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ME 274 Lecture 24

Particle Kinetics – Angular Impulse Momentum – Part 1

Eugenio “Henny” Frias-Miranda

03/11/26

Today's

4D1

4D3

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 23 (4.O and 4.P) due today!!**
2. Office hours are changing to ME2008B...
 - Second floor of renovated side of ME.
3. Bonus quiz grade at end of the semester if we get a good response rate to QR code surveys at the end of lecture.
 - If you are unable to attend lecture on that day/forget to fill it out:
 - Feel free to give feedback based on the content of that lecture's slides.
 - Way of you reviewing previous content and giving feedback to me.
4. Ugrad research and Dynamical Systems Concentration
5. Bonus points if you attend this Friday's lecture (the lecture before spring break)



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Purdue BSME Students: Do you enjoy studying Dynamical Systems?

Purdue BSME Students can pursue a
[Concentration in Dynamical Systems Modeling & Analysis](#)

- Develop skills in dynamics, vibrations, controls, and machine learning
- Gain experience in mathematical modeling and analysis of dynamical systems
- Become knowledgeable in the tools and methods required to succeed in advanced study fields and high-demand engineering careers
- Add research coursework and experience to meet your concentration requirements.



- Requirements:** 10 credit hours
- 1-credit **foundational course** in dynamical systems
 - **Two advanced courses** from a designated list focusing on dynamics, vibrations, or approved research, and choose
 - **One additional course** from an extended list covering control systems, signal processing, numerical methods, machine learning, nonlinear systems, aeroelasticity, research, or related topics



Talk to your ME [undergraduate advisor](#) to determine if this concentration is right for you!

Kinetics: Four-step problem solving method

1. FBDs:

- Draw appropriate FBD(s).
- Choose your coordinate system.

2. Kinetics:

- Choose what solution method for the particular problem at hand (**we will go over these in the coming days...**):
 - Newton/Euler (lectures 15-18) – **Analyzing an instant in time**
 - Work/Energy (lectures 19-20) – **Analyzing speed in terms of position**
 - Linear impulse/momentum (lectures 21-22) - **Analyzing change in velocity during a change in time**
 - Central Impact (lecture 23) - **Analyze velocities between two states**
 - **Angular impulse/momentum – (lectures 24-25) – Analyze Angular Velocity during a change in time**

3. Kinematics:

- Perform needed kinematic analysis (position/velocity/acceleration)
- Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem

4. Solve:

- Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
 - a) Draw more FBDs
 - b) You will need to do more kinematic analysis

Angular Impulse Momentum - Introduction

Objectives of this section/lecture:

- Develop the Angular Impulse-Momentum (AIM) equation for analyzing ***change in angular velocity during a change in time***
- Extend this idea for ***systems of particles***
- Concept of ***conservation of angular momentum***



Where:

- $\vec{H}_O = \vec{r}_{P/O} \times m\vec{v}_P$ is angular impulse momentum of particle P about a fixed-point O

Angular Impulse Momentum - Derivation

1. Newton's Second Law

$$\vec{R} = \sum \vec{F} = m\vec{a}$$

2. Moment Equation - manipulation

$$\vec{M}_O = \vec{r}_{P/O} \times \vec{R}$$

$$\vec{M}_O = \vec{r}_{P/O} \times \left(m \frac{d\vec{v}_P}{dt} \right)$$

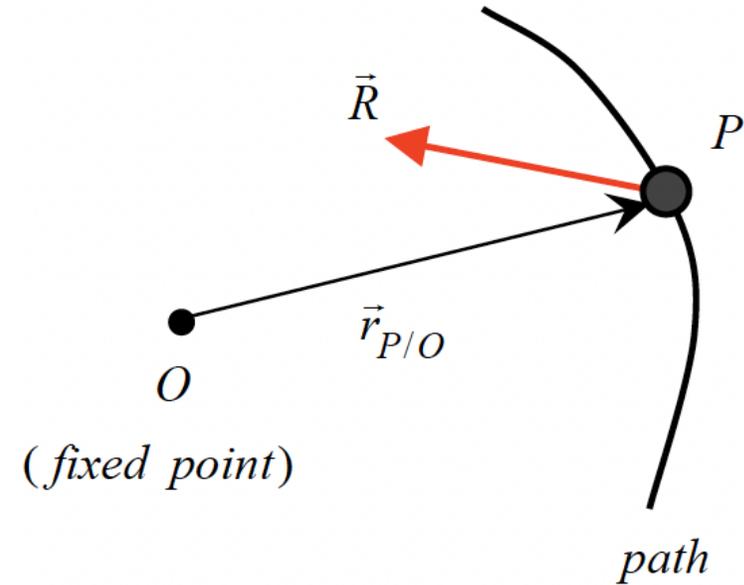
$$\vec{a} \times \frac{d\vec{b}}{dt} = \frac{d}{dt} (\vec{a} \times \vec{b}) - \frac{d\vec{a}}{dt} \times \vec{b}$$

$$\vec{M}_O = \frac{d}{dt} [\vec{r}_{P/O} \times m\vec{v}] - \frac{d\vec{r}_{P/O}}{dt} \times (m\vec{v})$$

$$= \frac{d}{dt} [\vec{r}_{P/O} \times m\vec{v}] - \vec{v} \times (m\vec{v})$$

$$\vec{M}_O = \frac{d}{dt} [\vec{r}_{P/O} \times m\vec{v}]$$

$$= \frac{d\vec{H}_O}{dt}$$



Angular Impulse Momentum - Discussion

$$\int_1^2 \vec{M}_O dt = \int_1^2 d\vec{H}_O = \vec{H}_{O2} - \vec{H}_{O1}$$

1. AIM equation relates the **change in** **to change in**

2. For *central force problems* all forces on the particle pass
Therefore, angular momentum

3. Angular momentum **can** be written in terms of $\vec{H}_O = m (r^2 \dot{\theta}) \hat{k}$

4. When Angular momentum is conserved, we have: $\vec{H}_{O2} = \vec{H}_{O1} \Rightarrow \dot{\theta}_2 = \frac{r_1^2}{r_2^2} \dot{\theta}_1$

- Angular velocity must decrease as radial distance, r , increases; in order for angular momentum to be conserved. Or vice versa.

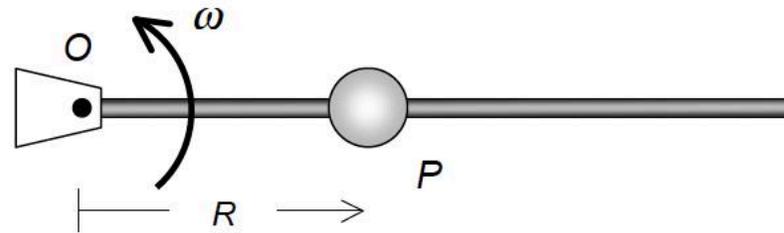
5. Angular momentum gives information of velocity in the

- If we want the (+ radial component of velocity) we usually use the for this. We will see this in example 4.D.3 today.

Example 4.D.1

Given: Particle P (weighing 2 lb) is able to slide on a smooth, lightweight horizontal arm that is rotating about a vertical axis. Initially, P is stationary relative to the arm when the arm is rotating at a rate of $\omega_1 = 20 \text{ rad/s}$ and P is $R = 3 \text{ in}$ from the rotation axis of the arm.

Find: Determine the angular speed of the arm when the P has moved outward to a position of $R = 24 \text{ in}$ from the axis of the shaft.

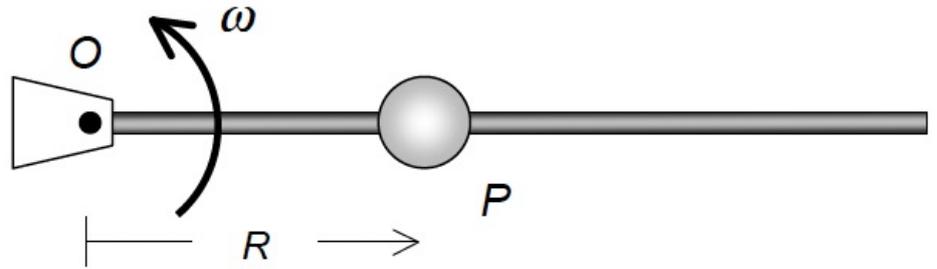


HORIZONTAL PLANE

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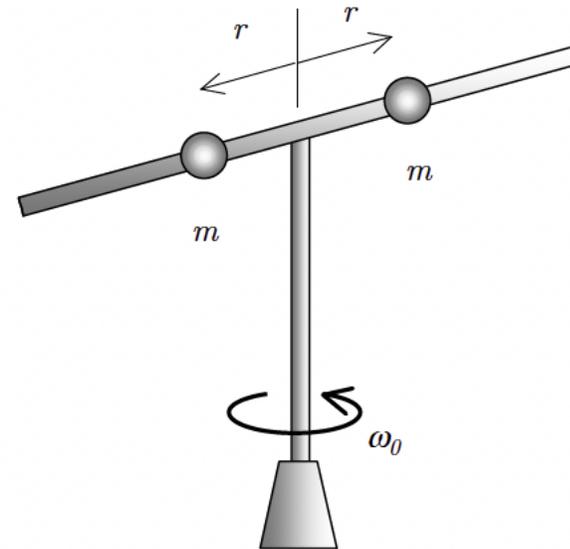


HORIZONTAL PLANE

Example 4.D.2

Given: The assembly shown is rotating with a speed of ω_0 with two particles held fixed relative to the rotating arm at a distance of r from the axis of rotation. The particles are then released and slide outward on the arm to a distance of $2r$.

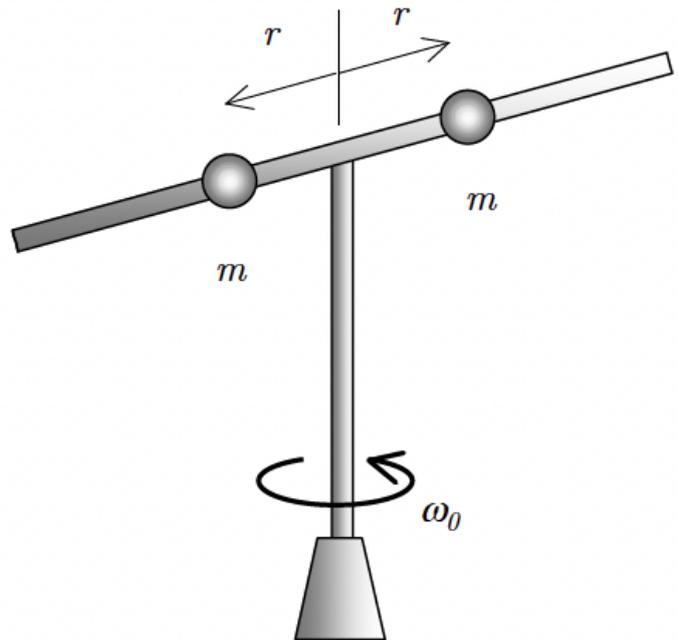
Find: Determine the angular speed of the arm at this new position.



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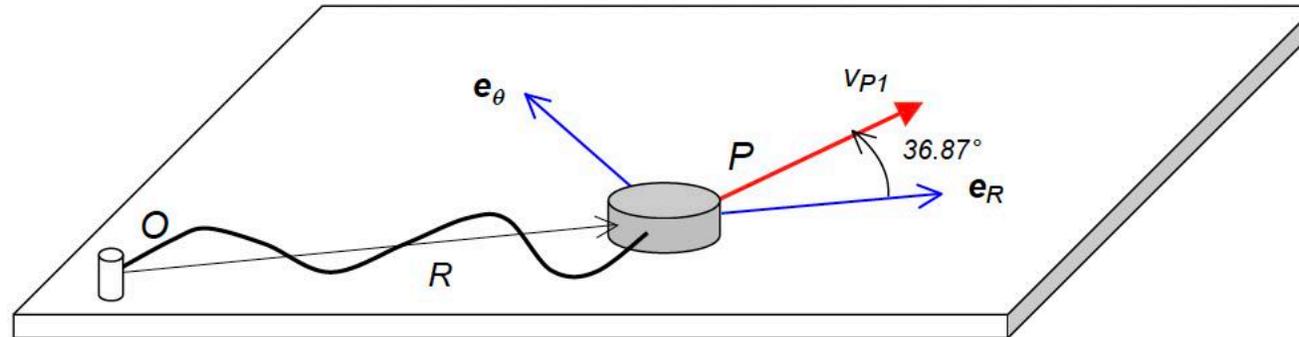
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Example 4.D.3

Given: Disk P having a mass of 0.2 kg is able to slide on a smooth, horizontal surface. A rubber band (having a stiffness of $k = 10 \text{ N/m}$ and unstretched length of 0.6 m) attaches B to a fixed peg at O. Disk P is set into motion with a speed of $v_{P1} = 15 \text{ m/s}$ in the direction shown with $R = 0.4 \text{ m}$.

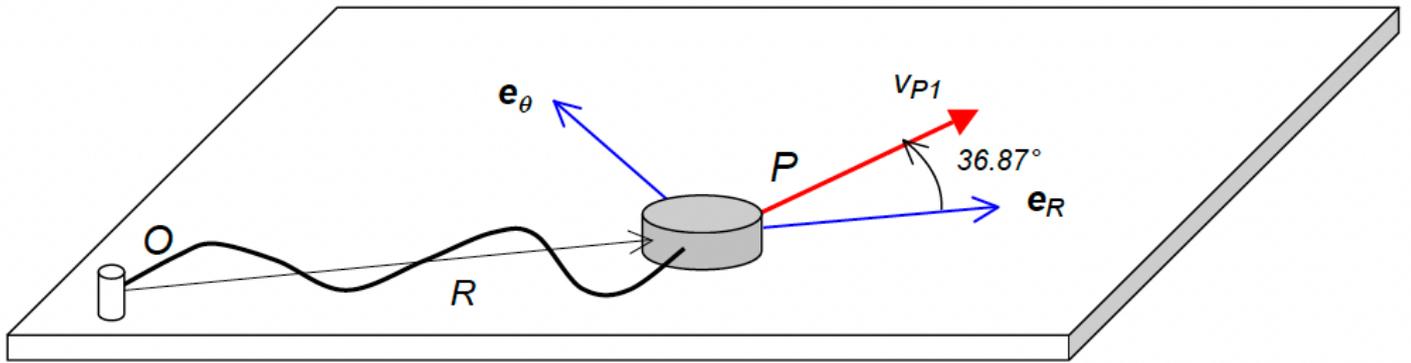
Find: Determine the polar coordinates of the velocity of P when P is a distance 1.5 m from O.



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Summary: Angular Impulse/Momentum Equation 1

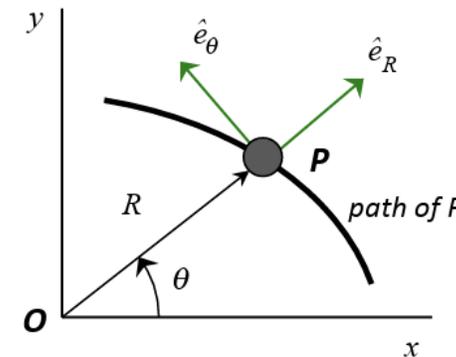
FUNDAMENTAL equation:

$$(\vec{H}_O)_2 = (\vec{H}_O)_1 + \int_1^2 \sum \vec{M}_O dt$$

COMPUTING angular momentum:

$$\begin{aligned} \vec{H}_O &= m\vec{r}_{P/O} \times \vec{v}_P && ; \text{ general} \\ &= m(x\hat{i} + y\hat{j}) \times (\dot{x}\hat{i} + \dot{y}\hat{j}) = m(xy - y\dot{x})\hat{k} && ; \text{ Cartesian} \\ &= m(r\hat{e}_r) \times (\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta) = mr^2\dot{\theta}\hat{k} && ; \text{ polar} \end{aligned}$$

NOTE: O must be a FIXED point! Why?



Lec 24 Short
Feedback Form:

