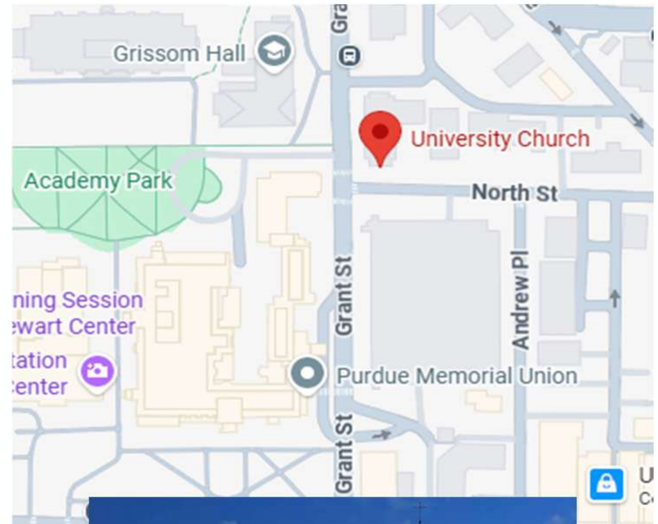
ME 274: Basic Mechanics II

Lecture 26: Particle Kinetics – Review

PURDUE UNIVERSITY | School of Mechanical Engineering

Announcements

- HW 25 (the one released Friday before Spring Break) DUE TONIGHT!
- Exam 2, Thursday, 4/2, 8:00 – 9:30pm
  - Topics covered: Lectures 11-26 (Moving reference frames – Angular impulse momentum)
  - Location: UC 114
  - DRC students – expect an email from Dr. Krousgrill
  - Let me know of any conflicts ASAP
- Review sessions:
  - Pi Tau Sigma: Tuesday, March 31, 6:30-7:30 PM, WTHR 104
  - Wednesday, April 1, 7:00 PM over Zoom
  - Recordings of both sessions will be posted



Kinetics: Four-Step Problem Solving Method

1) FBD
    - Draw appropriate FBD (note- you may have to draw more than 1)
    - Choose coordinate system (cartesian, path, polar)
2) Kinetics
    - Choose appropriate method to solve the problem – practice determining when to use each!
        - Newton/Euler
        - Work/Energy
        - Linear impulse/Momentum
        - Angular impulse/Momentum
3) Kinematics
    - Perform a kinematic analysis of the system using techniques we have developed in the previous chapters.
    - Use your kinetics equations from step 2 to determine what additional information you need to solve the problem
4) Solve
    - Count the number of equations and unknowns. Do they match?
    - If not:
        - Draw more FBDs
        - Do additional kinematic analysis

Newton's 2nd Law

When to use: relating forces to accelerations

Fundamental equations:

$$\sum F = m\vec{a}$$

Remember:

Break into scalar equations by component!

$$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

Work-Energy

When to use: relating change in **speed** to a change in **position**

Fundamental equations:

$$T_1 + V_1 + U_{1\to2}^{NC} = T_2 + V_2$$

Kinetic energy: $T = \frac{1}{2}mv^2$

Potential energy: gravity $\to V = mgh$

spring $\to V = \frac{1}{2}k(\Delta L)^2$

Non- conservative work: $U_{1\to2}^{NC} = \int_1^2 \left(\vec{R} \cdot \hat{e}_t\right) ds$

Remember:

- If $U_{1\to2}^{NC} = 0$, mechanical energy is conserved!
- Only force components tangential to the path contribute to $U_{1\to2}^{NC}$

Linear Impulse Momentum

When to use: relating change in **velocity** to a change in **time**

Fundamental equations:

$$\int_{t_1}^{t_2} \vec{R}\, dt = m\vec{v}_2 - m\vec{v}_1$$

Remember:

- If $\int_{t_1}^{t_2} \vec{R}\, dt = 0$, linear momentum is conserved!
- In multiparticle problems, make the system as large as possible so internal forces cancel.

Angular Impulse Momentum

When to use: relating change in **angular velocity** to a change in **time**

Fundamental equations:

$$\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_{O2} - \vec{H}_{O1}$$

Angular momentum: $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$

Remember:

- If $\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = 0$, linear momentum is conserved! (central force problems)

"Rapid fire" questions from Chapter 4 – Kinetics of Particles
ME 274 – Spring 2025
cmk

Attached is a set of problems from past exams in this course that are related to the material on particle kinetics found in Chapter 4. I would like for us to use these problems to sharpen our skills on two issues: 1) deciding which method(s) to use in solving the problem (Newton's 2^{nd} law, the work/energy equation, the linear impulse momentum equations and/or the angular impulse momentum equations), and 2) drawing the appropriate free body diagram(s) for solving the problems. I am calling these "rapid fire" questions in that these are two issues that you with which you need to be extremely comfortable, and that you can handle in a relatively short time at the beginning of the problem solution. Also attached here is page 352 of the course lecture book which can be helpful in guiding you on your choice of method(s) to use in solving.

With this, I challenge you to do these two steps on the following problems. You may continue on with the solution beyond these two steps of course; however, focus on developing skills in answering these two steps first before solving.

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $where \;\; T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$ $where \;\; T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \bullet \left(\vec{\omega} \times \vec{r}_{G/A}\right)$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\,dt = \vec{H}_{O2} - \vec{H}_{O1}$ $where \;\; \vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\,dt = \vec{H}_{A2} - \vec{H}_{A1}$ $where \;\; \vec{H}_A = I_A\vec{\omega}$

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Examination No. 2
PROBLEM NO. 1

Given: Pellet P having a mass of m is pulled through a barrel (having negligible mass) by means of radial force $F = 60R$, where F is in Newtons and R is in meters. The barrel is constrained to move in a HORIZONTAL plane by rotating about shaft passing through point O. The system is released with $R = R_1, \dot{R} = \dot{R}_1$ and $\dot{\theta} = \dot{\theta}_1$.

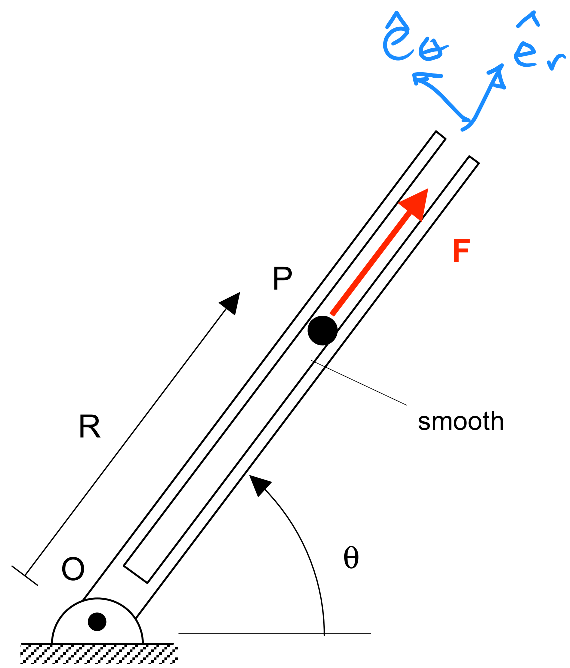
Find: For the instant when $R = R_2$:

a) determine the rotation rate of the barrel, $\dot{\theta}_2$.

b) determine the value of $\dot{R}_2$.

Use the following parameters in your analysis: $m = 20\,kg$, $R_1 = 1.5\ meters$, $\dot{R}_1 = 4\ m/sec$, $\dot{\theta}_1 = 8\ rad/sec(CCW)$ and $R_2 = 3\ meters$.

using our 4 step problem solving method:



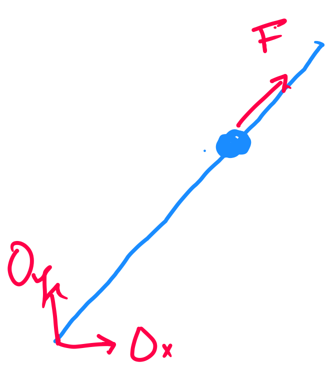
HORIZONTAL PLANE

Step 1: FBD & coord sys

From our kinematics section, we should be able to see that the radial positioning of the particle along the slot & the angle theta indicate POLAR cords

We will make our system the combined slot & particle

FBD:



From this FBD, we can see that this is a Central Force problem with no net moment.

$$\Sigma \vec{M}_O = 0$$

Step 2: Kinetics

To decide what solution method(s) we should use, lets look at the information we are given as well as what we're asked to solve for

in part (a) we are asked to find a change in angular velocity. Also, from our FBD, we know this is a central force problem. This is an indication that we should use AIM.

(Note: AIM generally is used to find a change in angular momentum over a change in time; however, if our net moment = 0, we can use AIM without an explicitly given time frame)

Remember that AIM only gives the velocity in the transverse θ direction, so we know We will have to use Work-Energy to solve for $\dot{R}$ in part (b).

part a

AIM: $\sum \vec{M}_o = 0 \rightarrow \vec{H}_{o2} = \vec{H}_{o1}$

$$m\vec{r}_2 \times \vec{v}_2 = m\vec{r}_1 \times \vec{v}_1$$

recall in polar coords: $\vec{r} = r\hat{e}_r$

$\vec{v} = \dot{R}\hat{e}_r + R\dot{\theta}\hat{e}_\theta$

$$m(R_2\hat{e}_r)\times(\dot{R}_2\hat{e}_r + R_2\dot{\theta}_2\hat{e}_\theta) = m(R_1\hat{e}_r)\times(\dot{R}_1\hat{e}_r + R_1\dot{\theta}_1\hat{e}_\theta)$$

$$mR_2^2\dot{\theta}_2\hat{k} = mR_1^2\dot{\theta}_1\hat{k}$$

$$\Rightarrow \boxed{\dot{\theta}_2 = \left(\frac{R_1}{R_2}\right)^2\dot{\theta}_1}$$

part b

In part b we apply WE

$$T_1 + \cancel{V_1} + U^{NC}_{1\to 2} = T_2 + \cancel{V_2}$$

v^2 in polar

$$T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} m \left(\dot{R}_1^2 + (R_1 \dot{\theta}_1)^2 \right)$$

$$U^{NC}_{1\to 2} = \int_{R_1}^{R_2} F \, dR = \int_{R_1}^{R_2} 60 R \, dR = 30 \left(R_2^2 - R_1^2 \right)$$

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} m \left(\dot{R}_2^2 + (R_2 \dot{\theta}_2)^2 \right)$$

from part A

$$\frac{1}{2} m \left(\dot{R}_1^2 + (R_1 \dot{\theta}_1)^2 \right) + 30 \left(R_2^2 - R_1^2 \right) = \frac{1}{2} m \left(\dot{R}_2^2 + (R_2 \dot{\theta}_2)^2 \right)$$

Solve for $\dot{R}_2$!

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Examination No. 2
PROBLEM NO. 3

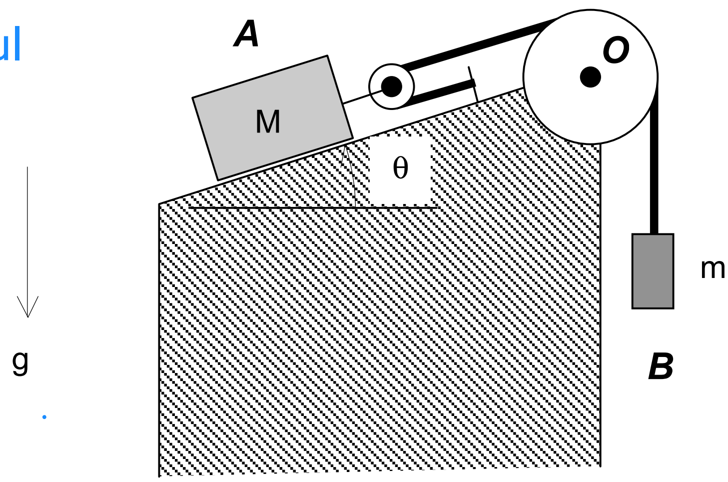
Given: Blocks A and B are connected by the cable-pulley shown. The system is released from rest. Consider all surfaces to be smooth and that the masses of the pulleys are small compared to the masses of A and B.

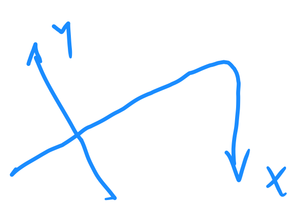
Find: Upon release,

a) determine the acceleration of block B. Write your answer as a vector.
b) determine the tension in the cable.

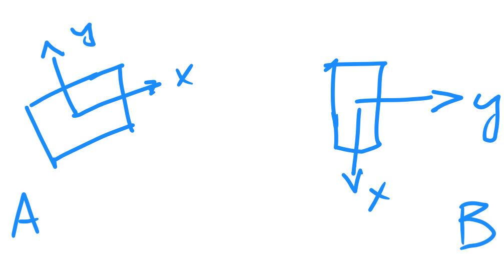
Use the following parameters in your analysis: $m = 5kg$, $M = 20kg$ and $\theta = 36.87°$.

For pully problems like this be careful
In defining your axis. You want
your axis to be continuous along
The length of the cable, so it should
"bend" around the pulley
So if for block A, your X
axis is aligned with the cable, it
should still be aligned with the cable at b.
You can draw that like with one axis like:



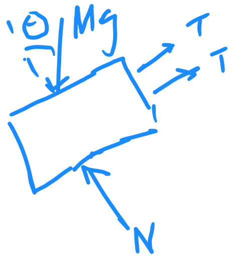


or with separate axes for each block



FBD

Block A



Block B

Kinetics → we are asked for the acceleration → Newton's 2nd

Block A

$\Sigma F_x = 2T - Mg\sin\theta = M a_{Ax}$ (1)

$\Sigma F_y = N - Mg\cos\theta = 0$ ← does not provide new info

Block B

$\Sigma F_x = mg - T = m a_{Bx}$ (2)

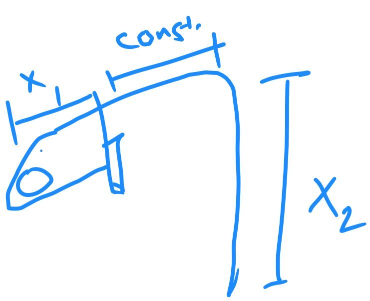
from our kinetics, we have 2 eqns,

unknowns: a_{Ax}, a_{Bx}, T

use cable constraints:

$L = 2x_1 + x_2 + \text{const.}$

$\frac{d^2L}{dt^2} = 0 = 2\ddot{x}_1 + \ddot{x}_2$



$\Rightarrow a_B = 2a_A$

solve!

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Examination No. 2

PROBLEM NO. 4

Given: Block B, having a mass of m, is pressed against a spring (of stiffness k) that is attached to cart A. Cart A (having a mass of M) rests on a horizontal surface. The system is released from rest with the spring compressed by an amount of Δ. After release, block B impacts A, with this impact having a coefficient of restitution of e. Assume all surfaces to be smooth. (Note that since B is simply pressed against the spring, the spring *can push but not pull* on B.)

Find: For this problem,

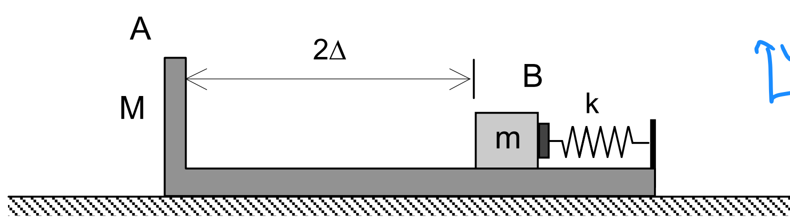
a) determine the velocities of A and B immediately BEFORE impact.

b) determine the velocities of A and B immediately AFTER impact

Write your answers as vectors.

Use the following parameters in your analysis: $m = 20 kg$, $M = 40 kg$, $k = 3000\ N/m$, $\Delta = 0.2\, meters$ and $e = 0.5$.

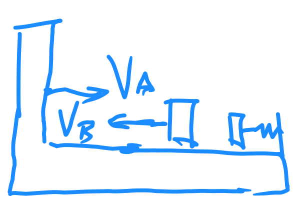
This problem asks for velocity at two points in time, but you really need to evaluate the transition between 3 states



State 0

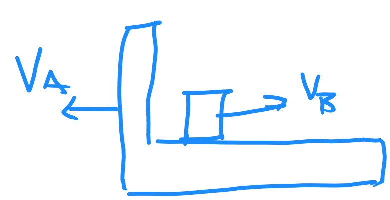
A & B stationary, spring compressed

State 1



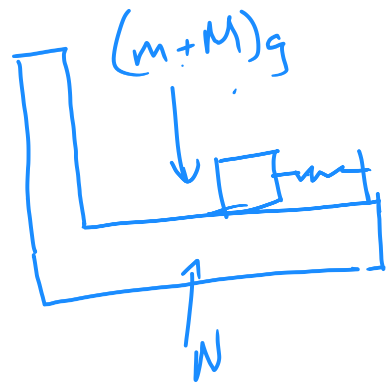
Spring released A + B moving before impact

State 2



post impact

from ① → ① use W-E



$$T_0 + V_0 + U^{NC}_{0\to1} = T_1 + V_1$$

our system is A & B

$T_0 = 0$ $\qquad T_1 = \frac{1}{2} m V_{B1}^2 + \frac{1}{2} M V_{A1}^2$

$V_0 = \frac{1}{2} k \Delta^2$ $\qquad V_1 = 0$

$U^{NC}_{0\to1} = 0$

$$\frac{1}{2} k \Delta^2 = \frac{1}{2} m V_{B1}^2 + \frac{1}{2} M V_{A1}^2$$

← from W-E we have 1 eqn, 2 unknowns

Now use LIM!

no external impulse → momentum conserved!

$$M \cancel{\vec{V}_{A0}}^{0} + m \cancel{\vec{V}_{B,0}}^{0} = M \vec{V}_{A1} + m V_{B1}$$

$$M \vec{V}_{A1} = - m \vec{V}_{B1} \Rightarrow V_{A1} = -\frac{m}{M} V_{B1}$$

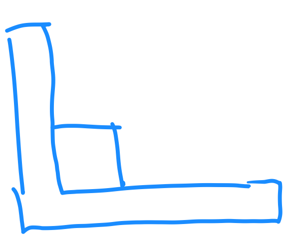
plug into WE

$$\frac{1}{2} k \Delta^2 = \frac{1}{2} m V_{B1}^2 + \frac{1}{2} M \left(-\frac{m}{M} V_{B1}\right)^2$$

Solve for V_{B1}, V_{A1}

from ① → ② use LIM & COR

FBD



→ no external forces on the system → momentum conserved

$$M v_{A1} + m V_{B1} = M v_{A2} + m V_{B2}$$

$$e = - \frac{V_{B2} - V_{A2}}{V_{B1} - V_{A1}}$$

2 eqn, 2 unknowns $\rightarrow$ Solve for V_{A2}, V_{B2}!

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Final Examination

PROBLEM NO. 1

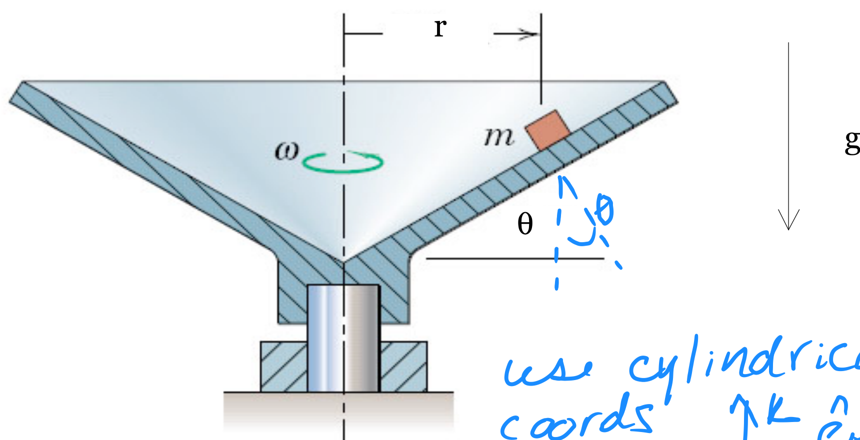
Given: A small object of mass m is placed on the inner surface of a conical dish that is rotating at a constant rate of ω. The coefficients of static and kinetic friction between the object and the dish are known to be μ_s and μ_k, respectively.

Find: Determine the *maximum* rotation rate ω for which the object does not slip on the dish.

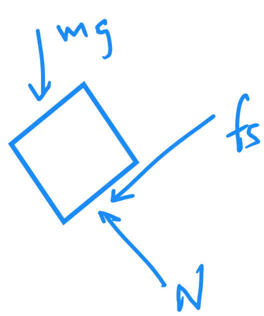
Implies if we increase rotation, the box will fly up out of the dish

Use the following parameters in your analysis: $m = 5kg$, $r = 0.92\ meters$, $\mu_s = 0.4$, $\mu_k = 0.1$ and $\theta = 36.87°$.

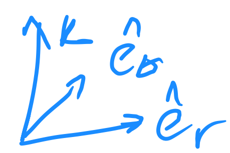
Solution Method:
Newton's 2nd!



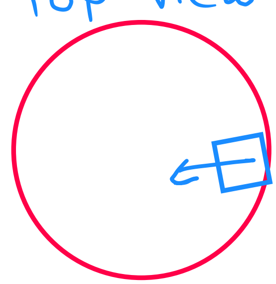
FBD



use cylindrical coords



Top view



$\vec{a} = -\omega^2 r \hat{e}_r$

$$\Sigma F_r = -f_s \cos\theta - N \sin\theta = m a_r$$

$$\Sigma F_k = -mg + N\cos\theta = 0$$

$$f_s = \mu_s N \qquad a_r = \omega^2 r$$

Solve for ω!

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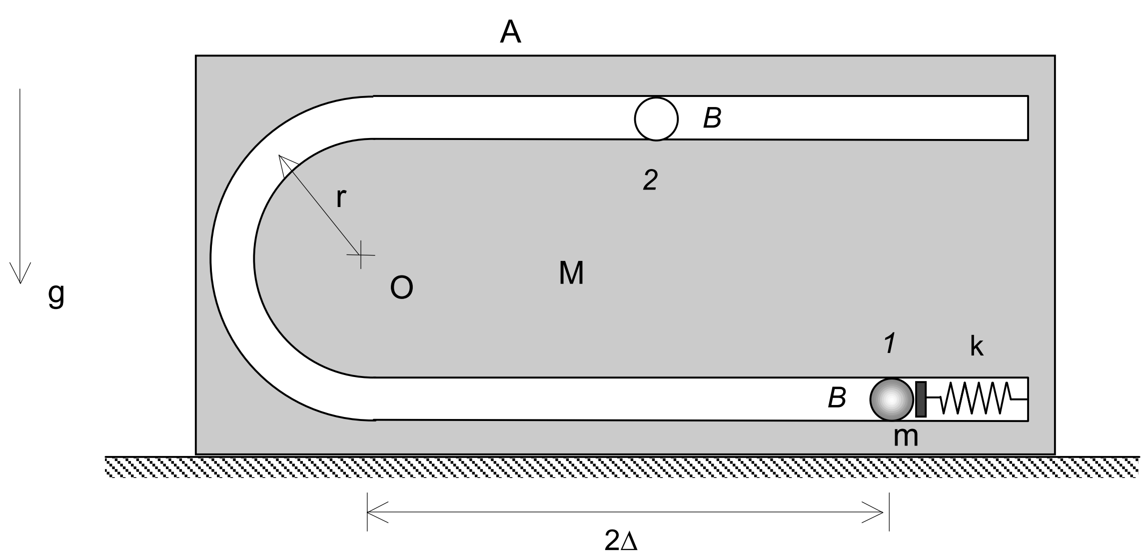
Final Examination
PROBLEM NO. 3

Given: Particle B, having a mass of m, is pressed against a spring (of stiffness k) that is attached to cart A. Cart A (having a mass of M) rests on a horizontal surface. The system is *released from rest* when B is at Position 1 with the spring compressed by an amount of Δ. After release, B travels within a slot cut into cart A, with the slot having straight horizontal and circular sections (the circular section has a radius of r and center at O). A position 2, B has reached the upper horizontal slot but has not yet impacted the cart at the right end of this slot.

Find: Determine the velocities of A and B when B is at Position 2. Write your answers as vectors.

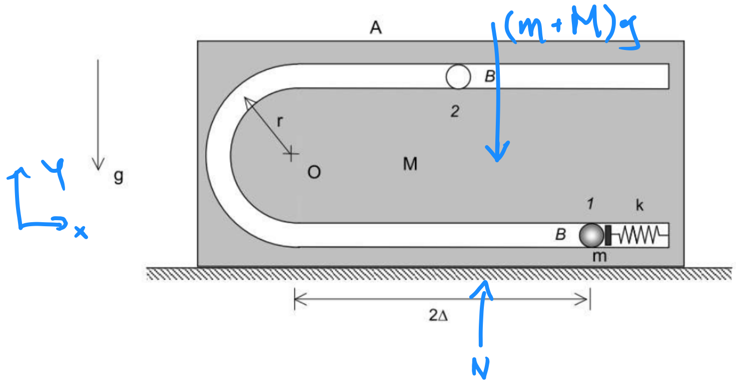
Note that since B is simply pressed against the spring, the spring *can push but not pull* on B. Assume all surfaces to be smooth.

Use the following parameters in your analysis: $m = 30 kg$, $M = 60 kg$, $k = 3000\ N/m$, $\Delta = 0.5\ meters$ and $r = 0.2\ meters$.



PLEASE START YOUR WORK ON THE NEXT PAGE.

Using a system of cart A & particle B, there are no net external forces



(the spring force & any reaction forces between A+B are internal)

→ Linear momentum conserved

<u>Soln Method</u> → WE + LIM

WE: we are told the slot is smooth, so there are no NC forces → mechanical energy conserved.

$$\cancel{T_1} + V_1 + \cancel{U^{NC}_{1\to 2}} = T_2 + V_2$$

$$\frac{1}{2} k \Delta^2 = \frac{1}{2} m v_{B2}^2 + \frac{1}{2} M v_{A2}^2 + mg(2r)$$

LIM: $m\vec{v}_{B1} + M\vec{v}_{A1} = m\vec{v}_{B2} + M\vec{v}_{A2}$

2 eqn, 2 unknowns → solve for v_{B2}, v_{A2}

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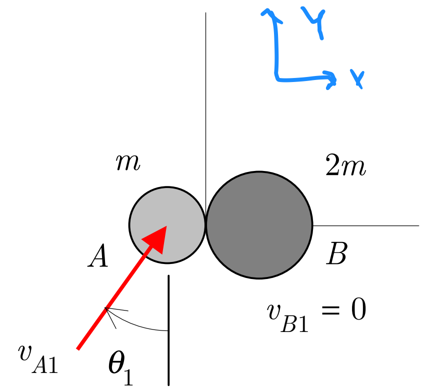
Examination No. 2
PROBLEM NO. 4d *(4 points max)*

Given: Particle A (of mass m) is traveling with a speed of v_{A1} in the direction shown below when it strikes a stationary particle B (of mass 2m). The coefficient of restitution for the impact of A with B is known to be $e = 0.5$.

Find: If $\theta_1 = 25°$, what is the direction of travel of particle A *AFTER* impacting B? Provide a mathematical justification for your answer.



Solution method:

LIM & COR

FBD System A&B

← in an impact, all reaction forces are internal & cancel

No net external forces → momentum is conserved

Kinetics → central impact

in this problem x is the normal direction & y is the tangent direction

X: $m V_{A1x} + 2m \cancel{V_{B1x}}^{\,0} = m V_{A2x} + 2m V_{B2x}$

$$V_{A1} \sin\theta = V_{A2x} + 2V_{B2x} \quad (1)$$

in central impact problems, the velocity in the tangent direction is constant

$\Rightarrow$ $V_{A1y} = V_{A1}\cos\theta = V_{A2y}$, $V_{By1} = 0 = V_{By2}$

COR

$$e = -\frac{V_{B2N} - V_{A2N}}{V_{B1N} - V_{A1N}}$$

remember this can only be used in the Normal direction → (x)

We now have 2 eqn, 2 unknowns, solve for V_{B2x} & V_{A2x}

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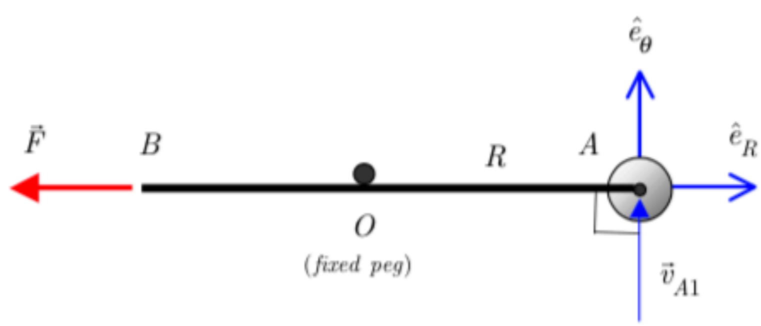
Examination No. 2
PROBLEM NO. 2

Given: Particle A (of mass m) slides upon a *smooth* HORIZONTAL surface. A flexible, inextensible cord is connected to A at one end and has a constant force $\vec{F}$ acting to the left on the other end. Initially, when A is at a radial distance of $R = R_1$ from O, the cord is in contact with a small, smooth peg at O. At this instant, A is moving perpendicular of line OA with a speed of v_{A1}, as shown in the figure.

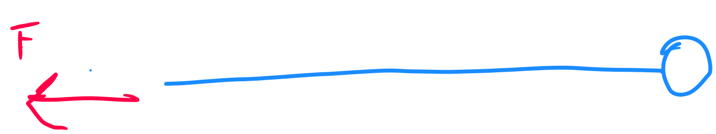
Find: When A is at a radial distance of $R = R_2$ from O, determine the speeds of ends A and B of the cord. Use the following parameter values in your work: $m = 10kg$, $R_1 = 2\ meters$, $R_2 = 3\ meters$, $|\vec{F}| = 280\ N$ and $v_{A1} = 15\,m/sec$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.



HORIZONTAL SURFACE

the system is the cord, particle, & peg



F is the only external force → central force problem $\vec{H}_{O1} = \vec{H}_{O2}$

we will use AIM to find the speed in the $\hat{e}_\theta$ direction & WE to find speed in $\hat{e}_R$

AIM

$$r_1 \times m v_{A1} = r_2 \times m v_{A2\theta}$$

$$V_{A2\theta} = \frac{r_1}{r_2} V_{A1}$$

WE

$$T_1 + \cancel{V_1} + U^{NC}_{1\to2} = T_2 + \cancel{V_2}$$

remember in polar coords $\vec{v} = \dot{r}\hat{e}_r + \underline{r\dot{\theta}}\hat{e}_\theta$

we found this with AIM

$= V_{A2\theta}$

$$T_1 = \tfrac{1}{2} m V_{A1}^2$$

$$U^{NC}_{1\to2} = \int_{R1}^{R2} F\,dr = F(R2 - R1)$$

$$T_2 = \tfrac{1}{2} m V_2^2 = \tfrac{1}{2} m\left(\dot{R}^2 + V_{A2R}^2\right)$$

$$\tfrac{1}{2} m V_{A1}^2 + F(R2 - R1) = \tfrac{1}{2} m\left(\dot{R}^2 + V_{A2R}^2\right)$$

solve for $\dot{R}$

$$V_A = \sqrt{\dot{R}^2 + V_{A2\theta}^2}$$

$$V_B = \dot{R}$$

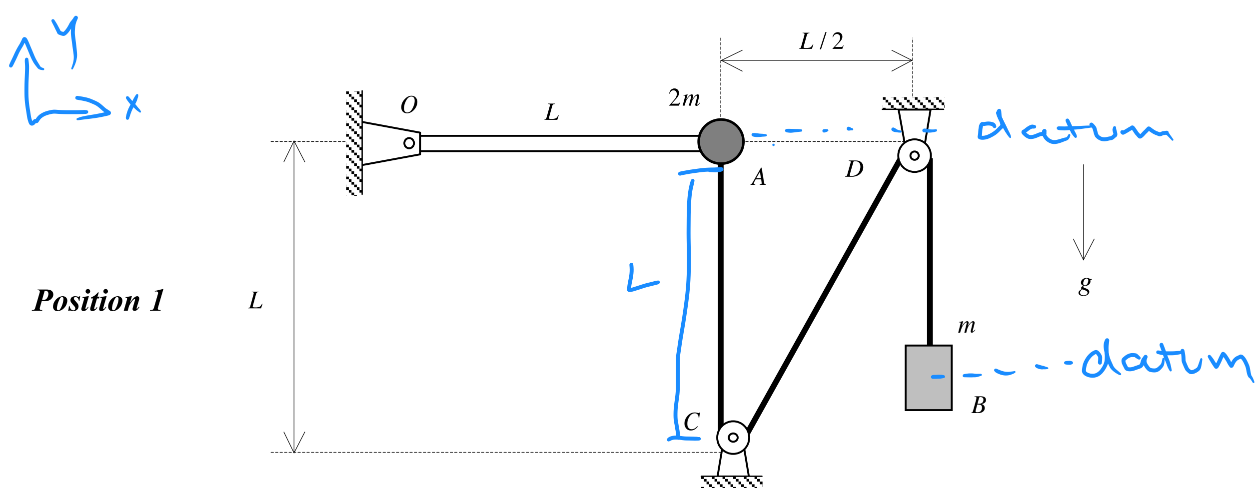
$\Big\}$ remember we are asked for speed

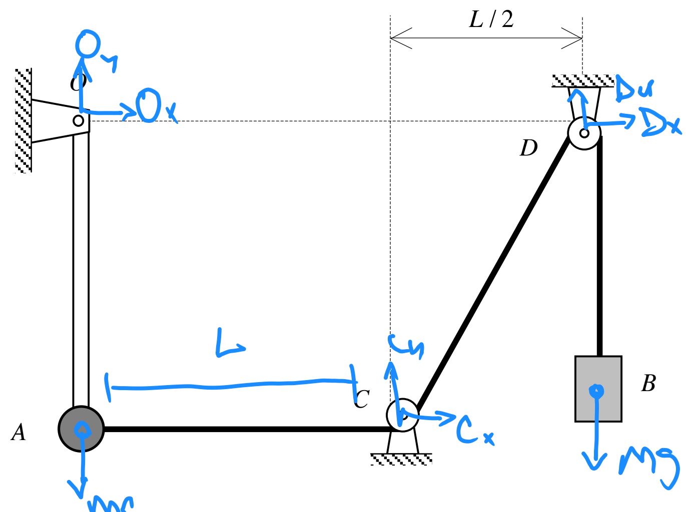
ME 274 – Summer 2011 **Name** ______________________________
Examination No. 2
PROBLEM NO. 3

Given: Particle A (of mass 2m) is attached to a rigid bar of negligible mass. Particle A is also connected to a cable that is wrapped around two pulleys and connected to particle B on its other end. The system is released from rest with OA being horizontal and with section AC of the cable being vertical. Assume that the radii of the pulleys to negligible.

Find: Determine the *angular velocity* of the bar at Position 2 where it has rotated 90° CW to a vertical orientation. (At Position 2, section AC of the cable is horizontal.) Use the following parameter values in your work: $m = 10kg$ and $L = 4\,meters$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.





PLEASE START YOUR ANALYSIS ON THE NEXT PAGE.

System : bar + cable + mass

$$\cancel{T_1} + \cancel{V_1} + \cancel{U^{NC}_{1\to 2}}^{\,0} = T_2 + V_2$$

$T_1 = 0$, RFR

$V_1 = 0$

$$T_2 = \tfrac{1}{2}(2m)v_A^2 + \tfrac{1}{2} m v_B^2$$

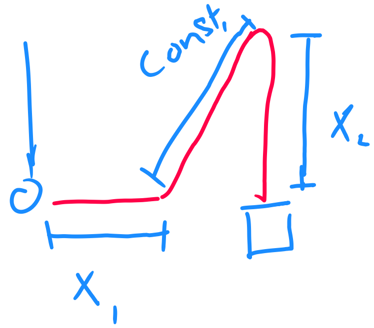
$V_2 = -2mgL$ ← note the block is at the same position at position 1 & 2

$$0 = \cancel{m} v_A^2 + \tfrac{1}{2}\cancel{m} v_B^2 - 2\cancel{m}gL$$

$$v_B = \sqrt{2v_A^2 - 4gL}$$

2 unknowns, 1 eqn → use Kinematics

→ inextensible cable



$L_{tot} = x_1 + x_2 + \text{const.}$

$$\frac{dL_{tot}}{dt} = \dot{x}_1 + \dot{x}_2 = 0$$

$v_A = -v_B$

↳ sub in & solve!

ME 274 – Summer 2012 **Name** ______________________________
Examination No. 2
PROBLEM NO. 3

Given: A box having a mass of M is constrained to move along a smooth, horizontal surface. Block A (having a mass of m) is able to slide along smooth, horizontal surface inside the box, as shown in the figure below. Block A is pressed against a spring having a stiffness of k. Initially, the system is at rest and the spring is compressed by an amount Δ_1. The coefficient of restitution between A and the box at end B is e.

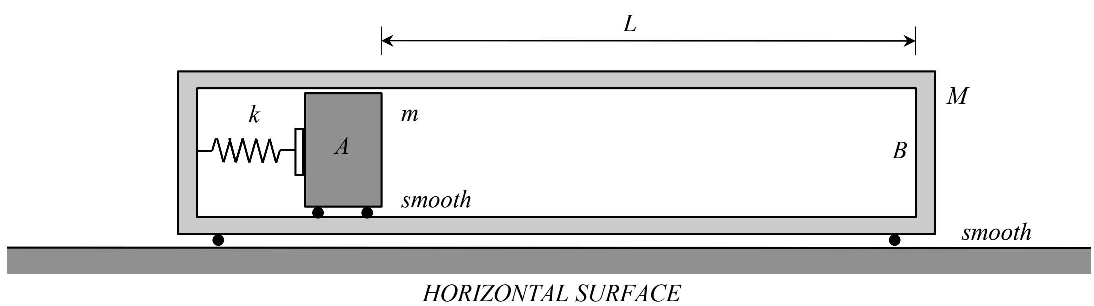
Find: After the spring is released:

a) Determine the speed of the box and the speed of block A immediately BEFORE A impacts the box at B. Write your answers as vectors.

b) Determine the speed of the box and the speed of block A immediately AFTER A impacts the box at B. Write your answers as vectors.

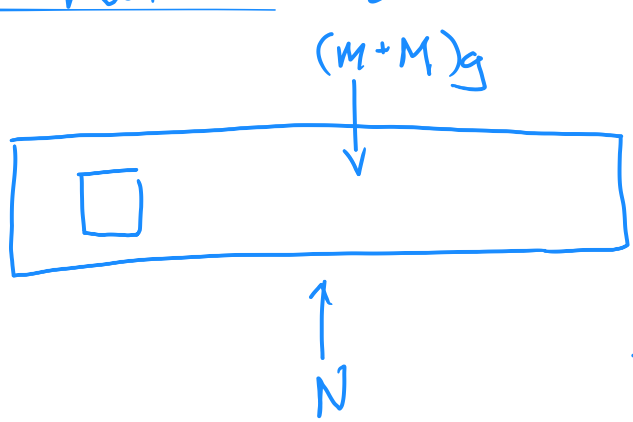
Use the following: $m = 3kg$, $M = 5kg$, $k = 12{,}000N/m$, $e = 0.5$, $\Delta_1 = 0.3m$ and $L = 2m$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.



Work appearing above this line will NOT be graded.

Solution method LIM & WE



System
Block A & cart B

No net external forces
→ lin. mo. conserved

We are looking at 3 configurations of our system

State 1 → system at rest, spring compressed

State 2 → carts A + B both moving after spring released, before impact

State 3 → carts A + B both moving after impact

① → ②

WE: $\frac{1}{2} K \Delta_1^2 = \frac{1}{2} m V_{A2}^2 + \frac{1}{2} M V_{B2}^2$

LIM: $m \cancel{V_{A1}}^{0} + M \cancel{V_{B1}}^{0} = m V_{A2} + M V_{B1} \Rightarrow V_{B1} = -\frac{m V_{A2}}{M}$

Solve for V_{B2} + V_{A2}

② → ③ no external forces during impact → Lin. mo. conserved

LIM: $m V_{A2} + M V_{B2} = m V_{A3} + M V_{B3}$

COR: $e = -\frac{V_{B3} - V_{A3}}{V_{B2} - V_{A2}}$

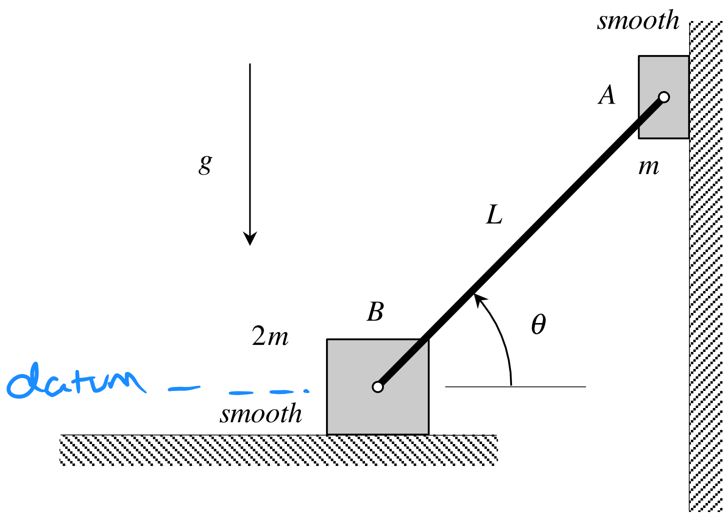
solve for V_{B3} + V_{A3}

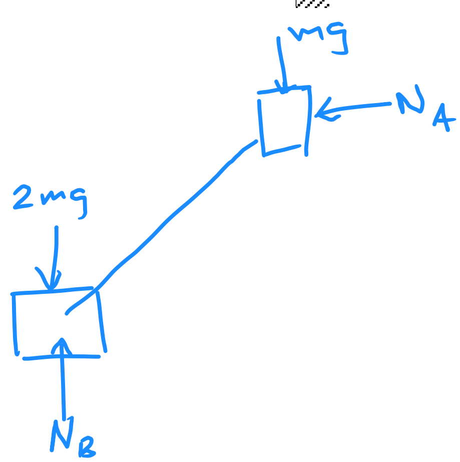
ME 274 – Summer 2015 **Name**________________________________
Examination No. 2
PROBLEM NO. 2 – 20 points

Given: Blocks A and B (having masses of m and 2m, respectively) are connected by rigid bar AB, with bar AB having negligible mass. Block A is constrained to move along a smooth, vertical wall, whereas block B moves along a smooth horizontal surface. The system is released from rest with $\theta = 53.13°$.

Find: Determine the speeds of blocks A and B at the position where $\theta = 0$.

Use the following parameter values: $L = 2\ m$ and $m = 10\ kg$.





Solution method: WE

System: Blocks A + B

Note reaction forces are internal + cancel

kinetics

WE: $T_1 + V_1 + U^{NC}_{1\to 2} = T_2 + V_2$

$T_1 = 0$ (RFR)

$T_2 = \frac{1}{2}(2m)v_B^2 + \frac{1}{2} m v_A^2$

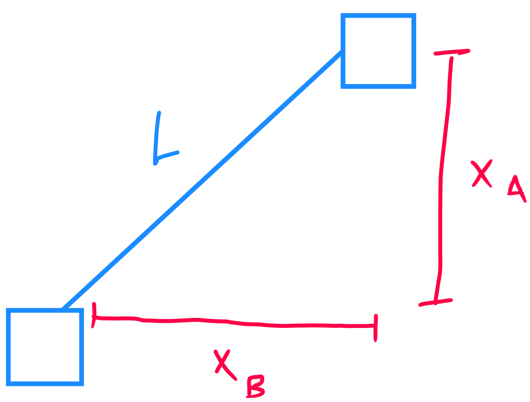
$V_1 = mgL\sin\theta$

$V_2 = 0$

$U^{NC}_{1\to 2} = 0$

$$\cancel{m}gL\sin\theta = \cancel{m}v_B^2 + \frac{1}{2}\cancel{m}v_A^2 \qquad (1)$$

kinetics:



$$L^2 = x_A^2 + x_B^2$$

$$\frac{d(L^2)}{dt} = 0 = 2x_A\dot{x}_A + 2x_B\dot{x}_B$$

$$x_A = L\cos\theta$$

$$x_B = L\sin\theta$$

$$0 = 2L\cos\theta\, v_A + 2L\sin\theta\, v_B$$

$$V_A = -\tan\theta\, V_B \qquad (2)$$

2 eqn, 2 unknown, solve