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ME 274 Lecture 29

Rigid body kinetics: Newton/Euler – Part 3

Eugenio “Henny” Frias-Miranda

03/30/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 28 (5.C and 5.D) due today!!**

2. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME.

3. Exam 2 Information:
    - Thursday, April 2, 8:00-9:30 PM
    - BHEE129
    - Coverage: Lectures 11-26 (up through angular impulse/momentum for particles)

4. Exam 2 Review sessions both videos to be posted on website
    - Pi Tau Sigma: Tuesday, March 31, 6:30-7:30 PM, WTHR 104 (WL in-person and Indy online)
    - ME 274 Instructor, CK: Wednesday, April 1, 7:00 PM, live on Zoom for both WL and Indy:
        - https://purdue.edu.zoom.us/j/94496659802?pwd=VMHo3NfyaHO3HbbmmLForgikls3PL7.1

Chapter 5: Planar Rigid Body Kinetics

- Newton/Euler (N/E) Equations for Planar Rigid Bodies (lectures 27-29)

$$\sum \vec{F} = m\vec{a}_G$$

$$\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$$

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ $\text{where } T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ $\text{where } T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \bullet (\vec{\omega} \times \vec{r}_{G/A})$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_{O2} - \vec{H}_{O1}$ $\text{where } \vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\, dt = \vec{H}_{A2} - \vec{H}_{A1}$ $\text{where } \vec{H}_A = I_A\vec{\omega}$

[pg. 252 and 295 content]

3 Special Forms of the Euler's Equation where $\vec{r}_{G/A} \times (m\vec{a}_A)$ = 0

$$\left(\sum \vec{M}_A\right)_{ext} = I_A\vec{\alpha} + \vec{r}_{G/A} \times (m\vec{a}_A)$$

1. If you choose A to be the **center of mass, G.** $\vec{r}_{G/A} = \vec{0}$.

$$\sum \vec{M}_G = I_G\vec{\alpha}$$

2. If you choose A to be a **fixed point on the body, O.** $\vec{a}_O = \vec{0}$.

$$\sum \vec{M}_O = I_O\vec{\alpha}$$

3. If you choose a point A which has an acceleration vector that is **parallel to** $\vec{r}_{G/A}$ **.** $\vec{r}_{G/A} \times \vec{a}_A = \vec{0}$

$$\sum \vec{M}_A = I_A\vec{\alpha}$$

Picking one of these 3 locations will simplify our problem, **it is recommended to do this as often as possible.**

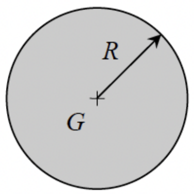
[pg. 298 content]

Mass Moment of Inertia of Commonly Used Objects and when to use Parallel Axis Theorem (P.A.T.)

- Often times you will reference a table such as below and get your **I_G.**
- If you are not using your Euler's Equation in **terms of G**, you will have to use **Parallel Axis Theorem (P.A.T.)**

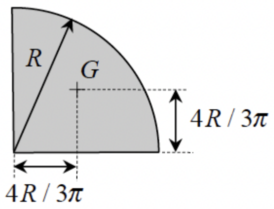
$$I_A = I_G + md_{AG}^2$$

Uniform disk of mass m



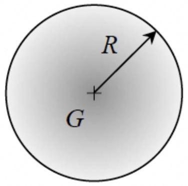
$$I_G = \frac{1}{2}mR^2$$

Quarter circular plate of mass m



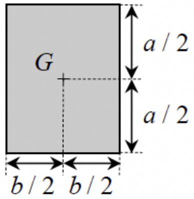
$$I_G = \left(\frac{9\pi^2 - 64}{18\pi^2}\right)mR^2$$

Uniform sphere of mass m



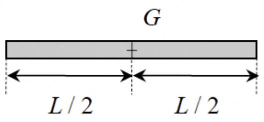
$$I_G = \frac{2}{5}mR^2$$

Uniform rectangular plate of mass m



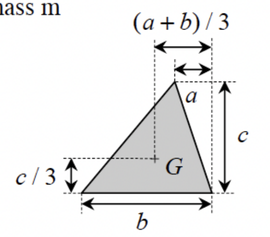
$$I_G = \frac{1}{12}m\left(a^2 + b^2\right)$$

Uniform thin bar of mass m



$$I_G = \frac{1}{12}mL^2$$

Uniform triangular plate of mass m



$$I_G = \frac{1}{18}m\left(a^2 + b^2 + c^2 - ab\right)$$

[pg. 303-304 content]

Radius of Gyration Concept

- Definition of Gyration: *act of moving in a circle or spiral around a fixed point/axis.*

- **In this course you will occasionally be given the radius of gyration about some point A.**

- In order to find the mass moment of inertia about point A, **use the equation below,** where 'k' is the radius of gyration about point A:

$$I_A = m{k_A}^2$$

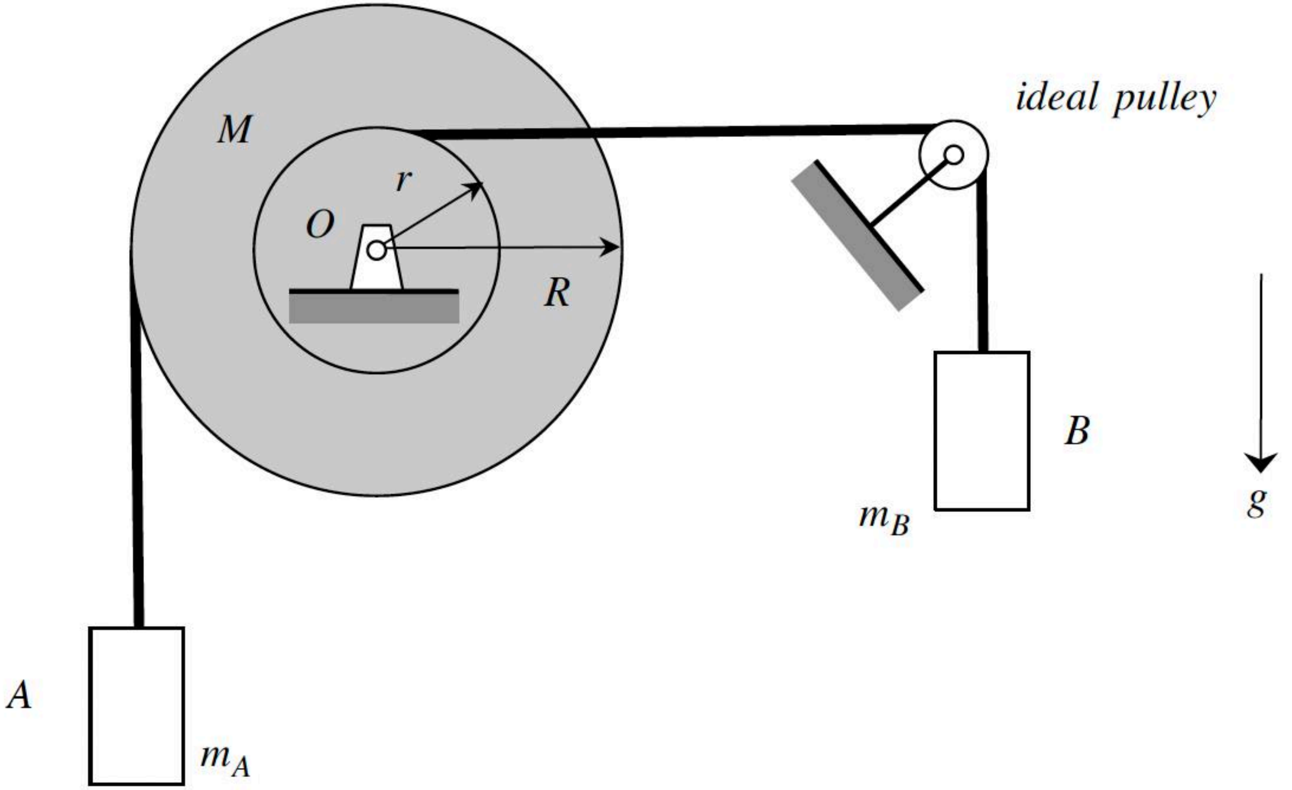
[pg. 306 content]

Example 5.A.11

Given: A stepped drum (having a mass of M and radius of gyration about its center O of k_O) is attached to a smooth shaft passing through its center O. A cable wrapped around the outer radius of the drum is attached to block A. A second cable is wrapped around the inner radius of the drum with this cable pulled over an ideal pulley and is attached to block B. Assume that the cables do not slip on the drum. The system is released from rest.

Find: Determine the angular acceleration of the drum on release. Write your answer as a vector.

Use the following parameters in your analysis: $m_A = 10$ kg, $m_B = 30$ kg, $M = 20$ kg, $r = 0.2$ m, $R = 0.4$ m and $k_O = 0.25$ m.



[pg. 318]

Example 5.A.11 p. 318 Similar to hwk due Fri, April 3rd

Given: A stepped drum (having a mass of M and radius of gyration about its center O of k_O) is attached to a smooth shaft passing through its center O. A cable wrapped around the outer radius of the drum is attached to block A. A second cable is wrapped around the inner radius of the drum with this cable pulled over an ideal pulley and is attached to block B. Assume that the cables do not slip on the drum. The system is released from rest.

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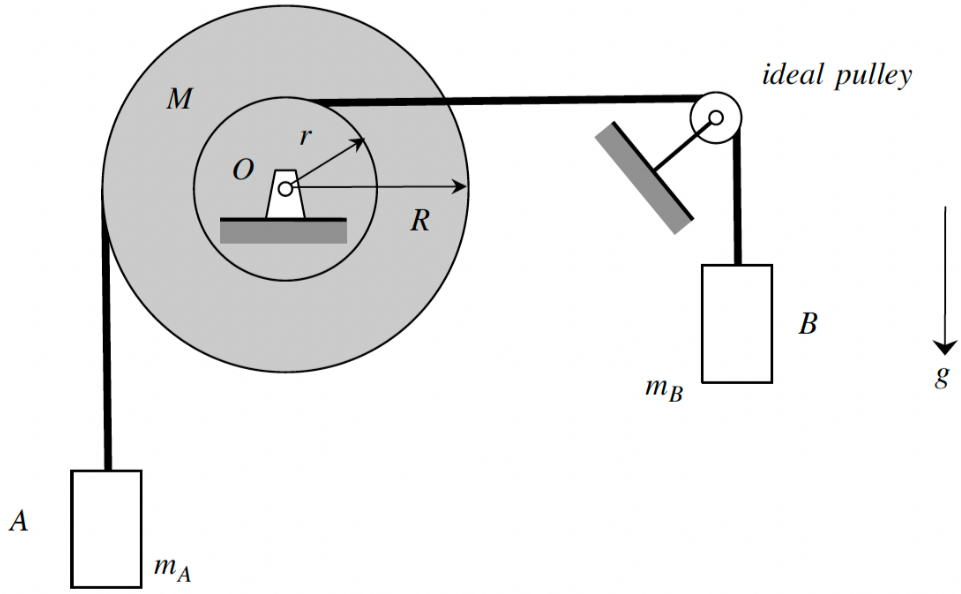
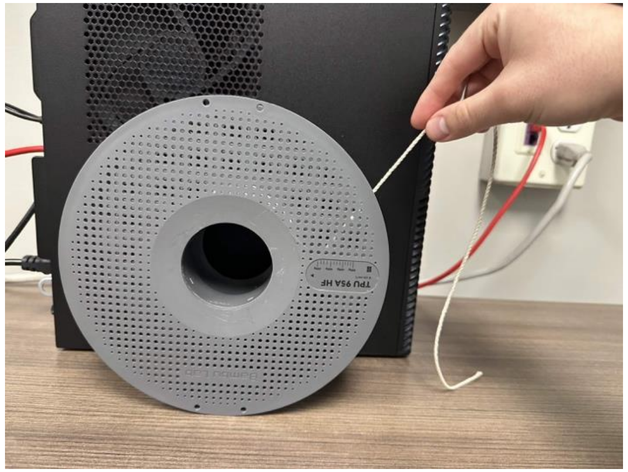
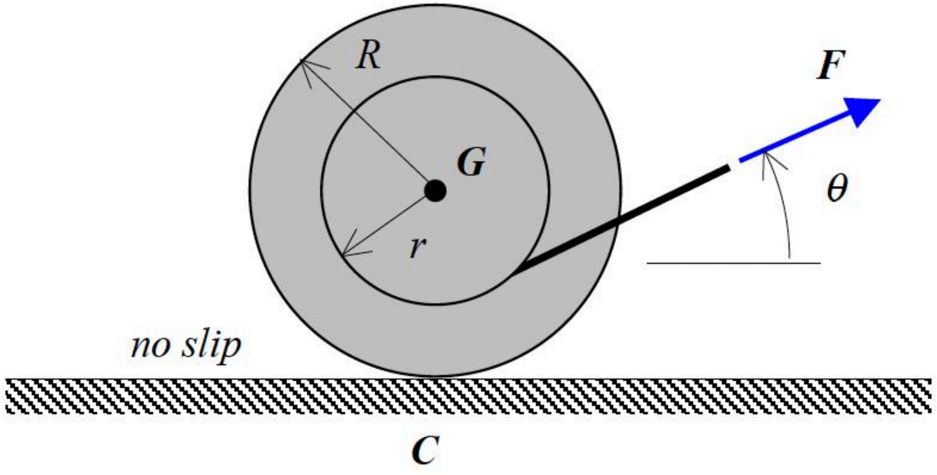


Example 5.A.12

Given: A spool has a mass of $m = 30$ kg, a centroidal radius of gyration $k_G = 0.25$ m, an outer radius of $R = 0.3$ m and an inner radius of $r = 0.1$ m. A constant force $F = 60$ N is applied at an angle of θ by a cord that is wrapped around the inner radius of the spool. The spool rolls without slipping on the rough horizontal surface.

Find: Determine the angular acceleration of the spool as a function of the angle θ.





[pg. 319]

Example 5.A.12 p.319 Lec 26 problem

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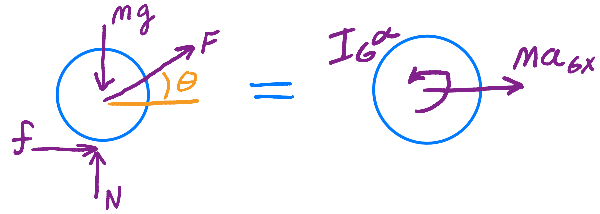
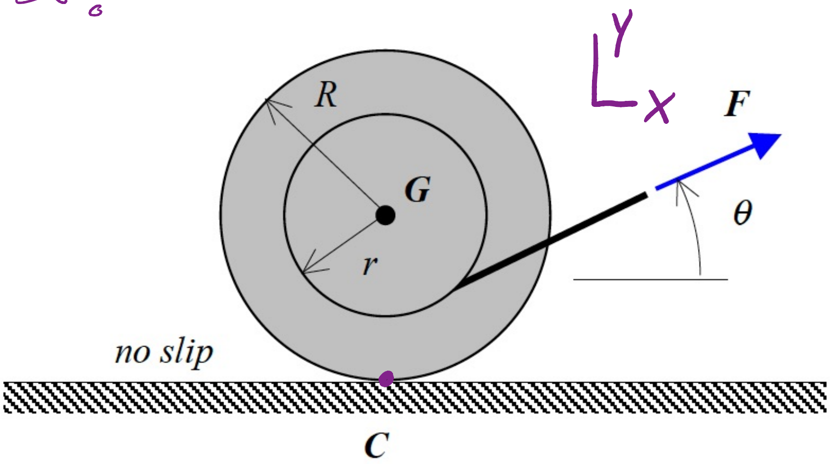
Find: Determine the angular acceleration of the spool as a function of the angle θ.

α_G?

① FBD





Kinetics

② Newton on x dir

$\Sigma F_x = m a_{Gx}$

$\Rightarrow f + F\cos\theta = m a_{Gx}$ (1)

③ Newton on y dir

$\Sigma F_y = 0$

$\Rightarrow N + F\sin\theta - mg = 0$ (2)

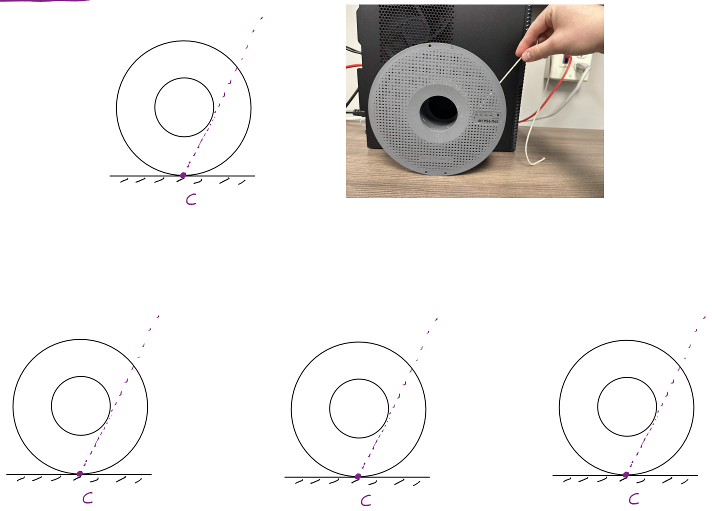
④ Euler on G

$\Sigma M_G = I_G \alpha$

radius of gyration eqn

$\Rightarrow fR + Fr = m k_G^2 \alpha$ (3)

⑤ 4 unkns 3 eqns



Kinematics

⑥ GC

$\vec{a}_G = \vec{a}_C + \vec{\alpha} \times \vec{r}_{G/C} - \omega^2 \vec{r}_{G/C}$

$\Rightarrow a_{Gx}\hat{i} = a_C\hat{j} + \alpha\hat{k} \times R\hat{j} - \omega^2 R\hat{j}$

$= a_C\hat{j} + -\alpha R\hat{i} - \omega^2 R\hat{j}$

$\hat{i}: a_{Gx} = -\alpha R$ (4)

$\hat{j}: 0 = a_C - \omega^2 R$

$\Rightarrow a_C = \omega^2 R$

Solve 4 unkns 4 eqns

⑦ Use (1) & (4).

(1) $\Rightarrow f + F\cos\theta = -m\alpha R$

$\Rightarrow f = -F\cos\theta - m\alpha R$

⑧ Use (3) & f.

(3) $\Rightarrow -F\cos\theta R - m\alpha R^2 + Fr = m k_G^2 \alpha$

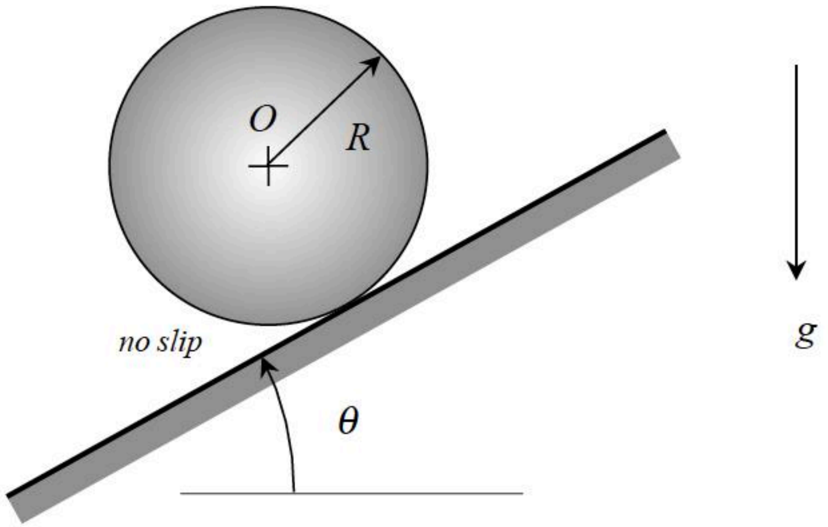
$\Rightarrow Fr - F\cos\theta R = m(R^2 + k_G^2)\alpha$

$\Rightarrow \vec{\alpha} = \frac{F(r - R\cos\theta)}{m(R^2 + k_G^2)}\hat{k}$

Example 5.A.15

Given: The homogeneous sphere (having a mass of m and radius R) is released from rest on a rough incline at an angle of θ. After release, the sphere rolls without slipping on the incline.

Find: Determine:
(a) The acceleration of the center of mass G of the sphere; and
(b) The force of friction acting on the sphere on release.



[pg. 322]

Example 5.A.15 p.322

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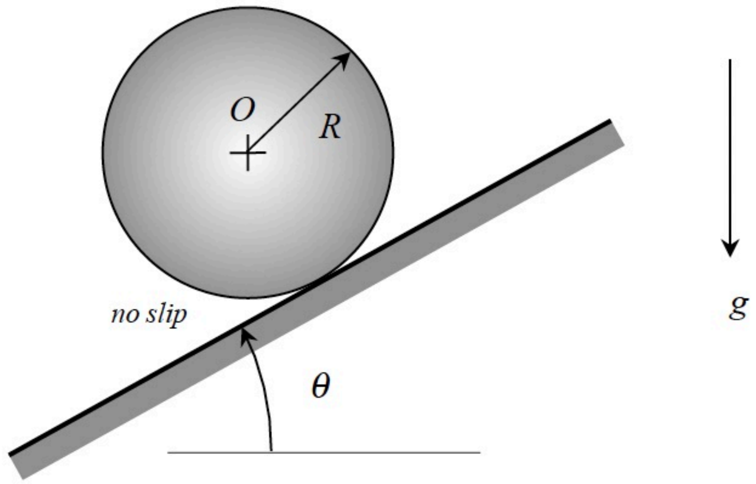
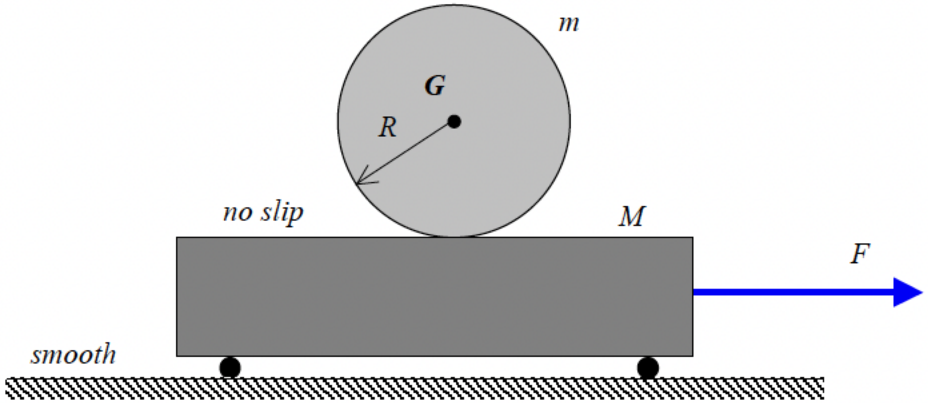


Example 5.A.16

Given: The system is initially at rest.

Find: Determine the acceleration of G when force F acts on the cart.

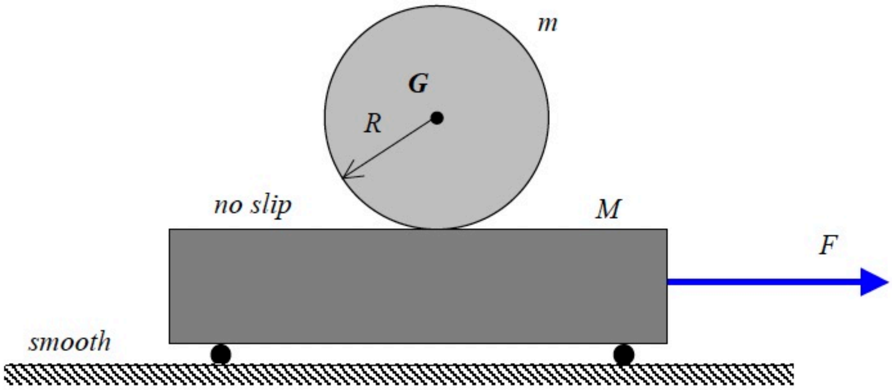


[pg. 323]

Example 5.A.16 p.323

Given: The system is initially at rest.

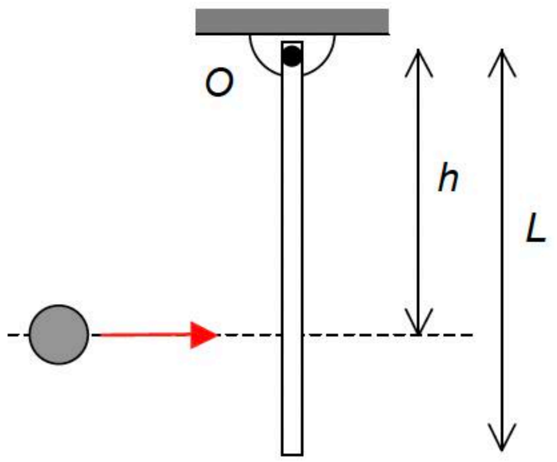
Find: Determine the acceleration of G when force F acts on the cart.



Example 5.A.17

Given: A ball of mass m strikes a homogeneous bar of length L and mass M at a distance h from the pivot point O.

Find: Determine the distance h such that the horizontal reaction force at O on the bar is zero. (This location on the bar is known as the body's "center of percussion". Can you think of a practical application of knowing the location of the center of percussion of a body?)



[pg. 324]

Example 5.A.17

p.324

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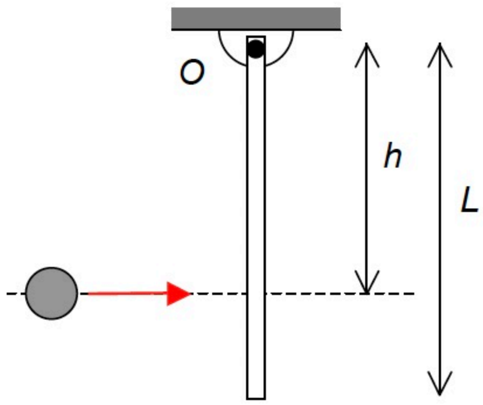


Summary: Newton/Euler Equations 3

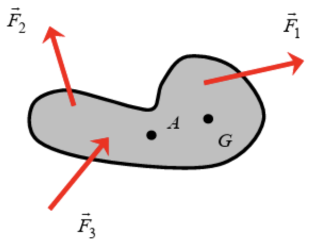
[pg. 305]

FUNDAMENTAL equations:

(1) $\sum \vec{F} = m\vec{a}_G$

(2) $\sum \vec{M}_{\boxed{A}} = I_{\boxed{A}}\vec{\alpha}$; A = c.m. *OR* fixed point *OR* $\vec{r}_{G/A} \parallel \vec{a}_A$

SAME point "A"!



Lec 29 Short Feedback Form:

SOLUTION METHOD: the four-step plan

Kinetics: Four-Step Problem Solving Method

The suggested plan of action for solving kinetics problems:

1. ***Free body diagram(s)***. Draw appropriate free body diagrams (FBDs) for the problem. Your choice of FBDs is problem dependent. For some problems, you will draw an FBD for each body; for others, you will draw an FBD for the entire system. An integral part of your FBDs is your choice of coordinate systems. For each FBD, draw the unit vectors corresponding to your coordinate choice.

2. ***Kinetics equations***. At this point, you will need to choose what solution method(s) that you will need to use for the particular problem at hand. In this section of the course we will study four basic methods: Newton/Euler, work/energy, linear impulse/momentum and angular impulse/momentum. Based on your choice of method(s), write down the appropriate equations from your FBD(s) from Step 1.

3. ***Kinematics***. Perform the needed kinematic analysis. A study of the equations in Step 2 above will guide you in deciding what kinematics are needed to find a solution to the problem.

4. ***Solve***. Count the number of unknowns and the number of equations from above. If you do not have enough equations to solve for your unknowns, then you either: (i) need to draw more FBDs, OR (ii) need to do more kinematic analysis. When you have sufficient equations for the number of unknowns, solve for the desired unknowns from the above equations.

Draw INDIVIDUAL free-body diagrams for Newton/Euler.

Be sure to use the correct mass moment of inertia for your choice of point "A". Use PAT if necessary.

Typically the most difficult step. Recall the rigid body kinematics from Chapter 2.

If you are short equations, go back to Step 3 – Kinematics.