# PARTICLE KINETICS

2/25/26

## KINETICS PROBLEMS

1] COORDINATE SYSTEM

2] FREE BODY DIAGRAMS

3] KINETICS EQUATIONS

if need be ?

3] KINEMATICS

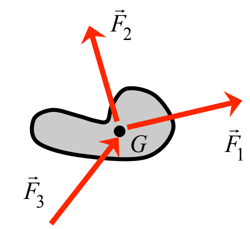
4] SOLVE FOR...

**SOLUTION**

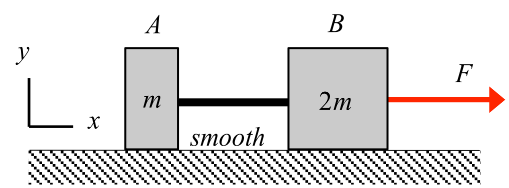
# *Summary*: Newton's Laws 2

*FUNDAMENTAL* equation: For a set of forces acting concurrently at the center of mass G of a body, we have Newton's 2$^{nd}$ Law:

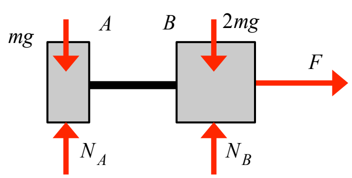
$$\sum \vec{F} = m\vec{a}_G$$




*DRAWING THE RIGHT FBD*: Be aware that, for good or bad, "internal forces" of an FBD will not appear in Newton's 2$^{nd}$ Law equation. Consider the system shown with a force *F* acting on block B.

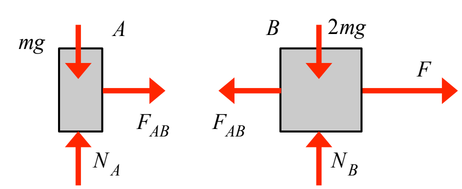



*Question 1: What is the acceleration of block B?*




$$\sum F_x = F = 3ma \quad \Rightarrow \quad a = \frac{F}{3m}$$

*Question 2: What is the force carried by member AB?*




$$B: \ \sum F_x = F - F_{AB} = 2ma \quad \Rightarrow \quad F_{AB} = F - 2ma = \frac{F}{3}$$

# Practice Problem

**Example 4.A.12**

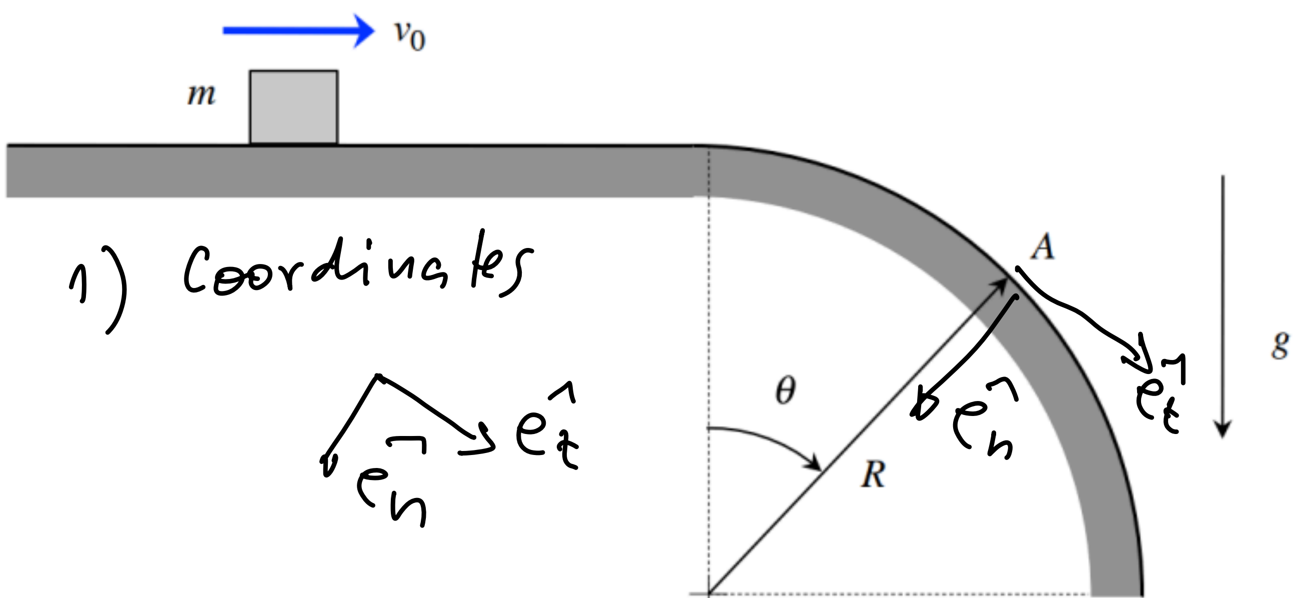
**Given:** A particle of mass $m$ moves along the smooth path shown.

**Find:**

(a) Determine the speed $v_0$ such that the particle loses contact with the semi-circular hill at $\theta =$ ~~53.13°~~ $36.87^\circ$

(b) Using the speed results from above, determine the normal contact force acting on the block by the hill when $\theta = 36.87^\circ$.

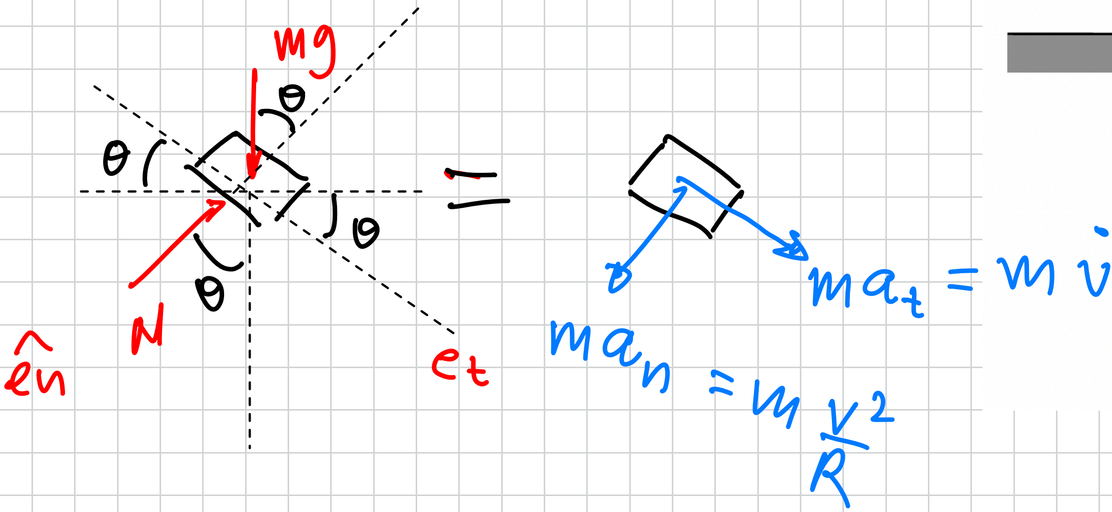
Use the following parameter in your analysis: $m = 10$ kg and $R = 2.5$ m.

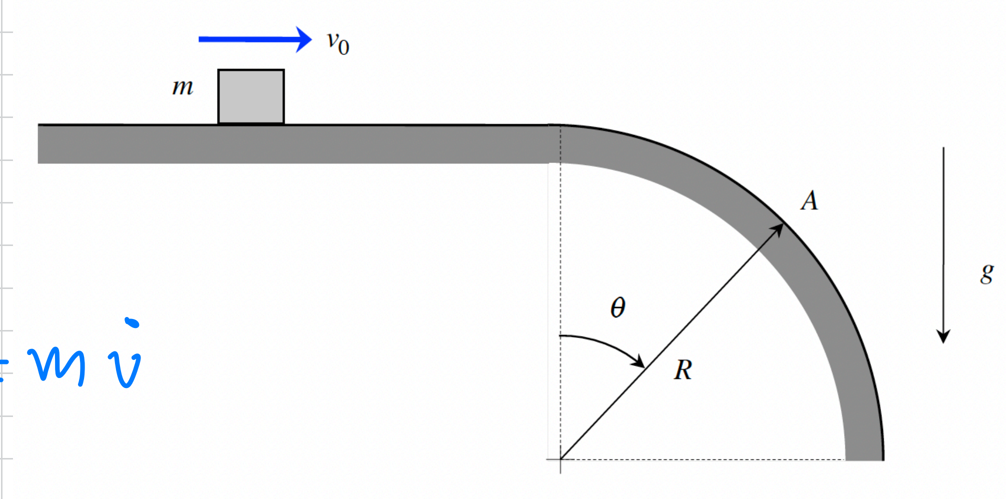



Solution 1) Coordinates

2) FBD

somewhere along hill




3) Kinetics

$$\Sigma F_t = m\dot{v} \qquad\qquad \Sigma F_n = m\frac{v^2}{R}$$

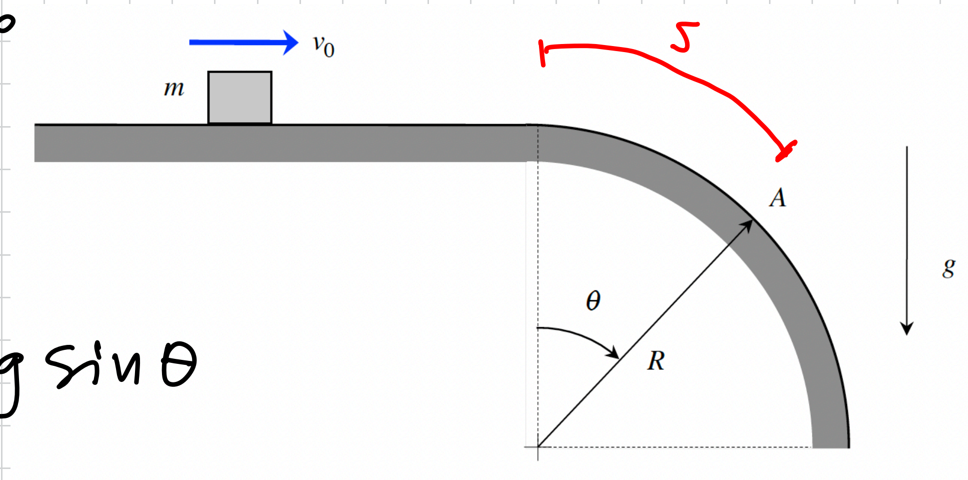
$$mg\sin\theta = m\dot{v} \quad (1) \qquad\qquad mg\cos\theta - N = \frac{mv^2}{R} \quad (2)$$

Critical speed when $N=0$

From (2) $\quad N=0 \quad \Rightarrow \quad \cancel{m}g\cos\theta = \frac{\cancel{m}v^2}{R}$

$$\Rightarrow \quad v = \sqrt{Rg\cos\theta} \quad (3) \quad \Rightarrow \quad v = 4.43 \text{ m/s}$$

part (a) asks for $v_0$




From (1)

$$\dot{v} = g\sin\theta$$

but $$\dot{v} = \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\cancelto{\frac{1}{R}}{\frac{d\theta}{ds}}\cdot\cancelto{v}{\frac{ds}{dt}} = g\sin\theta$$

$s = R\theta$

$ds = R\,d\theta$

$\frac{d\theta}{ds} = \frac{1}{R}$

$$\frac{dv}{d\theta}\cdot\frac{1}{R}\cdot v = g\sin\theta$$

$$\int_{v_0}^{v}\frac{1}{R}v\,dv = \int_0^{\theta} g\sin\theta\,d\theta$$

$$\frac{1}{2R}\left[v^2 - v_0^2\right] = g\left[-\cos\theta + \cos 0\right] = g(1-\cos\theta)$$

$$v_0^2 = v^2 - 2gR(1-\cos\theta)$$

using (3): $$v_0^2 = gR\cos\theta - 2gR + 2gR\cos\theta$$

$$v_0^2 = 3gR\cos\theta - 2gR$$

$$V_0 = \sqrt{3gR\cos\theta - 2gR}$$

$$\boxed{V_0 = 3.13 \ m/s}$$ ← ANSWER (a)

Part (b) From (2)

$$mg\cos\theta - N = \frac{mv^2}{R}$$

$$N = mg\cos\theta - \frac{mv^2}{R}$$

$$N = (10)(9.81)(0.8) - \frac{(10)(4.43)}{2.5}$$

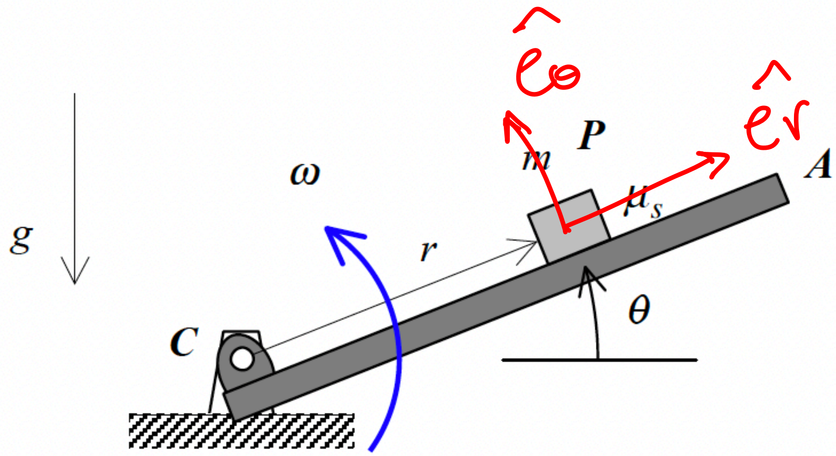
$$\boxed{N = 60.76 \ N}$$ ← ANSWER (b)

**Example 4.A.13**

**Given:** Arm CA is rotating in a vertical plane with a constant rate of $\omega$. When CA is horizontal, particle P (having a mass of $m$) is stationary with respect to CA. However, P is known to start slipping with respect to CA when CA has reached an angle of $\theta_{slip}$. The coefficient of static friction between P and the arm is $\mu_S$.

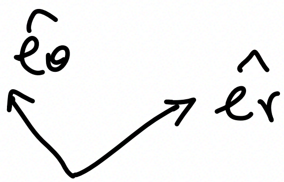
**Find:** The value for the coefficient of static friction $\mu_S$ between P and CA.

Use the following parameters in your analysis: $\omega = 2$ rad/s, $r = 3$ ft and $\theta_{slip} = 53.13°$.

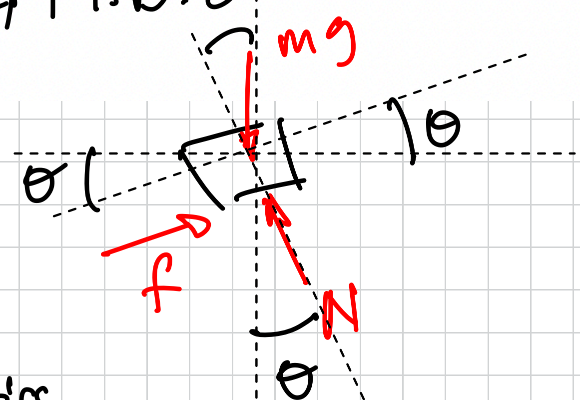


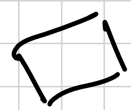

Solution

1) Coord




2) FBD. $\theta$



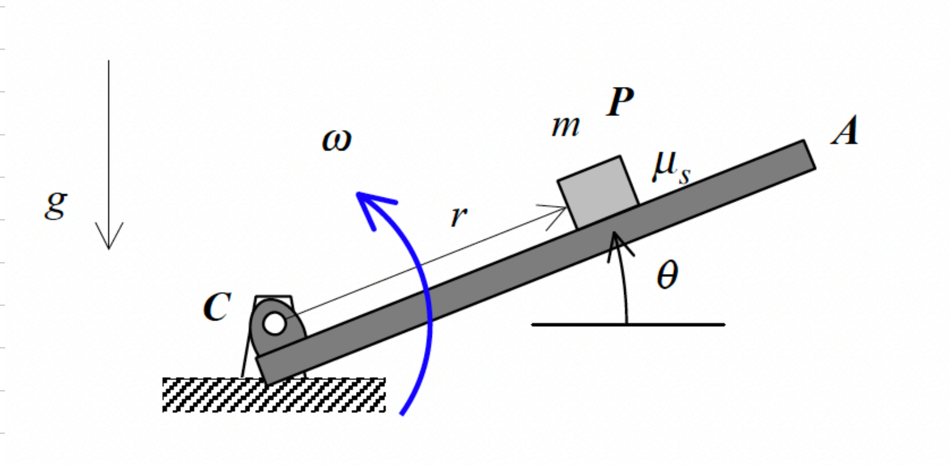
3) Kinetics

$$\Sigma F_r = m a_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\Sigma F_\theta = m a_\theta = m(2\dot{r}\dot{\theta} + r\ddot{\theta})$$

$$f - mg\sin\theta = m(\cancel{\ddot{r}}^{0} - r\dot{\theta}^2) \quad ①$$

$$N - mg\cos\theta = m(\cancel{2\dot{r}}^{0}\dot{\theta} + \cancel{r\ddot{\theta}}^{0}) \quad ②$$




$$f = \mu_s N \quad ③$$

Problem asks for $\mu_s$

4) Kinematics

- Instant before slipping: $\dot{r} = 0$, $\ddot{r} = 0$
- Constant angular speed $\omega$: $\dot{\theta} = \omega$, $\ddot{\theta} = 0$

5) Solve. From ② & ③ $N = mg\cos\theta_{sL} \Rightarrow f = \mu_s mg\cos\theta_{sL}$

Plug into ① $\mu_s \cancel{m} g\cos\theta_{sL} - \cancel{m} g\sin\theta_{sL} = -\cancel{m} r\omega^2$

$$\mu_s g\cos\theta_{sL} - g\sin\theta_{sL} = -r\omega^2$$

Solve for $\underline{\mu_s}$