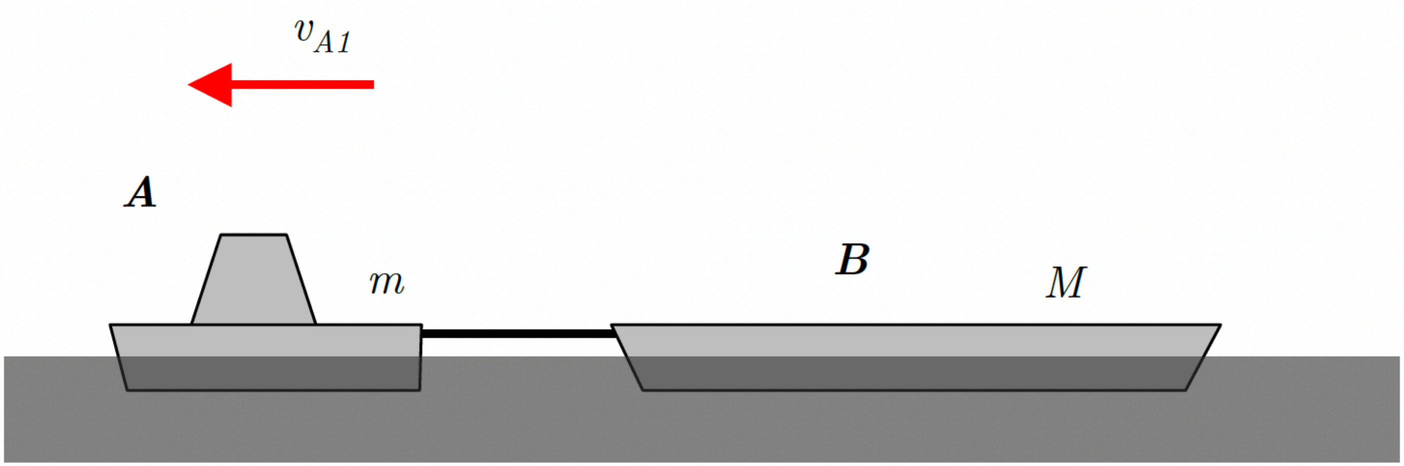
SYSTEM CHOICE: Make your choice of system as "large" as reasonable − you want to make as many forces as possible INTERNAL to the system.

Example 4.C.3

Given: A tug having a mass of m is towing a coal barge (mass M) with a speed of v_{A1}. During a short period of time Δt when the engines of the tug are turned off, the stern winch takes in the towing cable at a rate of $v_{B/A}$.

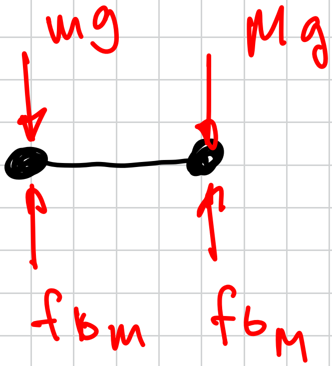
Find: Determine:
(a) The speed of the tug during this period of time; and
(b) The average value of the tension in the towing cable during this time.

Ignore the water resistance on the tug and barge during this time period. Also, assume that the towing cable remains taut at all times.



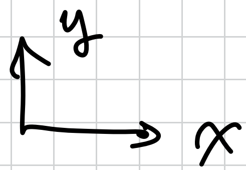
- Need to know changes in speed after changes in time → LIM
- When engine shuts off ⇒ no External Forces.

1) FBD



$\Rightarrow$ Momentum in x conserved.

2) Coord



3) Kinetics LIM in x

$$(m+M)\, v_{A1} + \cancel{\int_1^2 \vec{F}_{ext} \cdot dt}^{\,0} = m\, v_{A2} + M\, v_{B2}$$

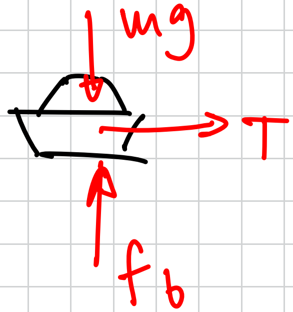
4) Kinematics. $v_{B/A} = v_B - v_A \Rightarrow v_{B2} = v_{A2} + v_{B/A}$

5) Solve For v_{A2}

$$\underset{\checkmark}{(m+M)\, v_{A1}} = m\, \underset{?}{v_{A2}} + M\left(\underset{?}{v_{A2}} + \underset{\checkmark}{v_{B/A}}\right)$$

$$\boxed{v_{A2} = \frac{(m+M)\,v_{A1} - M\,v_{B/A}}{m+M}}$$

b)



LIM in x

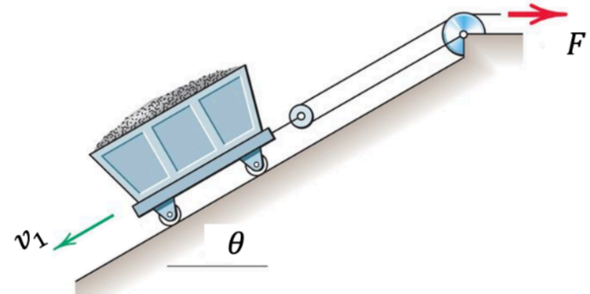
$$m\,v_{A1} + \int_0^{\Delta t} T\,dt = m\,v_{A2}$$

$$m\,v_{A1} + T_{AVG}\cdot\Delta t = m\,v_{A2}$$

$$\boxed{T_{AV} = \frac{m\,(v_{A2} - v_{A})}{\Delta t}}$$

Practice Problem

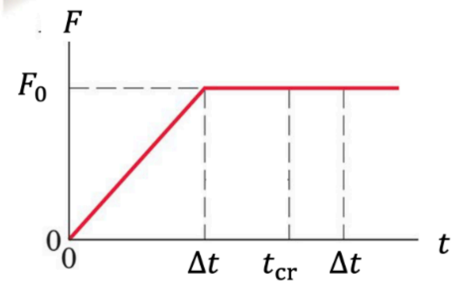
The loaded skip of mass m is rolling down the incline with a speed of v_1 when a force F is applied to the cable, as shown at time $t = 0$. The force F is increased uniformly over time until it reaches F_0 at Δt, after which it remains constant at this value. Ignore the effects of friction on the wheels.



Find:

(a) The time t_{cr} at which the skip reverses its direction.

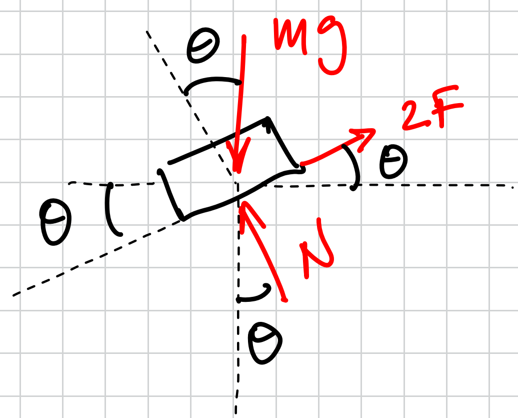
(b) The velocity v of the skip at $2\Delta t$. Treat the skip as a particle.



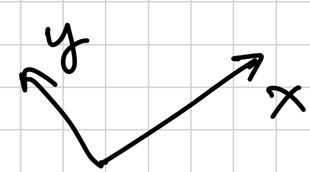
Use the following numerical values in your analysis: $m = 150$ kg, $v_1 = 4$ m/s, $F_0 = 600$ N, $\Delta t = 4$ s

Solution: Need to Find velocity after certain time: LIM

1) FBD



2) Coord



3) We have external forces in $x \Rightarrow$ LIM in x-dir

(a) LIM in x-dir:

$$m v_{1x} + \int_1^2 F_x\,dt = m v_{2x}$$

$$m(-v_1) + \int_0^{\Delta t} \left(2\frac{F_0}{\Delta t} t - mg\sin\theta\right) + \int_{\Delta t}^{t_{cr}} (2F_0 - mg\sin\theta) = m\cancel{v_{2x}}^{\,0}$$

$$-mv_1 + \left[\frac{F_0}{\Delta t}t^2 - mg\sin\theta\, t\right]_0^{\Delta t} + \left[2F_0 t - mg\sin\theta\, t\right]_{\Delta t}^{t_{cr}} = 0$$

$$-mv_1 + \frac{F_0}{\cancel{\Delta t}}\Delta t^{\cancel{2}} - \cancel{mg\sin\theta}\,\Delta t + 2F_0 t_{cr} - 2F_0\Delta t - mg\sin\theta\, t_{cr} + \cancel{mg\sin\theta}\,\Delta t = 0$$

$$-mv_1 - F_0\Delta t + 2F_0 t_{cr} - mg\sin\theta\, t_{cr} = 0$$

$$t_{cr} = \frac{mv_1 + F_0\Delta t}{2F_0 - mg\sin\theta} = 6.45\ s$$

(b)

$$m v_1 + \int_0^{2\Delta t} \Sigma F_x \, dt = m v_2$$

$$-mv_1 + \int_0^{\Delta t} \left(2\frac{F_0}{\Delta t} t - mg\sin\theta\right) + \int_{\Delta t}^{2\Delta t} \left(2F_0 - mg\sin\theta\right) = mv_2$$

$$-mv_1 + \frac{F_0}{\cancel{\Delta t}} \Delta t^{\cancel{2}} - mg\sin\theta\,\Delta t + 2F_0(2\Delta t) - mg\sin(2\Delta t)$$
$$- 2F_0\Delta t + mg\sin\Delta t = mv_2$$

$$-mv_1 + 3F_0\Delta t - 2mg\sin\theta\,\Delta t = m v_2$$

$$\vec{v}_2 = \left(-v_1 + \frac{3}{m}F_0\Delta t - 2g\sin\theta\,\Delta t\right)\hat{i}$$

$$\vec{v}_2 = 4.76\hat{i} \quad m/s$$