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ME 274 Lecture 23

Particle Kinetics – Central Impact

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03/09/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 22 (4.M and 4.N) due today!!**
2. Office hours are changing to ME2008B...
 - Second floor of renovated side of ME.
3. Exam 2 equation sheet on course website

ME 274 – Exam No. 2

April 2, 2026

Name (print) _____

(Last)

(First)

EQUATIONS

$$\begin{aligned}\vec{v} &= \dot{x}\hat{i} + \dot{y}\hat{j} \\ &= v\hat{e}_t \\ &= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta\end{aligned}$$

$$\begin{aligned}\vec{a} &= \ddot{x}\hat{i} + \ddot{y}\hat{j} \\ &= \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n \\ &= (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta\end{aligned}$$

$$\begin{aligned}\vec{v}_B &= \vec{v}_A + \vec{v}_{B/A} \\ &= \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A} \\ &= \vec{v}_A + (\vec{v}_{B/A})_{rel} + \vec{\omega} \times \vec{r}_{B/A}\end{aligned}$$

$$\begin{aligned}\vec{a}_B &= \vec{a}_A + \vec{a}_{B/A} \\ &= \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A}) \\ &= \vec{a}_A + (\vec{a}_{B/A})_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})\end{aligned}$$

$$\Sigma \vec{F} = m\vec{a}$$

$$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$$

$$V_{gr} = mgh$$

$$V_{sp} = \frac{1}{2}k\Delta^2$$

$$T = \frac{1}{2}mv^2$$

$$U_{1 \rightarrow 2}^{(nc)} = \int_1^2 (\vec{F} \cdot \hat{e}_t) ds$$

$$\int_1^2 \Sigma \vec{F}_{ext} dt = \left(\sum_i m_i \vec{v}_i \right)_2 - \left(\sum_i m_i \vec{v}_i \right)_1$$

$$\int_1^2 \Sigma \vec{M}_O dt = \left(\sum_i (\vec{H}_O)_i \right)_2 - \left(\sum_i (\vec{H}_O)_i \right)_1$$

$$(\vec{H}_O)_i = \vec{r}_{i/O} \times (m_i \vec{v}_i)$$

Kinetics: Four-step problem solving method

1. FBDs:

- Draw appropriate FBD(s).
- Choose your coordinate system.

2. Kinetics:

- Choose what solution method for the particular problem at hand (**we will go over these in the coming days...**):
 - Newton/Euler (lectures 15-18) – **Analyzing an instant in time**
 - Work/Energy (lectures 19-20) – **Analyzing speed in terms of position**
 - Linear impulse/momentum (lectures 21-22) - **Analyzing change in velocity during a change in time**
 - **Central Impact (lecture 23)**
 - Angular impulse/momentum – (lectures 24-25) -

3. Kinematics:

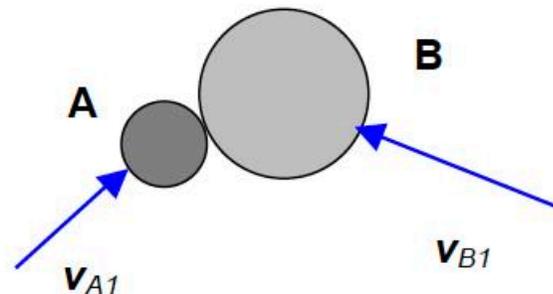
- Perform needed kinematic analysis (position/velocity/acceleration)
- Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem

4. Solve:

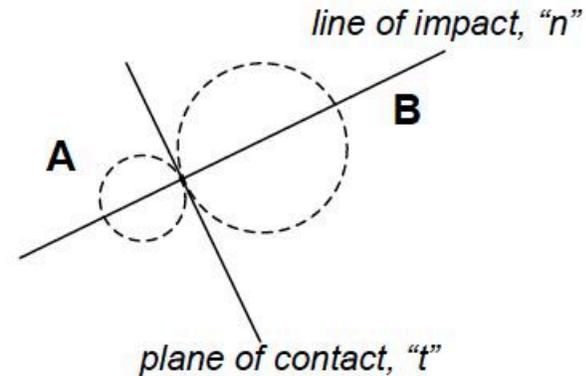
- Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
 - a) Draw more FBDs
 - b) You will need to do more kinematic analysis

Central Impact of a Pair of Particles

- **Why/Motivation?** Helps us relate/find between two states. (Pre and Post collision)
- **Condition to use this concept:** *The line of impact passes through the centers of mass of each body involved in the impact. We are given a Coefficient of Restitution/'e' for the problem.*
- **Relevant/important definitions:**
 1. A plane that is tangent to the contact surfaces of A and B. Denoted as 't'
 2. A line that is perpendicular to the plane of contact for A and B. Denoted as 'n'.
 3. An impact in which the line of impact passes through the centers of mass of each body involved in the impact.



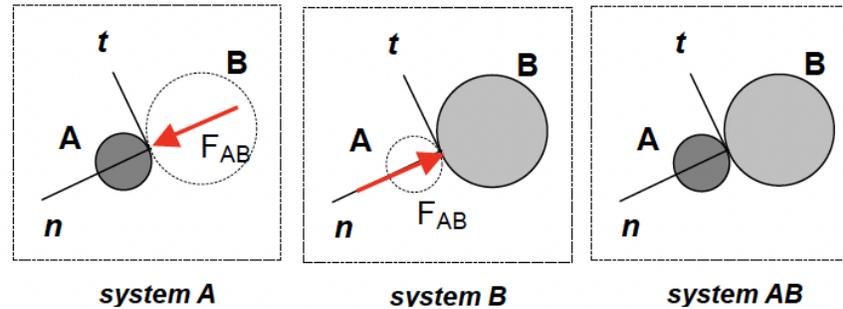
(a) central impact



(b) coordinate axes

Central Impact Problems – How to/general outline

1. FBDs. Usually 3:



2. System A and System B, separately:

(1) $\sum F_t = 0 \Rightarrow m_A v_{At2} = m_A v_{At1} \Rightarrow$ $\sum F_t = 0 \Rightarrow m_B v_{Bt2} = m_B v_{Bt1} \Rightarrow$

$\sum F_n = F_{AB} \neq 0 \Rightarrow$ momentum NOT conserved in “n” direction for A $\sum F_n = -F_{AB} \neq 0 \Rightarrow$ momentum NOT conserved in “n” direction for B

3. System A and B together

$\sum F_t = 0 \Rightarrow m_A v_{At2} + m_B v_{Bt2} = m_A v_{At1} + m_B v_{Bt1}$

(3) $\sum F_n = 0 \Rightarrow$

4. Coefficient of Restitution:

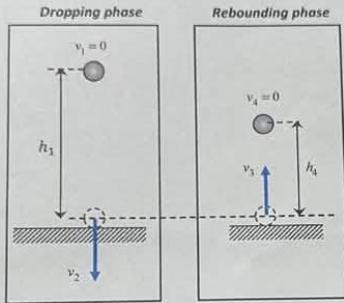
(4)

In class demo – calculating Coefficient Of Restitution (COR)



Drop tests and the coefficient of restitution

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Dropping phase

$$\frac{1}{2}mv_2^2 = mgh_1 \Rightarrow v_2 = -\sqrt{2gh_1}$$

Rebounding phase

$$\frac{1}{2}mv_3^2 = mgh_4 \Rightarrow v_3 = \sqrt{2gh_4}$$

Coefficient of restitution (COR)

$$e = -\left(\frac{v_3}{v_2}\right) = \sqrt{h_4 / h_1}$$

Stress ball

happy face

h_1	steel ball		plastic ball		ping pong ball		tomato		eyeball	
	h_4	e	h_4	e	h_4	e	h_4	e	h_4	e
36"			22"	0.78						
30"			19"	0.80						
24"			15"	0.79						
18"			12"	0.72						



Central Impact Problems – Discussion

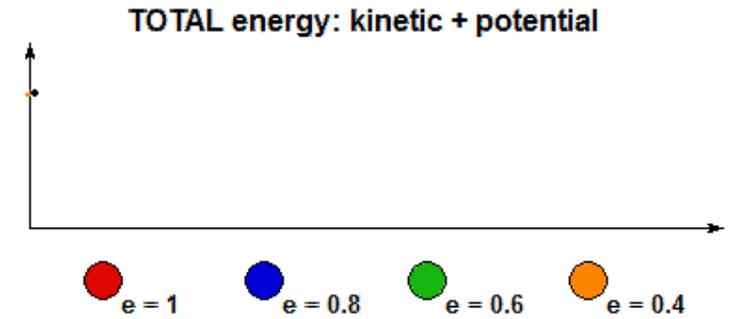
1. Equations Used in this type of problems:

$$(1) \quad v_{At2} = v_{At1}$$

$$(2) \quad v_{Bt2} = v_{Bt1}$$

$$(3) \quad m_A v_{An2} + m_B v_{Bn2} = m_A v_{An1} + m_B v_{Bn1}$$

$$(4) \quad e = -\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}$$



2. Central impact = line of impact passes through centers of mass of bodies involved.

3. Reminder: The coefficient of restitution equation (COR) above (equation 4) is valid only for velocity components in the normal direction, n.

4. Mechanical energy in System AB is not conserved during impact. Except in special case of $e = 1$ (which is physically impossible). See figure in top left or link:

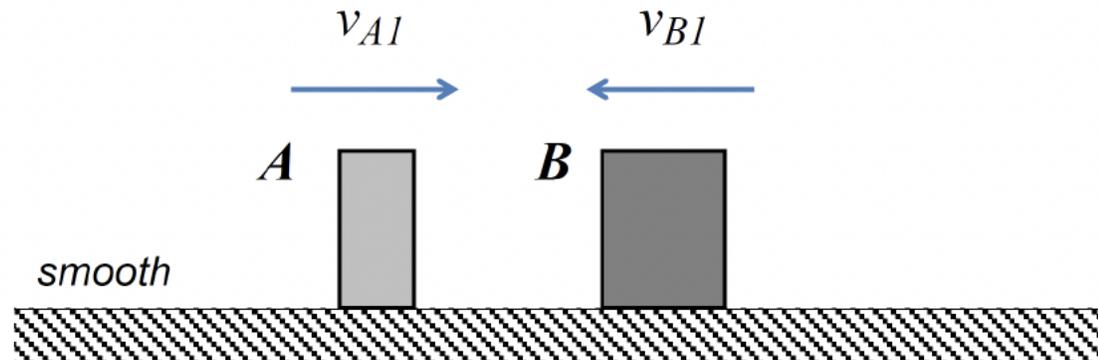
- <https://www.purdue.edu/freeform/me274/course-material/animations/impacts-and-energy/>

Example 4.C.10

Given: Blocks A and B (having masses of m_A and m_B , respectively) are initially moving to the right and left, respectively, on a smooth horizontal surface with speeds of v_{A1} and v_{B1} , respectively. At some instant in time, A strikes B. The coefficient of restitution of this impact is e . As a result of the impact, block B becomes stationary.

Find: Determine the initial speed v_{B1} of block B.

Use the following parameters in your analysis: $m_A = 2$ kg, $m_B = 3$ kg, $v_{A1} = 1.5$ m/s and $e = 0.4$.



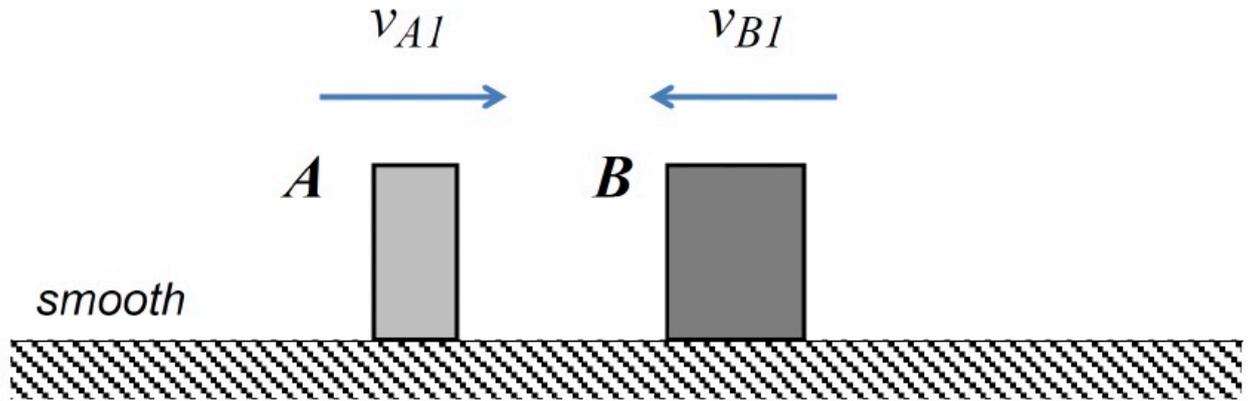
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p.245

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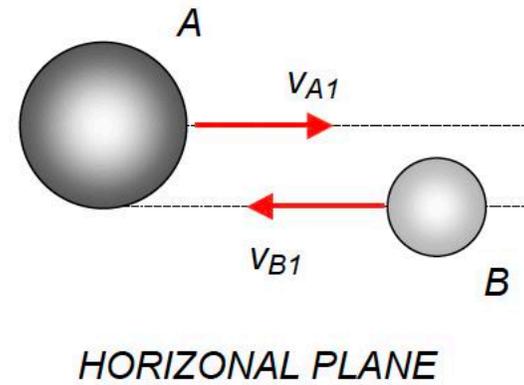


Example 4.C.11

Given: Sphere A has a mass of 20 kg and a radius of 75 mm, while B has a mass of 5 kg and a radius of 48 mm. The coefficient of restitution for the impact of A and B is known to be $e = 0.6$.

Find: Determine the velocities of the spheres immediately after impact.

Use the following parameters in your analysis: $v_A = 5$ m/s and $v_B = 15$ m/s.

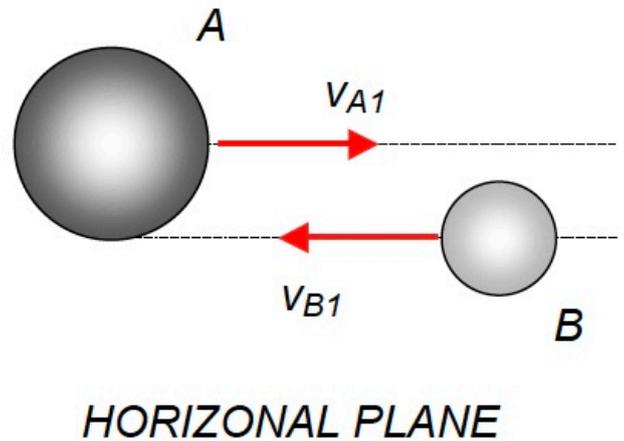


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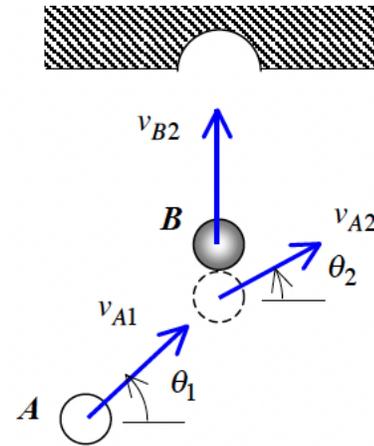


Example 4.C.12

Given: Cue ball A strikes a stationary object ball B, with a speed v_{A1} as shown in the figure below. The coefficient of restitution for this impact is e . After impact, A moves along a line defined by the angle θ_2 , and B moves directly to the side pocket.

Find: Determine the numerical value of the rebound angle θ_2 of A, assuming the masses of A and B are the same.

Use the following parameters in your analysis: $\theta_1 = 45^\circ$ and $e = 0.9$.

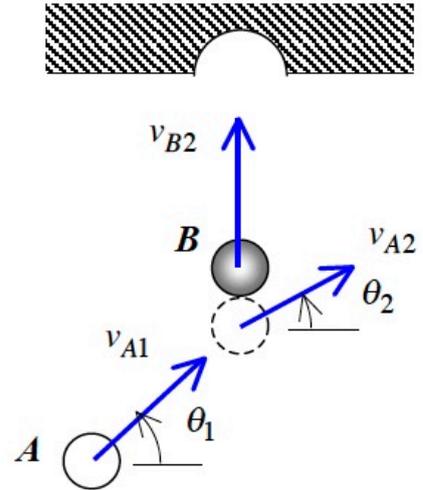


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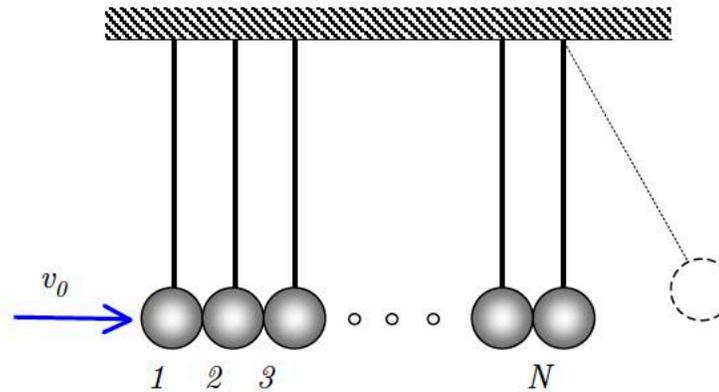


Example 4.C.13

Given: A “Newton’s Cradle” is made up of N identical pendulums. The particle for pendulum 1 has a horizontal velocity of v_0 when it strikes the particle for pendulum 2 (all pendulums except 1 are at rest prior to this impact). The coefficient of restitution for the impacts of all particles is e .

Find: Determine:

- (a) The speed of particle N immediately after the final impact; and
- (b) The maximum height h reached by particle N .

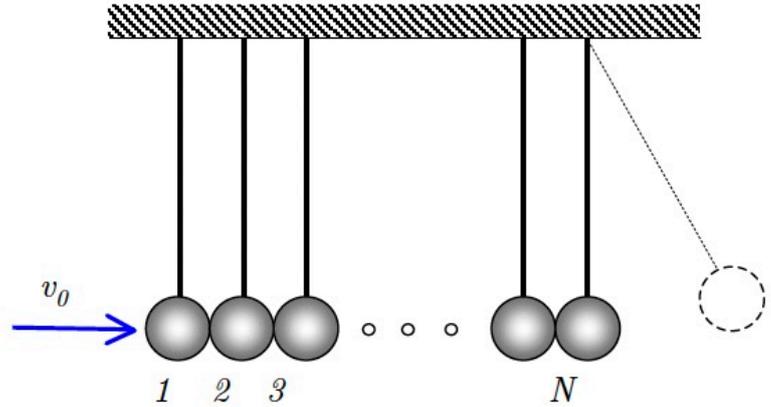


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Summary: Central Impact Problems

FUNDAMENTAL equations: the linear impulse-momentum equations and coefficient of restitution (COR) equation

$$A: \sum F_t = 0 \Rightarrow m_A v_{A1t} = m_A v_{A2t} \Rightarrow v_{A2t} = v_{A1t}$$

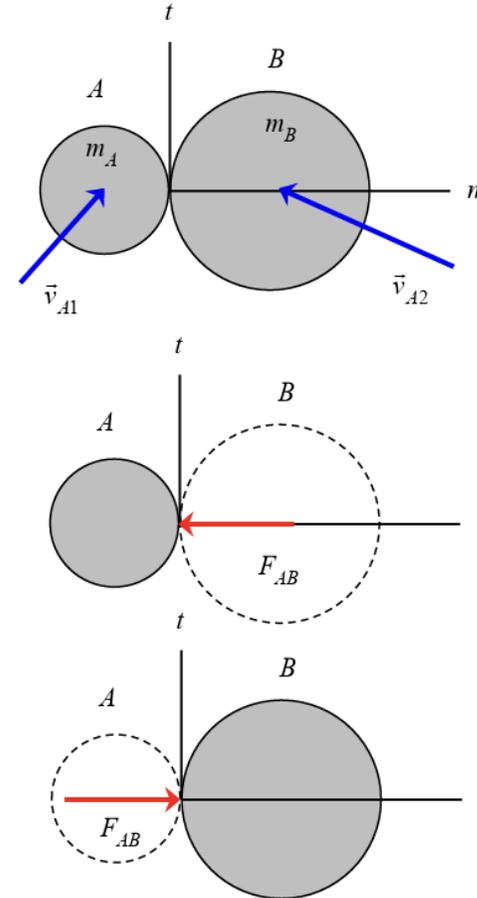
$$B: \sum F_t = 0 \Rightarrow m_B v_{B1t} = m_B v_{B2t} \Rightarrow v_{B2t} = v_{B1t}$$

$$A+B: \sum F_n = 0 \Rightarrow m_A v_{A1n} + m_B v_{B1n} = m_A v_{A2n} + m_B v_{B2n}$$

$$COR: e = -\left(\frac{v_{B2n} - v_{A2n}}{v_{B1n} - v_{A1n}} \right)$$

COMMENTS:

- The COR equation is valid for **ONLY** the n-components of velocity.
- Energy is **NOT** conserved during impact, except for $e = 1$.



Lec 23 Short
Feedback Form:

