ME 274: Basic Mechanics II

Lecture 21: Particle Kinetics – LIM, Central Impact

PURDUE UNIVERSITY | School of Mechanical Engineering

Applying LIM to Impacts

LIM equation $m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \vec{R}\, dt$

$m\vec{v}$ - linear momentum of the particle
$\int_1^2 \vec{R}\, dt$ - impulse from the net force acting on the particle

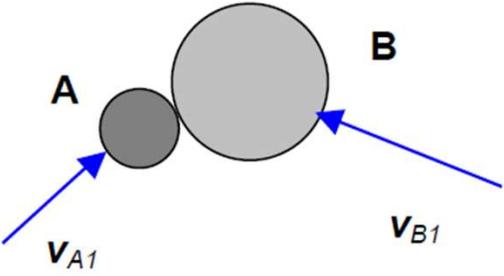**Impacts** occur over a very short time interval
- The impact force between particles is referred to as the **impulsive force** – large magnitude, short duration
- We **neglect** the contributions of **non-impulsive forces** (for example, due to weight or springs during our impact interval Δt)
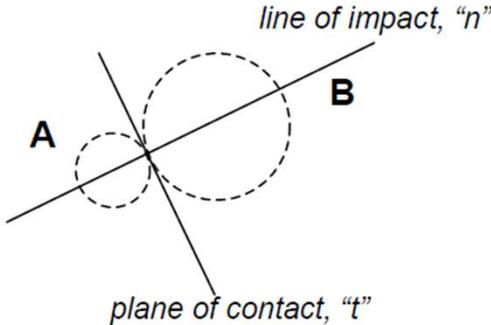

Impulse on Baseball

Force (kN)

Time (ms)
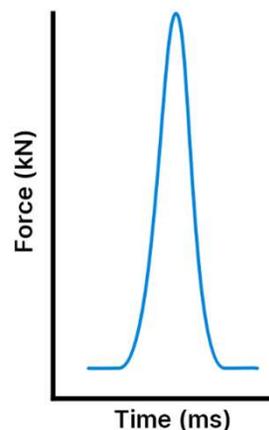
Central impact of two particles

(a) central impact

line of impact, "n"

plane of contact, "t"

(b) coordinate axes

Objective: relate the velocities of particles A and B immediately before $(\vec{v}_{A1}, \vec{v}_{B1})$ and immediately after $(\vec{v}_{A2}, \vec{v}_{B2})$ impact
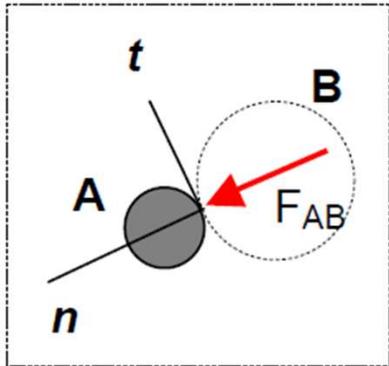
Definitions:
Plane of contact: The plane tangent to the surfaces of A and B
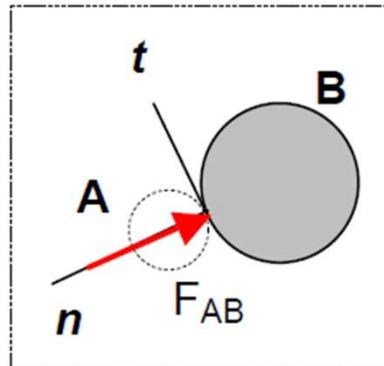**Line of impact:** The line perpendicular to the plane of contact for A and B
Central impact: an impact in which the line of impact passes through the center of mass of each body

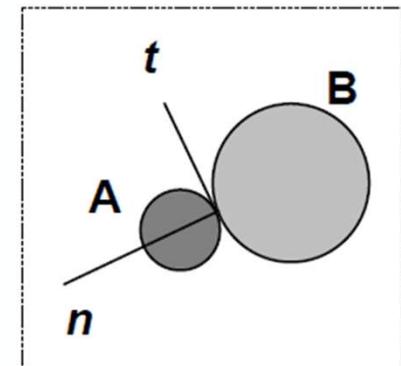Note: in these problems you will be using the $n - t$ coordinate axis

Systems within a contact problem

system A

Tangent direction:
$$\sum F_t = 0$$
$$m_A v_{At2} = m_A v_{At1} \quad \Rightarrow v_{At2} = v_{At1}$$

Normal direction:
$$\sum F_n = F_{AB} \neq 0$$

system B

Tangent direction:
$$\sum F_t = 0$$
$$m_B v_{Bt2} = m_B v_{Bt1} \quad \Rightarrow v_{Bt2} = v_{Bt1}$$

Normal direction:
$$\sum F_n = -F_{AB} \neq 0$$

system AB

Tangent direction:
$$\sum F_t = 0$$
$$m_A v_{At2} + m_B v_{Bt2} = m_A v_{At1} + m_B v_{Bt1}$$
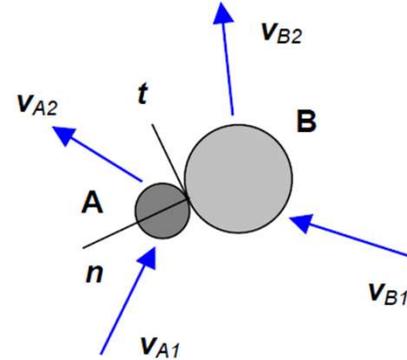
Normal direction:
$$\sum F_n = 0$$
$$m_A v_{An2} + m_B v_{Bn2} = m_A v_{An1} + m_B v_{Bn1}$$

Coefficient of restitution (COR):
$$e = -\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}$$

Central impact problems – Discussion

(1) $v_{At2} = v_{At1}$

(2) $v_{Bt2} = v_{Bt1}$

(3) $m_A v_{An2} + m_B v_{Bn2} = m_A v_{An1} + m_B v_{Bn1}$

(4) $e = -\dfrac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}$

- Equations 1-4 can be used in cases where the impact force is the **only impulse force** between A and B
- The COR equation is only valid in the **normal direction.**
- Mechanical energy in system AB is **not conserved** during impact - you should **never** use work-energy in impact analysis

What is the coefficient of restitution?
The COR is a measure of the elasticity of a collision
- COR = 1 – perfectly elastic collision – energy is conserved (non-physical)
- COR = 0 – perfectly inelastic collision – particles stick together

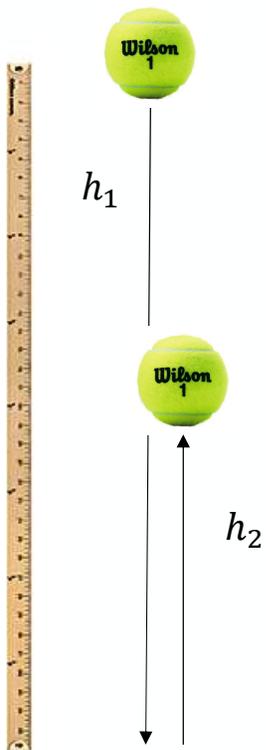$$e = -\dfrac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}$$

Demo: Calculating the coefficient of restitution (COR) of two balls

We drop a tennis and ping-pong ball from the same height and measure their max height after bouncing. How do we determine the COR?

Weight tennis ball: – 0.125 lbf
Weight ping-pong ball: – 0.006 lbf
Drop height, $h_1 = 3'$

h_1

h_2

Example 4.C.10

Given: Blocks A and B (having masses of m_A and m_B, respectively) are initially moving to the right and left, respectively, on a smooth horizontal surface with speeds of v_{A1} and v_{B1}, respectively. At some instant in time, A strikes B. The coefficient of restitution of this impact is e. As a result of the impact, block B becomes stationary.

Find: Determine the initial speed v_{B1} of block B.

Use the following parameters in your analysis: $m_A = 2$ kg, $m_B = 3$ kg, $v_{A1} = 1.5$ m/s and $e = 0.4$.
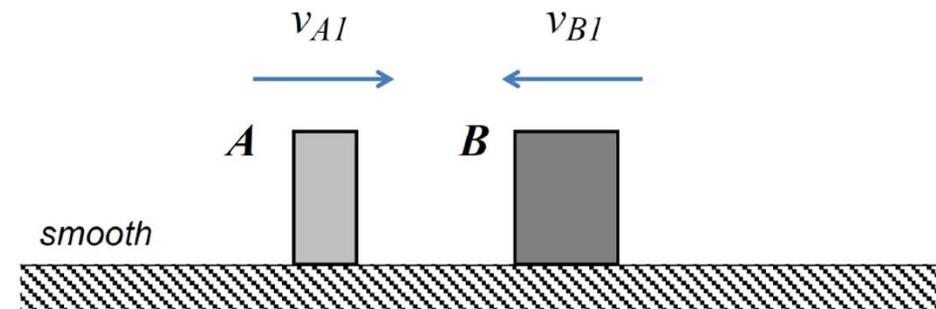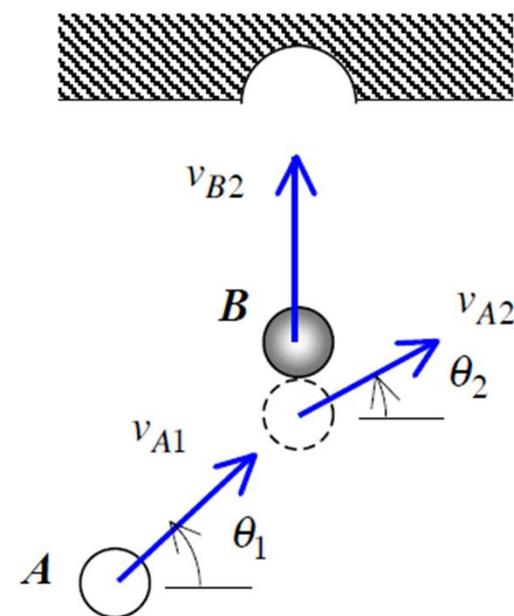
Example 4.C.12

Given: Cue ball A strikes a stationary object ball B, with a speed v_{A1} as shown in the figure below. The coefficient of restitution for this impact is e. After impact, A moves along a line defined by the angle θ_2, and B moves directly to the side pocket.

Find: Determine the numerical value of the rebound angle θ_2 of A, assuming the masses of A and B are the same.

Use the following parameters in your analysis: $\theta_1 = 45°$ and $e = 0.9$.

Example 4.C.13

Given: A "Newton's Cradle" is made up of N identical pendulums. The particle for pendulum 1 has a horizontal velocity of v_0 when it strikes the particle for pendulum 2 (all pendulums except 1 are at rest prior to this impact). The coefficient of restitution for the impacts of all particles is e.

Find: Determine:

   (a) The speed of particle N immediately after the final impact; and

   (b) The maximum height h reached by particle N.