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ME 274 Lecture 26

Particle Kinetics – Summary

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03/23/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 25 (4.S and 4.T) due today!!**
2. Quiz 5 due next Wednesday
3. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME.
4. Bonus quiz grade at end of the semester if we get a good response rate to QR code surveys at the end of lecture.
    - If you are unable to attend lecture on that day/forget to fill it out:
        - Feel free to give feedback based on the content of that lecture's slides.
        - Way of you reviewing previous content and giving feedback to me.

Kinetics Summary Handout

Kinetics Table *(page 352 of the lecture book)*

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot \left(\vec{\omega} \times \vec{r}_{G/A}\right)$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\, dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A\vec{\omega}$

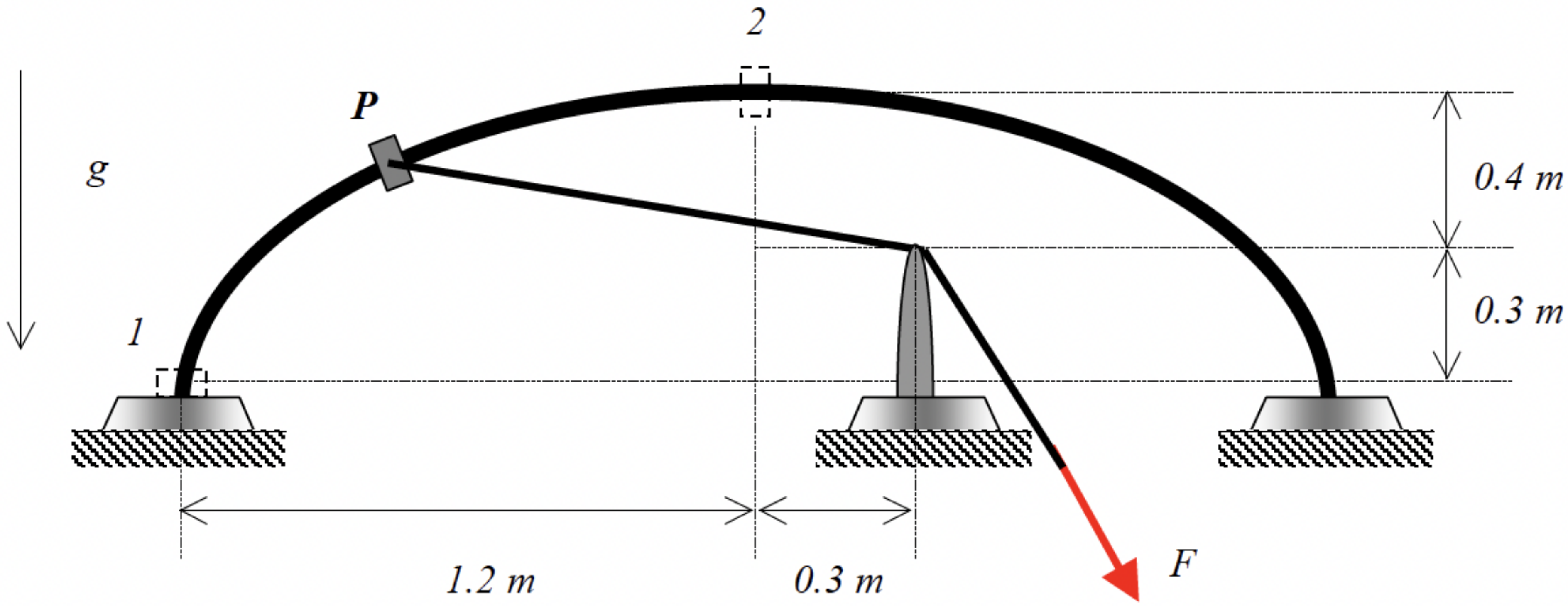
Notes on the *four-step* method:

1. ***FBD(s)***
   - Newton/Euler (*N/E*): typically individual FBDs
   - Work/energy (*W/E*), linear impulse momentum (*LIM*) and angular impulse/momentum (*AIM*): typically make it "BIG" (include all moving bodies)
2. ***Kinetics*** (see suggestions in the table to the right as to which method(s) to use)
   - Choose coordinate system(s) based on the *"Given"* and *"Find"*, including consideration of any motion constraints in the system.
   - *N/E*: point G in the Newton equation must be the center of mass for the body
   - *W/E*: mechanical energy is conserved if $U_{1\to2}^{(nc)} = 0$ (no non-conservative work being done on your system)
   - *LIM*: linear momentum in the *x*-direction is conserved if $\sum F_x = 0$ (no net force in the *x*-direction for your system)
   - *AIM*: angular momentum in the *z*-direction is conserved if $\sum M_{Oz} = 0$ (no net moment in the *z*-direction about point O for your system).For particles, O must be a fixed point. For rigid bodies, O can be either a fixed point or the center of mass.
3. ***Kinematics***
   - *N/E*: typically need *acceleration* kinematics
   - *W/E, LIM and AIM:* typically need *velocity* kinematics
4. ***Solve***

Example 4.B.5

Given: Slider P, having a mass of $m = 0.6$ kg, moves freely along the fixed, smooth, curved rod from position A to position B in the vertical plane under the action of the constant $F = 20$ N tension in the cord.

Find: Determine the speed of the slider at position 2, if the slider is released from rest at position 1.



[pg. 223]

Example 4.B.5 Similar to H.4.V pg. 223

Given: Slider P, having a mass of $m = 0.6$ kg, moves freely along the fixed, smooth, curved rod from position A to position B in the vertical plane under the action of the constant $F = 20$ N tension in the cord.

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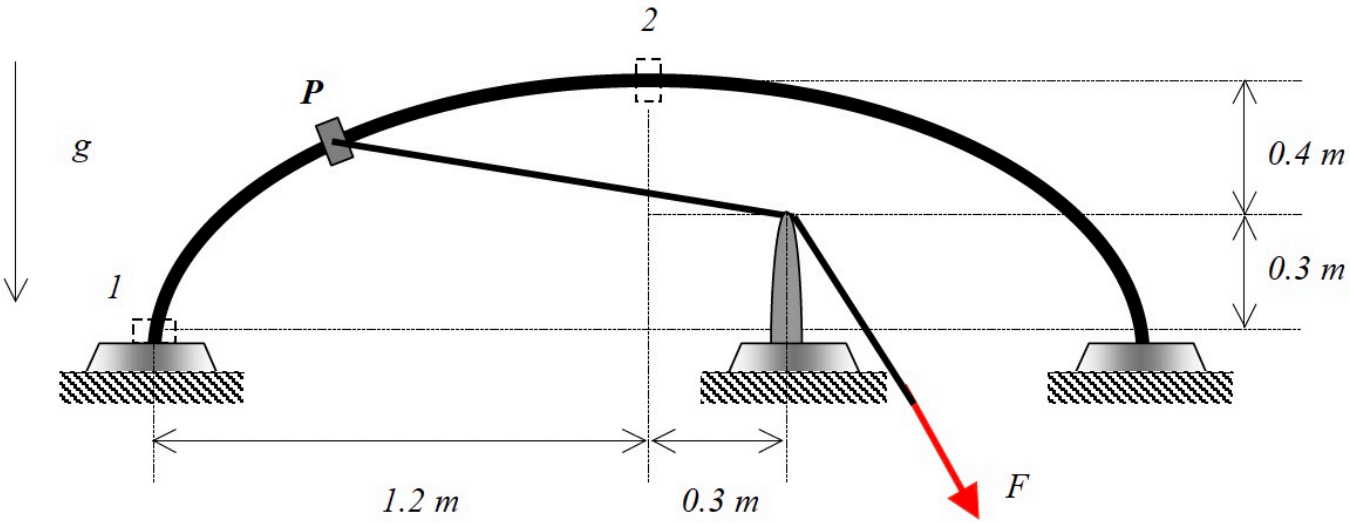
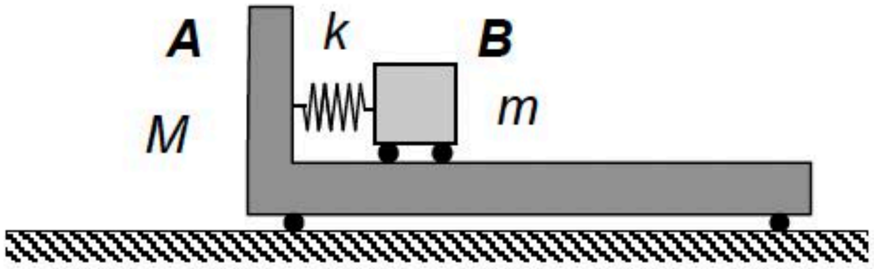
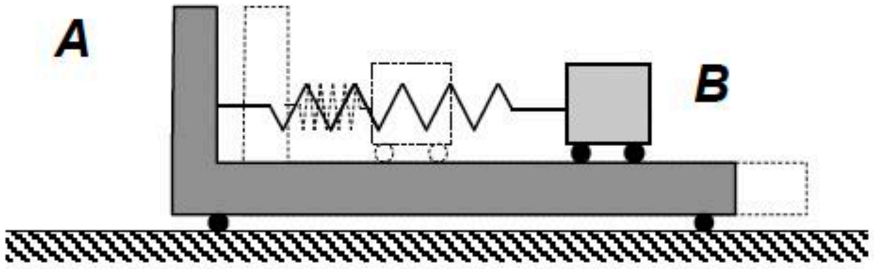


Question C4.7

Cart A and block B (having masses of $M = 4$ kg and $m = 2$ kg, respectively, are connected by a spring of stiffness $k = 300$ N/m. The system is released from rest with the spring being compressed 0.2 m (Position 1). Find the speed of the cart at Position 2 at the instant when the spring is uncompressed/unstretched. Consider all surfaces to be smooth.



Position 1
(at rest)



Position 2
(both A and B moving)

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Question C4.7 p.274

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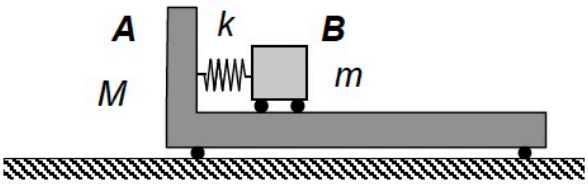
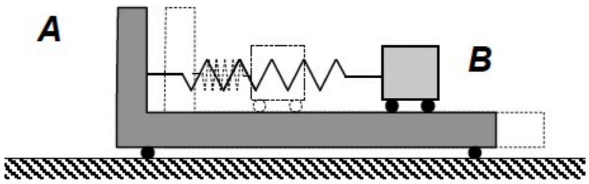


Position 1
(at rest)

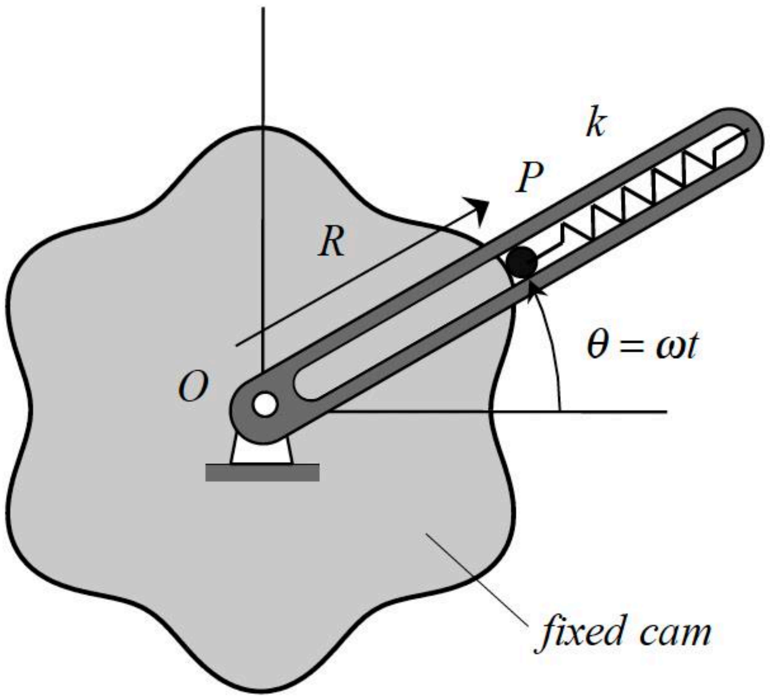


Position 2
(both A and B moving)

Example 4.A.11

Given: A slotted arm rotates about a vertical shaft passing through O that is at the center of a FIXED cam, as shown in the figure. A particle P, having a mass of $m = 0.2$ kg moves within the slot in the arm and remains in contact with the surface of the cam under the action of a spring attached between P and the outer end of the arm. The shape of the cam is such that the radial distance from O to P is given by the equation $R = R_0 - R_1 \cos(6\theta)$ where $R_0 = 0.5$ m and $R_0 = 0.1$ m. The spring has a stiffness of $k = 500$ N/m and is compressed by an amount of $\Delta = 0.2$ m when $\theta = 0$. The arm rotates at a constant rate of $\omega = 10$ rad/s.

Find: Determine the force acting on P by the cam when P passes over the top of the lobe in the position shown.



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Example 4.A.11 p.204 Lec18 problem

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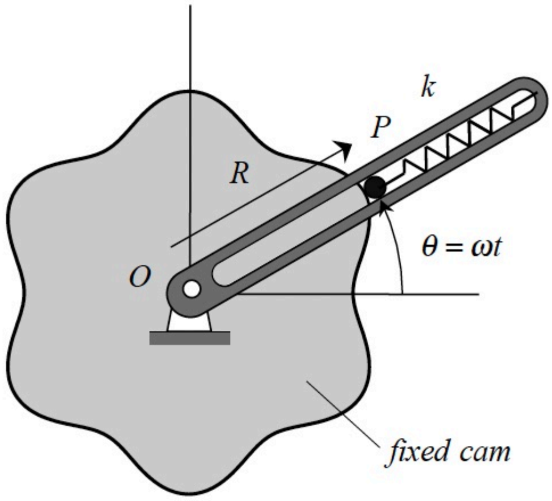
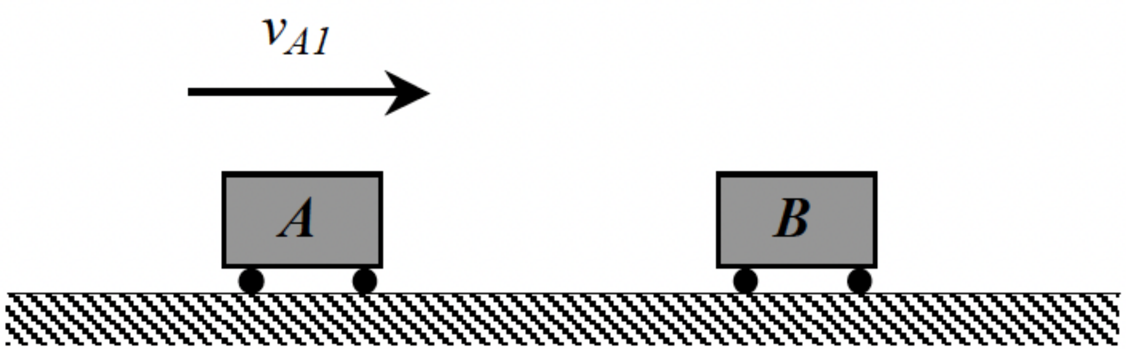


Example 4.C.4

Given: Car A (having mass of m_A) travels to the right with an initial speed of v_{A1}. Car A then impacts a stationary car B (having a mass of m_B). After impact, the two cars stick together.

Find: The change in kinetic energy for the system of A and B together after the impact and resulting coupling. What fraction is this change to the initial kinetic energy of the system of A and B?



[pg. 239]

Example 4.C.4

p. 239

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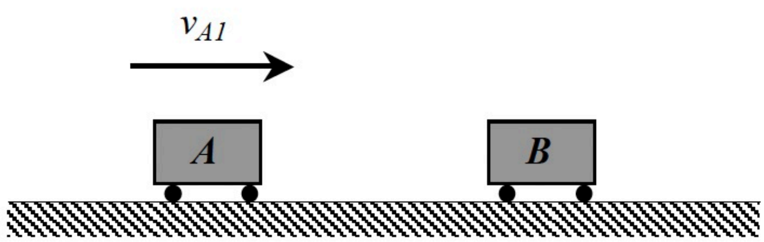


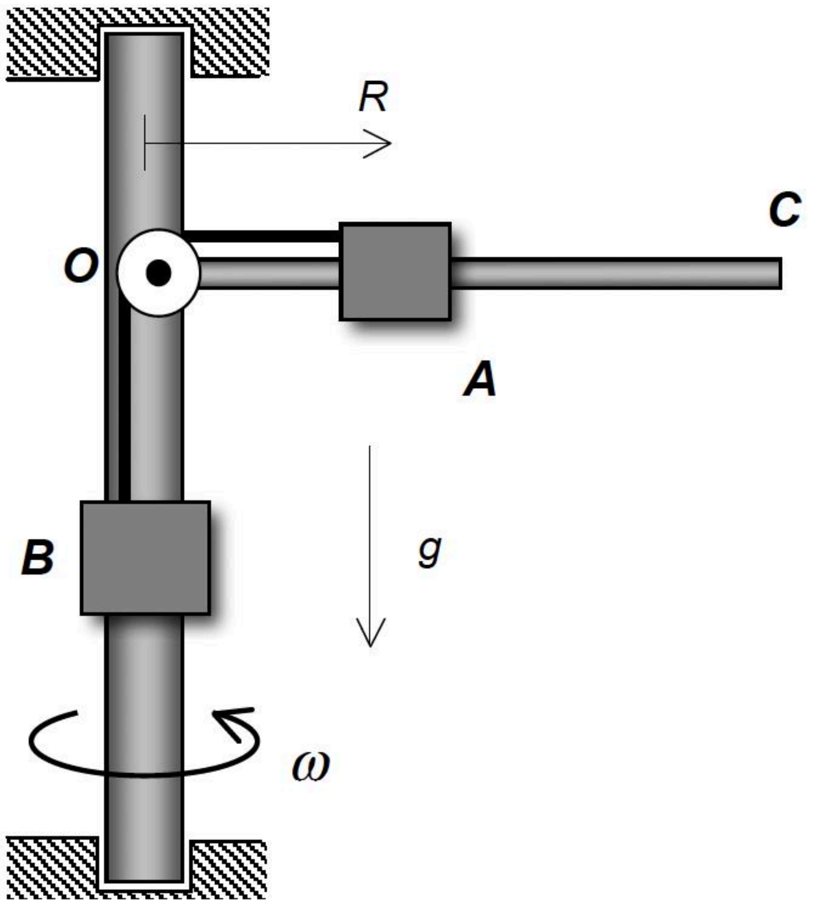
Example 4.D.4

Given: Particles A and B, whose masses are 0.5 kg each, are NOT sliding over their smooth guides at a position of $R = 0.8$ m with $\omega = 6$ rad/s. Assume the mass of the guides and pulley to the negligible and that the influences of friction are negligible everywhere.

Find: For the position where $R = 1.2$ m, find:

(a) The angular speed of arm OC; and

(b) The speed of particle B.



[pg. 261]

Example 4.D.4 p.261

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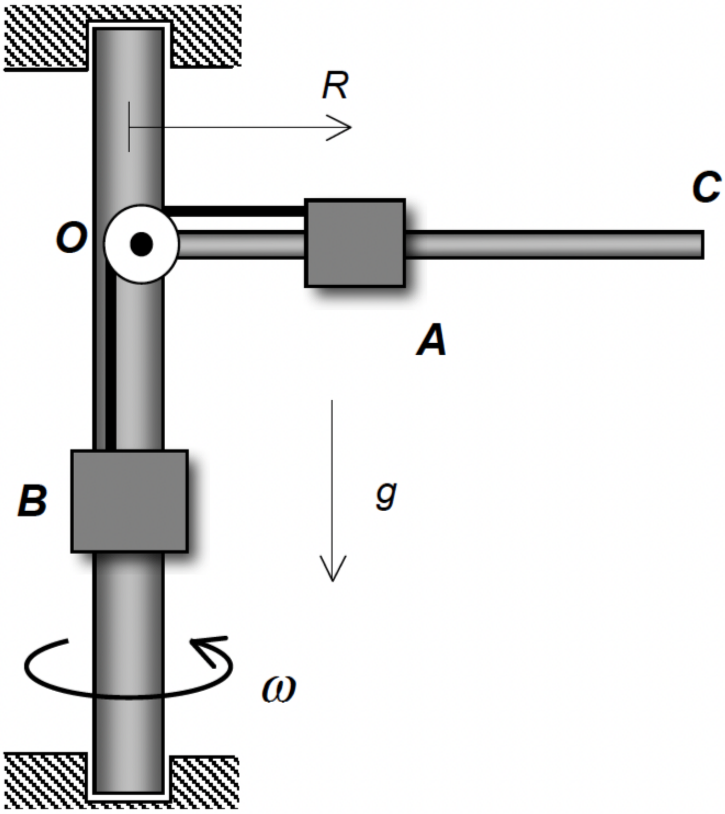

Summary: Particle Kinetics

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WHICH TOOL(s) TO USE?

Put effort up front deciding on which method(s) to use: *Newton*, *work/energy*, *linear impulse momentum* or *angular impulse momentum*. Use the Kinetics Table in Section 5.D of the lecture book as a guide.

THE FOUR-STEP PLAN: Follow it…it is your friend!

Kinetics Table

Method	Body model	Fundamental equations
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Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_2 - m\vec{v}_1$
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Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\, dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A\vec{\omega}$

Lec 26 Short Feedback Form: