ME 274: Basic Mechanics II

Lecture 21: Particle Kinetics – Linear Impulse-Momentum

PURDUE UNIVERSITY | School of Mechanical Engineering

Linear Impulse-Momentum Key Takeaways:

LIM equation $$m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \vec{R}\,dt$$

$m\vec{v}$ - linear momentum of the particle

$\int_1^2 \vec{R}dt$ - impulse from the net force acting on the particle

- Linear impulse-momentum relates the change in linear momentum to the impulse (force applied over time) acting on the particle.
- a **vector** equation → we can resolve it into components!

$$mv_{x2} = mv_{x1} + \int_1^2 R_x dt$$
$$mv_{y2} = mv_{y1} + \int_1^2 R_y dt$$

- If there is no resultant force, the linear momentum is said to be **conserved**: $mv_{x2} = mv_{x1}$
- Conservation of momentum does not imply conservation of energy (or vice versa)
- For systems of multiple particles, **internal forces cancel** → make your system as large as feasibly possible!

Question C4.9

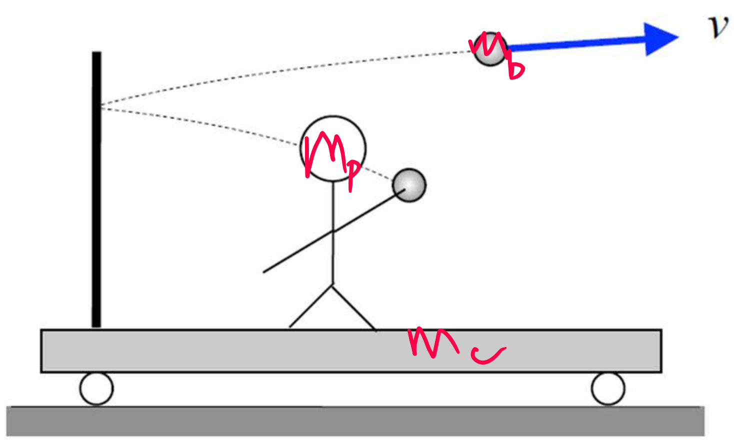
You are on a cart that is initially at rest on a smooth track. You throw a ball at a partition that is rigidly mounted on the cart. If the ball bounces off the partition as shown in the figure, then at the instant shown in the figure:

(a) The cart is moving to the right

(b) The cart is stationary

(c) The cart is moving to the left

(d) More information is needed about the impact of the ball with the partition in order to answer this question



@ t_1 $(m_b + m_p + m_c) v = 0$

@ t_2 $m_b v_b + (m_p + m_c) V_{p+c} = 0$

$(m_p + m_c) V_{p+c} = -m_b V_b$

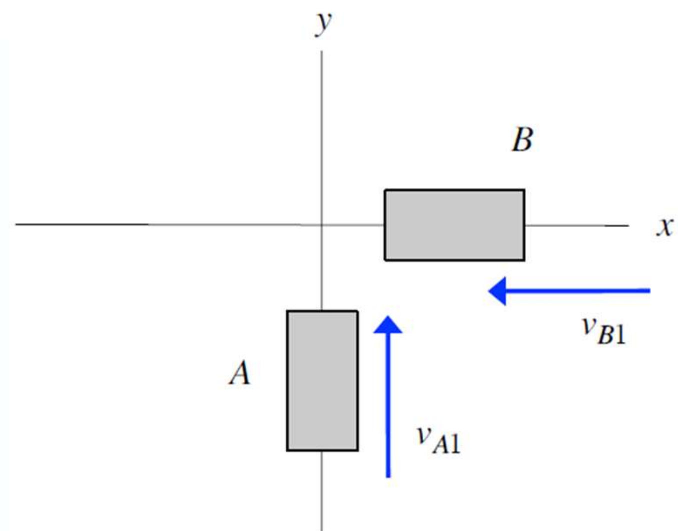
V_{p+c} opposite direction to V_b

Example 4.C.7

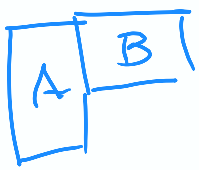
Given: Cars A and B (having weights of W_A and W_B, respectively) strike each other with speeds of v_{A1} and v_{B1}. After a short collision time, the cars stick together.

Find: Determine the x and y components of velocity for the cars after the collision.

Use the following parameters in your analysis: $W_A = 3000$ lb, $W_B = 4000$ lb, $v_{A1} = 40$ mph and $v_{B1} = 25$ mph.



FBD



→ no forces → linear momentum conserved in x + y

Kinetics

LIM x-dir:

$$-\frac{W_B}{g} V_{B1} = \frac{(W_B + W_A)}{g} V_{2x} \Rightarrow V_{2x} = \frac{-W_B V_{B1}}{W_B + W_A}$$

LIM y-dir:

$$\frac{W_A}{g} V_{A1} = \frac{(W_B + W_A)}{g} V_{2y} \Rightarrow V_{2y} = \frac{W_A V_{A1}}{W_B + W_A}$$

Example 4.C.8

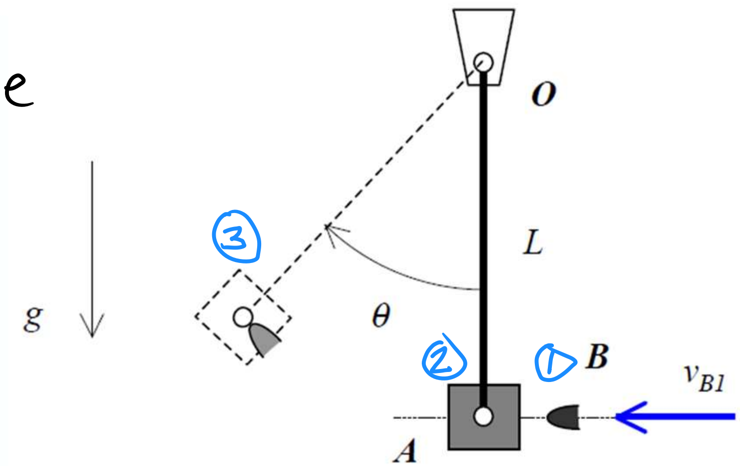
Given: Block A (having a weight of W) is suspended by a cord from fixed point O. Bullet B (having a weight of w) strikes the stationary block A with a speed of v_{B1}. On impact, the bullet sticks to block A.

Find: Determine:

(a) The maximum elevation angle θ of the cord after impact; and

(b) The energy lost during the impact of B with A.

Use the following parameters in your analysis: $w = 0.2$ lb, $W = 75$ lb, $L = 5$ ft and $v_{B1} = 1800$ ft/s.



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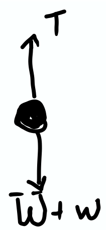


cord cannot carry a transverse load

(short time frame)

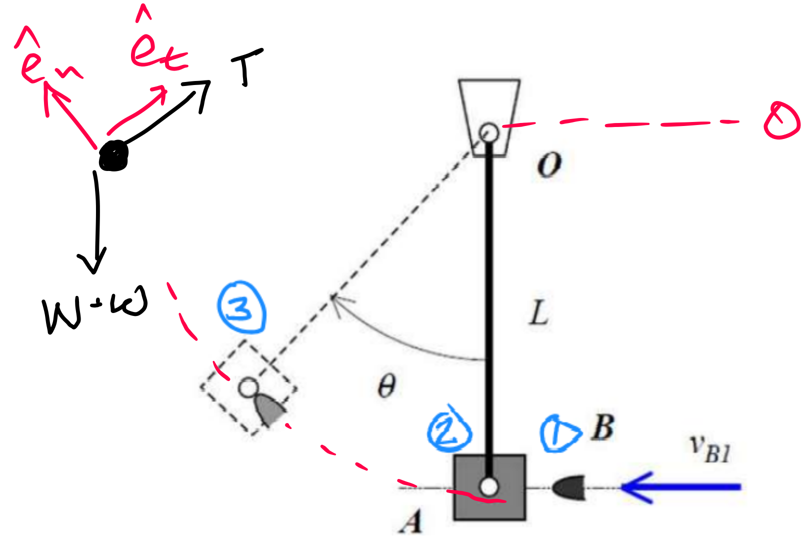
1–2 → impact, LIM

$$\frac{w}{g} v_{B1} = \frac{w+W}{g} v_{2x}$$

$$v_2 = \frac{w v_{B1}}{(w+W)}$$

2-3 → change in velocity, change in Position → W-E

$$T_2 + V_2 + U^N_{2\to3} = T_3 + V_3$$



$$T_2 = \frac{1}{2}\left(\frac{W+w}{g}\right)V_2^2 = \frac{1}{2}\frac{(wV_{B1})^2}{g(w+W)}$$

$$V_2 = -(W+w)L$$

$U^N_{2\to3} = 0$ → tension always Normal

$$T_3 = 0$$

$$V_3 = -(W+w)L\cos\theta_{max}$$

$$\frac{1}{2}\frac{(wV_{B1})^2}{g(w+W)} - (W+w)L = -(w+W)L\cos\theta_{max}$$

→ solve for θ_{max}

Energy Loss from $1 \to 2$

total energy at each time = Kinetic + potential

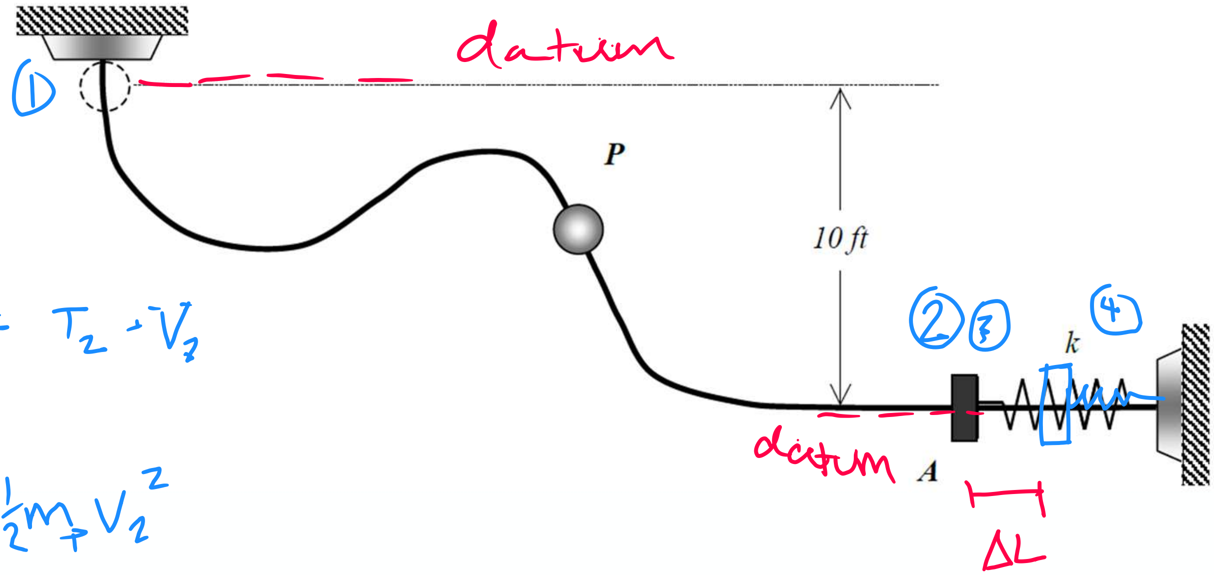
$$\Delta \text{Energy} = \left(T_2 + \cancel{V_2}\right) - \left(T_1 + \cancel{V_1}\right)$$

$$= \frac{1}{2}\frac{(w V_{B1})^2}{g(w+W)} - \frac{1}{2}\frac{w}{g}V_{B1}^2$$

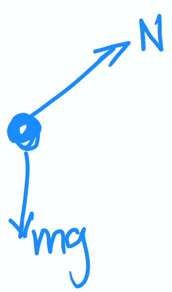
Example 4.C.6

Given: Particle P (weighing 10 lb) is released from rest and slides down a smooth, curved rod and sticks to block A (weighing 5 lb).

Find: Determine the maximum deflection of the spring attached to A, if the spring has a stiffness of $k = 100$ lb/ft.



$\underline{1 \to 2}$ sliding down guide W-E



$$T_1 + V_1 + U_{1\to 2} = T_2 + V_2$$

$T_1 = 0$ (RFR) $\qquad T_2 = \frac{1}{2} m_P v_2^2$

$V_1 = 0$ $\qquad V_2 = -m_P g h$

$U_{1\to 2} = 0$

$$\Rightarrow \quad 0 = \frac{1}{2} m_P v_2^2 - m_P g h$$

$$v_2 = \sqrt{2gh}$$

2 → 3 impact event → LIM

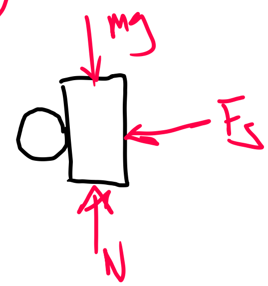
assume short duration → no spring compression → no resultant force, linear momentum conserved

before impact (only p moving)   after impact

$$m_p V_2 + \cancel{\int F_x \, dt} = (m_p + m_A) V_3$$

$$V_3 = \frac{m_p V_2}{(m_p + m_A)} = \frac{m_p \sqrt{2gh}}{(m_p + m_A)}$$

3 → 4 spring compression → WE



position 3

$$T_3 = \frac{1}{2}(m_p + m_A) V_3^2$$

$$= \frac{m_p^2 g h}{m_p + m_A}$$

$$V_3 = 0$$

$$U^{NC}_{3\to4} = 0$$

position 4

$$T_4 = 0$$

$$V_4 = \frac{1}{2} k (\Delta L)^2$$

Work - Energy

$$\frac{m_p^2 g h}{m_p + m_A} = \frac{1}{2} k \Delta L$$

$$\Delta L = \frac{2 m_p^2 g h}{k(m_p + m_A)}$$