ME 274: Basic Mechanics II

Lecture 21: Particle Kinetics – Linear Impulse-Momentum

PURDUE UNIVERSITY | School of Mechanical Engineering

Kinetics: Four-Step Problem Solving Method

1) FBD
   - Draw appropriate FBD (note- you may have to draw more than 1)
   - Choose coordinate system (cartesian, path, polar)
2) Kinetics $\sum F = ma$
   - Sum forces and break into components
   - Choose appropriate method to solve the problem – we will learn these in upcoming sections!
     - Newton/Euler
     - Work/Energy → V as it varies s & F(s)
     - Linear impulse/Momentum → V // t & F(t)
     - Angular impulse/Momentum)
3) Kinematics
   - Perform a kinematic analysis of the system using techniques we have developed in the previous chapters.
   - Use your kinetics equations from step 2 to determine what information you need to solve the problem
4) Solve
   - Count the number of equations and unknowns. Do they match?
   - If not:
     - Draw more FBDs
     - Do additional kinematic analysis

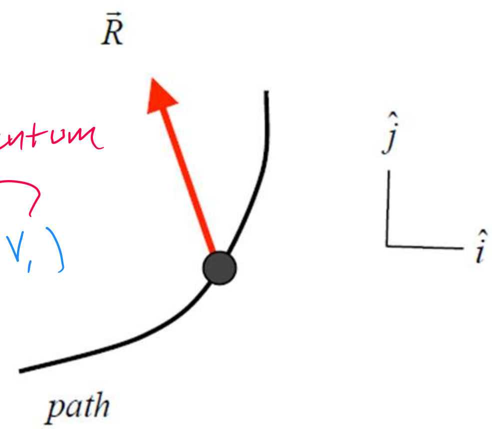
Linear impulse momentum - Analyzing changes in velocity resulting in forces over an elapsed time

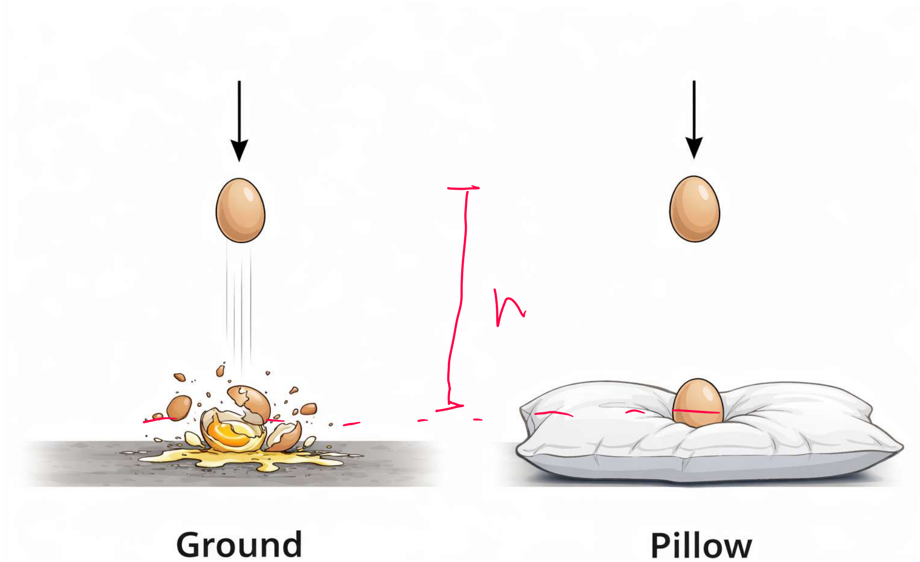
Remember our resultant force $\vec{R}$:

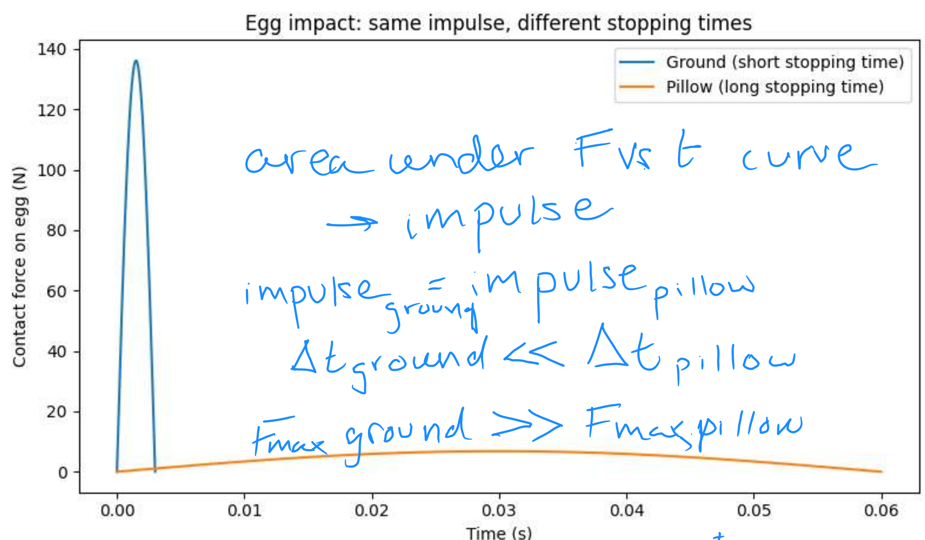
$$\vec{R} = \sum \vec{F} = m\vec{a} = m\frac{dv}{dt}$$

Separate & integrate:

$$\int_{t_1}^{t_2} \vec{R}\,dt = \int_{v_1}^{v_2} m\,dv \implies \underbrace{\int_{t_1}^{t_2} \vec{R}\,dt}_{\text{impulse}} = \underbrace{m(v_2 - v_1)}_{\text{momentum}}$$







area under F vs t curve → impulse

$\text{impulse}_{ground} = \text{impulse}_{pillow}$

$\Delta t_{ground} \ll \Delta t_{pillow}$

$F_{max\,ground} \gg F_{max\,pillow}$

$v_1 \rightarrow$ right before impact $= \sqrt{2gh}$
(you can solve using W-E)

$v_2 = 0 \rightarrow$ egg fully stopped

LIM $\Delta p = m(v_2 - v_1) = \int_{t_1}^{t_2} \vec{R}$

change in momentum is the same for both cases

what is different? → Δt

Important considerations for Linear Impulse-Momentum: $m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \vec{R}\,dt$

$m\vec{v}$ - linear momentum of the particle

$\int_1^2 \vec{R}dt$ - impulse from the net force acting on the particle

- Linear impulse-momentum relates the change in linear momentum to the impulse acting on the particle.
- Note that **impulse is a function of time**, so this effectively gives the **change in particle velocity with respect to the time** a force is acting on that particle.
- Also note $m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \vec{R}dt$ is a **vector** equation (in contrast to work-energy) → we can resolve it into components!

i.e. in the cartesian description:

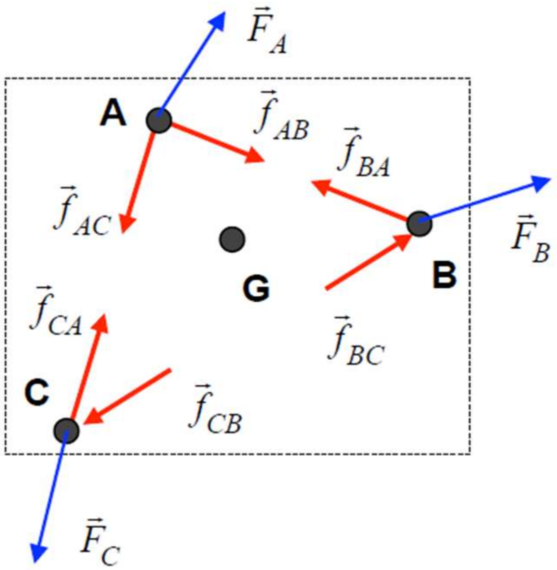
2 scalar eqn

$$mv_{x2} = mv_{x1} + \int_1^2 R_x dt$$

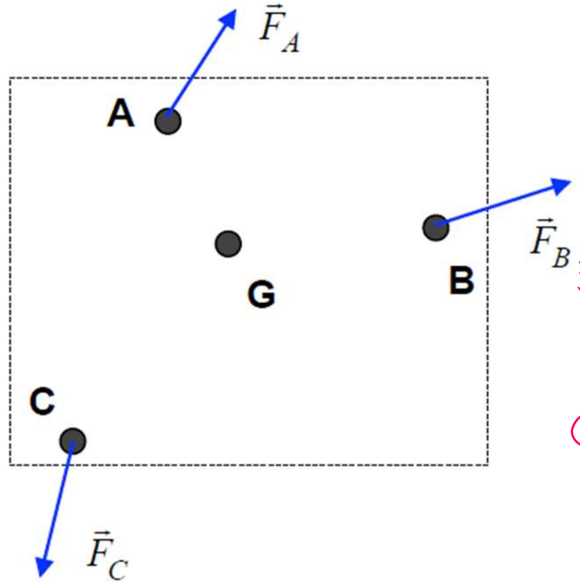
$$mv_{y2} = mv_{y1} + \int_1^2 R_y dt$$

- If there is no resultant force, the linear momentum is said to be conserved: $mv_{x2} = mv_{x1}$
- Conservation of momentum does not imply conservation of energy (or vice versa)

Linear Impulse-Momentum Equations for a System of Particles



all forces acting on system



"external" forces acting on system

Solving for impulse and momentum for each particle:

A $$m_A\vec{v}_{A2} = m_A\vec{v}_{A1} + \int_1^2 \left[\vec{F}_A + \vec{f}_{AB} + \vec{f}_{AC}\right] dt$$

B $$m_B\vec{v}_{B2} = m_B\vec{v}_{B1} + \int_1^2 \left[\vec{F}_B + \vec{f}_{BA} + \vec{f}_{BC}\right] dt$$

$$= m_B\vec{v}_{B1} + \int_1^2 \left[\vec{F}_B - \vec{f}_{AB} + \vec{f}_{BC}\right] dt$$

C $$m_C\vec{v}_{C2} = m_C\vec{v}_{C1} + \int_1^2 \left[\vec{F}_C + \vec{f}_{CA} + \vec{f}_{CB}\right] dt$$

$$= m_C\vec{v}_{C1} + \int_1^2 \left[\vec{F}_C - \vec{f}_{AC} - \vec{f}_{BC}\right] dt$$

Summing for entire system:

$$m_A\vec{v}_{A2} + m_B\vec{v}_{B2} + m_C\vec{v}_{C2} = m_A\vec{v}_{A1} + m_B\vec{v}_{B1} + m_C\vec{v}_{C1} + \int_1^2 \left[\vec{F}_A + \vec{F}_B + \vec{F}_C\right] dt \quad \Rightarrow$$

$$\sum m_j\vec{v}_{j2} = \sum m_j\vec{v}_{j1} + \int_1^2 \left[\sum \vec{F}_j^{ext}\right] dt$$

Key takeaway: make system as large as possible to include as many internal forces as you can!

Example 4.C.1

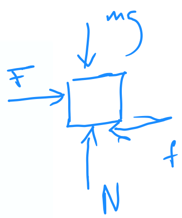
Find: Determine:

(a) The speed of the block at $t = \Delta t$; and

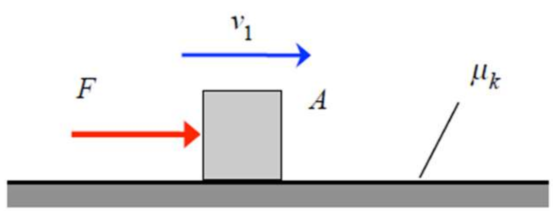
(b) The speed of the block at $t = 2\Delta t$.

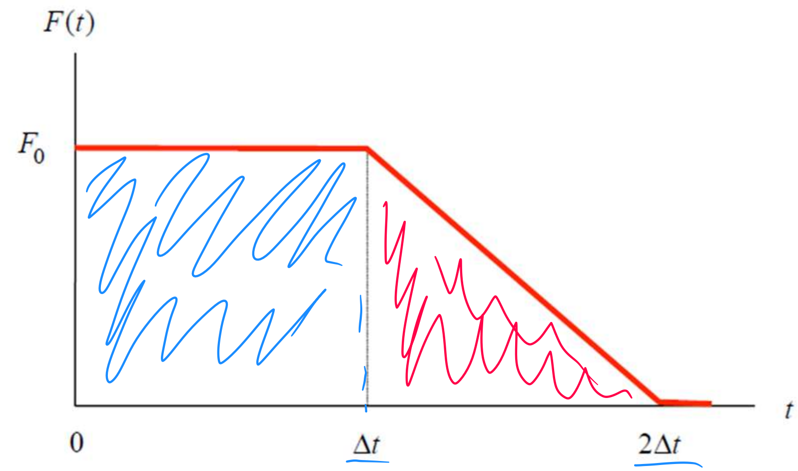
Use the following parameters in your analysis: $m = 8$ kg, $F_0 = 20$ N, $v_1 = 15$ m/s, $\mu_k = 0.15$ and $\Delta t = 0.2$ s.

1) FBD



$f = \mu_k mg$





Kinetics → LIM

$$mv_2 = mv_1 + \int_{t_1}^{t_2} \Sigma F_x \, dt$$

$$= mv_1 + \int_{t_1}^{t_2} (F(t) - \mu_k mg) \, dt$$

area under the curve

$0 \to \Delta t$

$$\int_0^{\Delta t} F_0 \, dt = F_0 \Delta t$$

$0 \to 2\Delta t$

$$F_0 \Delta t + \underbrace{\int_{\Delta t}^{2\Delta t} F(t) \, dt}_{1/2 \, F_0 \Delta t} = \tfrac{3}{2} F_0 \Delta t$$

integrate friction force $\rightarrow$ $\int_0^{\Delta t} \mu_k mg \, dt = \mu_k mg \Delta t$

$\int_0^{2\Delta t} \mu_k mg \, dt = 2\Delta t \, \mu_k mg$

a) $mv_2 = mv_1 + F_0 \Delta t - \mu_k mg \Delta t$ $\rightarrow$ solve for V_2

b) $mv_2 = mv_1 + \frac{3}{2} F_0 \Delta t - 2\mu_k mg \Delta t$ $\rightarrow$ solve for V_2

Example 4.C.3

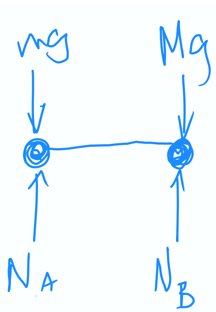
Given: A tug having a mass of m is towing a coal barge (mass M) with a speed of v_{A1}. During a short period of time Δt when the engines of the tug are turned off, the stern winch takes in the towing cable at a rate of $v_{B/A}$. $= v_B - v_A$

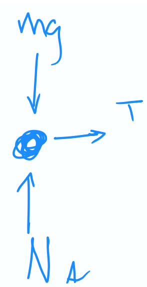
Ignore the water resistance on the tug and barge during this time period. Also, assume that the towing cable remains taut at all times.

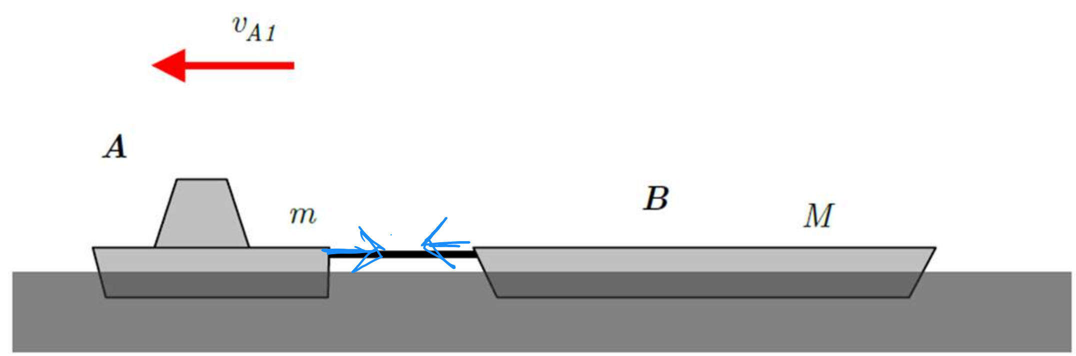
Find: Determine:

(a) The speed of the tug during this period of time; and

(b) The average value of the tension in the towing cable during this time.







a) LIM for combined tug/barge system → internal forces cancel

$$m v_{A1} + M v_{A1} = m v_{A2} + M v_{B2}$$

$$(m+M) v_{A1} = m v_{A2} + M(v_{B/A} + v_{A2}) \longrightarrow v_{A2} = v_{A1} + \frac{M v_{B/A}}{(m+M)}$$

b) tension → just tug boat as system

$$\underline{m v_{A2}} = m v_{A1} + \int_0^{t_1} T\,dt$$

$$m\left(v_{A1} + \frac{M v_{B/A}}{(m+M)}\right) = m v_{A1} + T\Delta t$$

$$T = \frac{m}{\Delta t}\left(\frac{M v_{B/A}}{(m+M)}\right)$$