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ME 274 Lecture 28

Rigid body kinetics: Newton/Euler – Part 2

Eugenio “Henny” Frias-Miranda

03/27/26

t. do in lectures

SA13

SA12

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. HW 27 (5.A and 5.B) due today!!

2. Office hours are changing to ME2008B...

- Second floor of renovated side of ME.

3. Exam 2 Information:

- Thursday, April 2, 8:00-9:30 PM
- BHEE129
- Coverage: Lectures 11-26 (up through angular impulse/momentum for particles)

4. Exam 2 Review sessions both videos to be posted on website

- Pi Tau Sigma: Tuesday, March 31, 6:30-7:30 PM, WTHR 104 (WL in-person and Indy online)
- ME 274 Instructor, CK: Wednesday, April 1, 7:00 PM, live on Zoom for both WL and Indy:
 - <https://purdue.edu.zoom.us/j/94496659802?pwd=VMHo3NfyaHO3HbbmmLForgikls3PL7.1>

Chapter 5: Planar Rigid Body Kinetics

- Newton/Euler (N/E) Equations for Planar Rigid Bodies (lectures 27-29)

$$\sum \vec{F} = m\vec{a}_G$$

$$\sum \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$$

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler (relating forces to accelerations)	particle	$\sum \vec{F} = m\vec{a}$
	rigid body (G = c.m. and A = any point on body)	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A \vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy (relating change in speed to change in position)	particle	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body (G = c.m. and A = any point on body)	$T_1 + V_1 + U_{1 \rightarrow 2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$
Linear impulse-momentum (relating change in velocity to change in time)	particle	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body (G = c.m.)	$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum (relating change in angular velocity to change in time)	particle (O = fixed point)	$\int_{t_1}^{t_2} \sum \vec{M}_O dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body (A = fixed point or c.m.)	$\int_{t_1}^{t_2} \sum \vec{M}_A dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A\vec{\omega}$

3 Special Forms of the Euler's Equation where

$$\vec{r}_{G/A} \times (m\vec{a}_A) = \vec{0}$$

$$\left(\sum \vec{M}_A\right)_{ext} = I_A \vec{\alpha} + \vec{r}_{G/A} \times (m\vec{a}_A)$$

1. If you choose A to be the **center of mass, G**. $\vec{r}_{G/A} = \vec{0}$.

$$\sum \vec{M}_G = I_G \vec{\alpha}$$

2. If you choose A to be a **fixed point on the body, O**. $\vec{a}_O = \vec{0}$.

$$\sum \vec{M}_O = I_O \vec{\alpha}$$

3. If you choose a point A which has an acceleration vector that is **parallel to** $\vec{r}_{G/A}$. $\vec{r}_{G/A} \times \vec{a}_A = \vec{0}$.

$$\sum \vec{M}_A = I_A \vec{\alpha}$$

Picking one of these 3 locations will simplify our problem, **it is recommended to** *do this as often as possible*

Mass Moment of Inertia For a Rigid Body

1. **Mass Moment of Inertia is a term that comes up in Euler's Equation that** measures an object's resistance to changes in its rotational speed about a specific axis.

- Which we often use "I" to refer to

$$I_A = \int r^2 dm = \int \rho r^2 dV$$

2. We can represent the Mass Moment of Inertia in different descriptions, since dV depends on the coordinate system used.

a. polar (having thickness h)

$$I_A = \int \int \rho h r^3 dr d\theta$$

b. cartesian (having thickness h)

$$I_A = \int \int \rho r^2 h dx dy = \int \int \rho h (x^2 + y^2) dx dy$$

c. for a thin body (x-sectional area of A)

$$I_A = \int \rho A x^2 dx$$

3. **Above expressions are used to finding Mass Moment of Inertia.**
Some examples are on the next slide.

We often will reference a table

Mass Moment of Inertia of Commonly Used Objects and when to use Parallel Axis Theorem (P.A.T.)

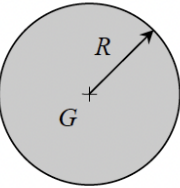
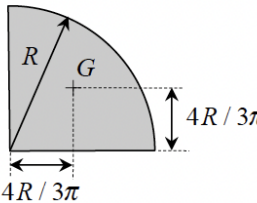
1. Derivation of some of these equations (x2) are found on page 302

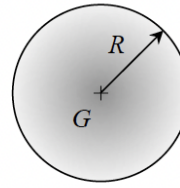
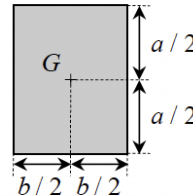
2. Often times you will reference a table such as below and get your I_G

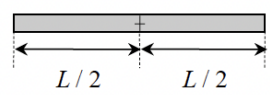
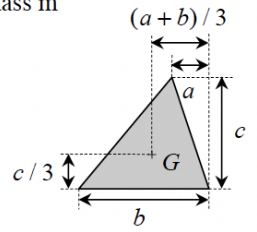
3. **Note:** These values below are presented in reference to G.

- If you are not using your Euler's Equation in terms of G you will have to use *Parallel Axis Thm (P.A.T.)*

$$I_A = I_G + md_{AG}^2$$

<p>Uniform disk of mass m</p>  $I_G = \frac{1}{2}mR^2$	<p>Quarter circular plate of mass m</p>  $I_G = \left(\frac{9\pi^2 - 64}{18\pi^2} \right) mR^2$
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<p>Uniform sphere of mass m</p>  $I_G = \frac{2}{5}mR^2$	<p>Uniform rectangular plate of mass m</p>  $I_G = \frac{1}{12}m(a^2 + b^2)$
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<p>Uniform thin bar of mass m</p>  $I_G = \frac{1}{12}mL^2$	<p>Uniform triangular plate of mass m</p>  $I_G = \frac{1}{18}m(a^2 + b^2 + c^2 - ab)$
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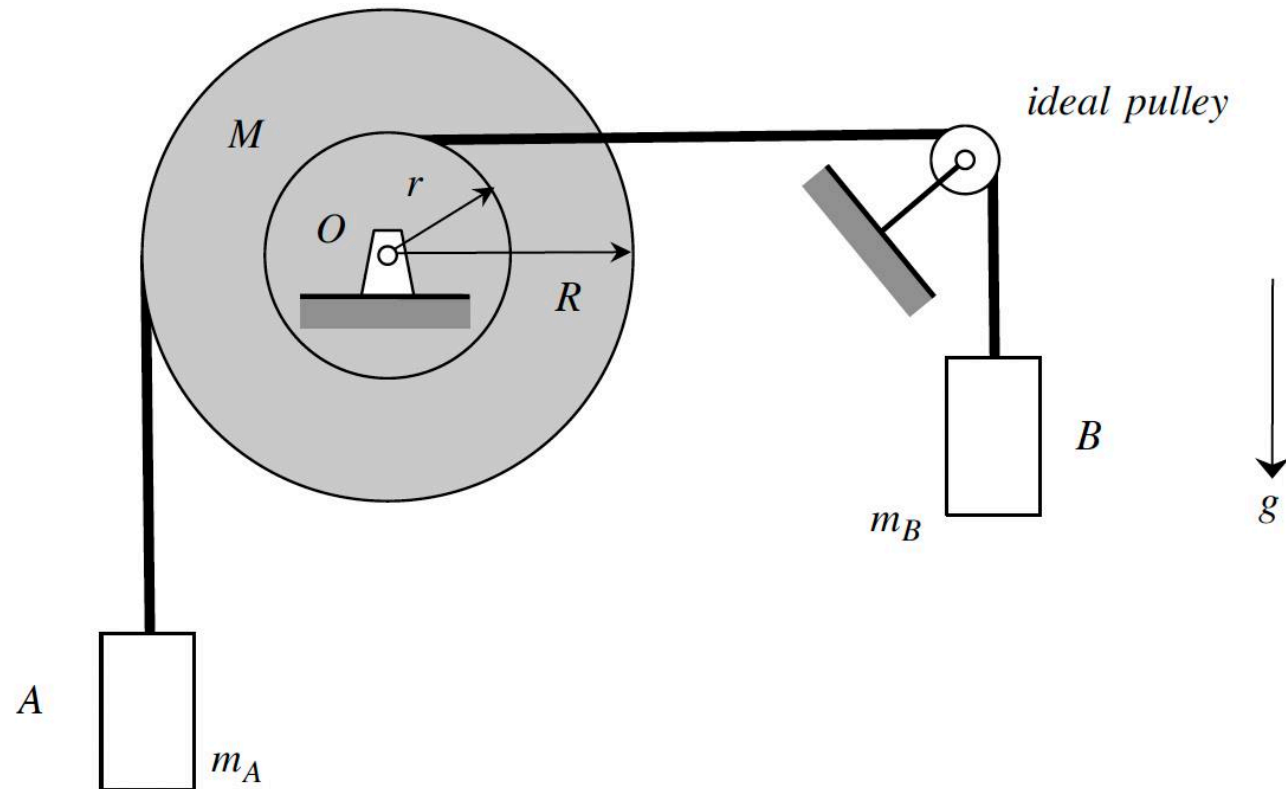
[pg. 303-304 content]

Example 5.A.11

Given: A stepped drum (having a mass of M and radius of gyration about its center O of k_O) is attached to a smooth shaft passing through its center O . A cable wrapped around the outer radius of the drum is attached to block A. A second cable is wrapped around the inner radius of the drum with this cable pulled over an ideal pulley and is attached to block B. Assume that the cables do not slip on the drum. The system is released from rest.

Find: Determine the angular acceleration of the drum on release. Write your answer as a vector.

Use the following parameters in your analysis: $m_A = 10$ kg, $m_B = 30$ kg, $M = 20$ kg, $r = 0.2$ m, $R = 0.4$ m and $k_O = 0.25$ m.



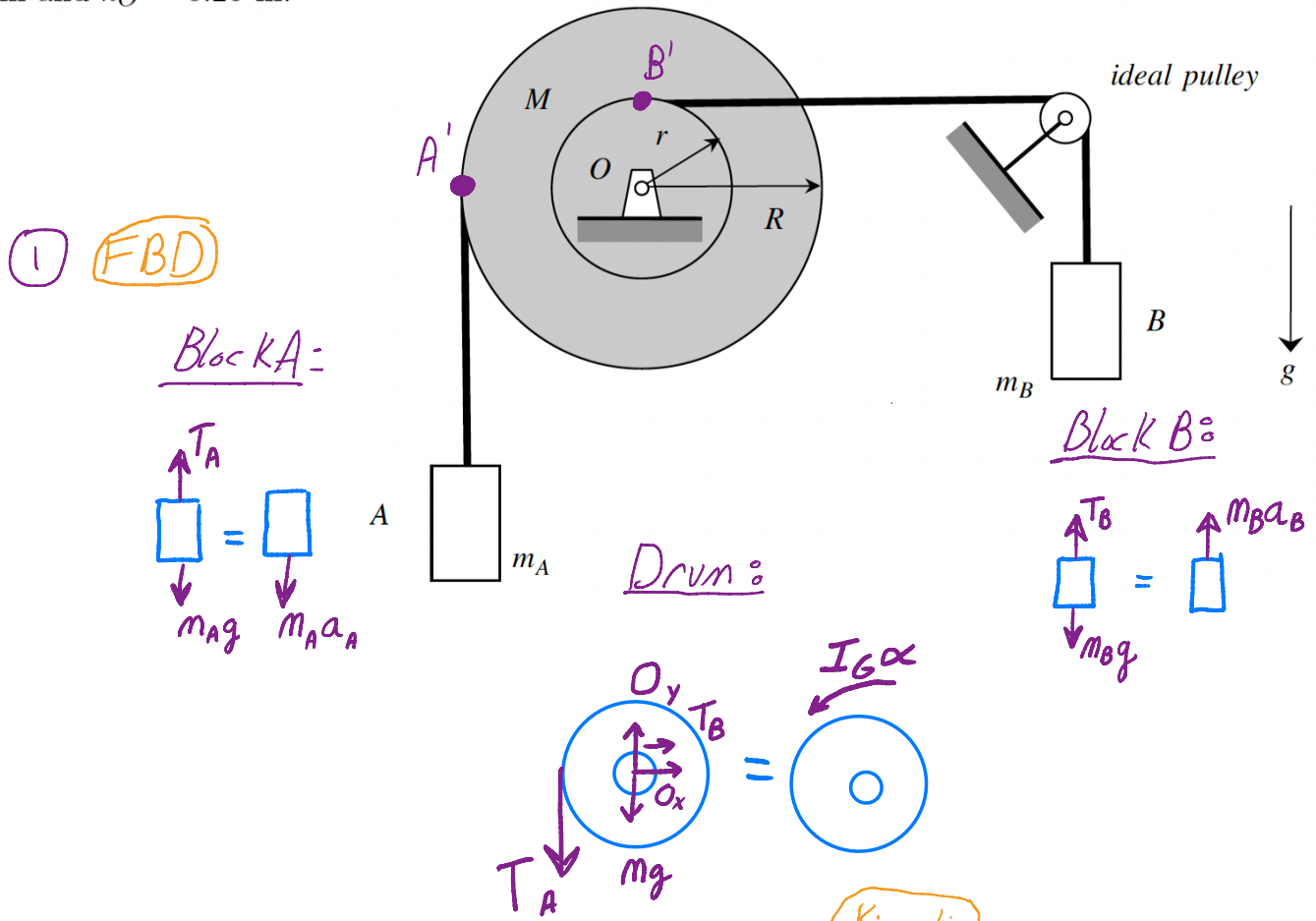
Example 5.A.11

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Given: A stepped drum (having a mass of M and radius of gyration about its center O of k_O) is attached to a smooth shaft passing through its center O . A cable wrapped around the outer radius of the drum is attached to block A. A second cable is wrapped around the inner radius of the drum with this cable pulled over an ideal pulley and is attached to block B. Assume that the cables do not slip on the drum. The system is released from rest.

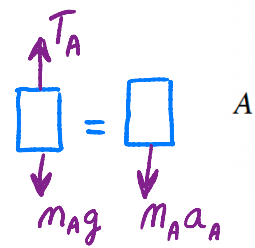
Find: Determine the angular acceleration of the drum on release. Write your answer as a vector.

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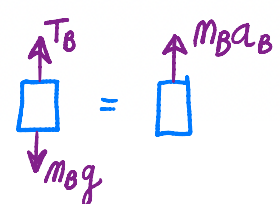


① FBD

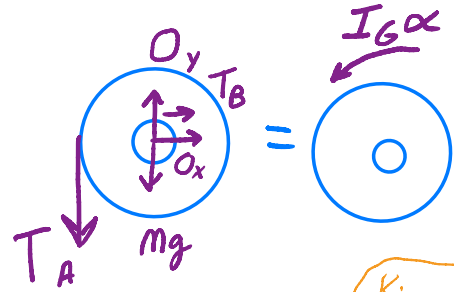
Block A:



Block B:



Drum:



Kinetics

② Newton on x dir

$$\sum M_O = I_G \alpha$$

$$\Rightarrow T_A R - T_B r = M k_O^2 \alpha \quad (1)$$

③ Newton on y dir

$$\sum F = m_A a_A$$

$$\Rightarrow m_A g - T_A = m_A a_A \quad (2)$$

④ Euler on G

$$\sum F = m_B a_B$$

$$\Rightarrow T_B - m_B g = m_B a_B \quad (3)$$

⑤ 5 unkns 3 eqns

Kinematics

⑥ OA'

$$\vec{a}_A = \vec{a}_O + \vec{\alpha} \times \vec{r}_{A/O} - \omega^2 \vec{r}_{A/O}$$

$$\Rightarrow a_A \hat{i} + a_A' \hat{j} = \vec{0} + \alpha \hat{k} \times R \hat{j} = -\alpha R \hat{j}$$

⑦ sep into \hat{j}

$$\hat{j} : a_A' = -\alpha R$$

$$\Rightarrow a_A = \alpha R \quad (4)$$

⑧ OB'

$$\vec{a}_B = \vec{a}_O + \vec{\alpha} \times \vec{r}_{B/O} - \omega^2 \vec{r}_{B/O}$$

$$\Rightarrow a_B \hat{i} + a_B' \hat{j} = \vec{0} + \alpha \hat{k} \times r \hat{j} = -\alpha r \hat{i}$$

⑨ sep into \hat{i}

$$\hat{i} : a_B' = -\alpha r$$

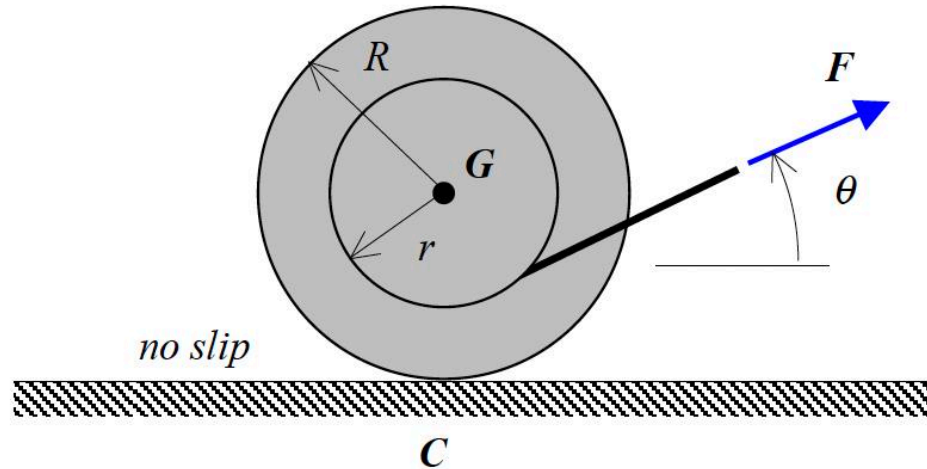
$$\Rightarrow a_B = \alpha r \quad (5)$$

⑩ Solve $\Rightarrow \vec{\alpha}$

Example 5.A.12

Given: A spool has a mass of $m = 30$ kg, a centroidal radius of gyration $k_G = 0.25$ m, an outer radius of $R = 0.3$ m and an inner radius of $r = 0.1$ m. A constant force $F = 60$ N is applied at an angle of θ by a cord that is wrapped around the inner radius of the spool. The spool rolls without slipping on the rough horizontal surface.

Find: Determine the angular acceleration of the spool as a function of the angle θ .



Example 5.A.12

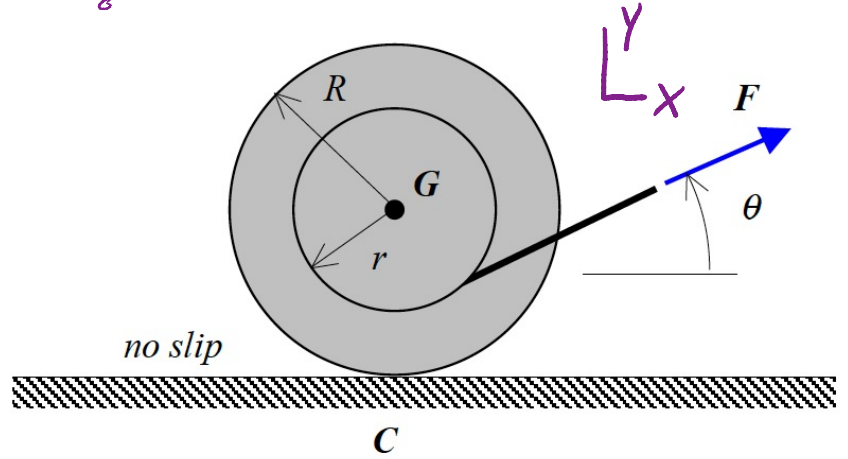
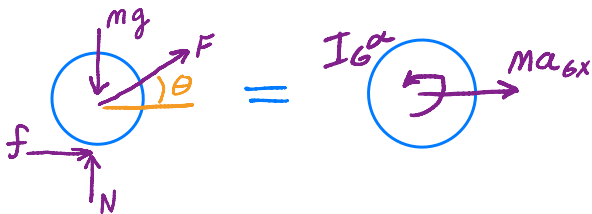
p. 319

Given: A spool has a mass of $m = 30$ kg, a centroidal radius of gyration $k_G = 0.25$ m, an outer radius of $R = 0.3$ m and an inner radius of $r = 0.1$ m. A constant force $F = 60$ N is applied at an angle of θ by a cord that is wrapped around the inner radius of the spool. The spool rolls without slipping on the rough horizontal surface.

Find: Determine the angular acceleration of the spool as a function of the angle θ .

$\alpha?$

① FBD



Kinetics

② Newton on x dir

$$\sum F_x = ma_{Gx}$$

$$\Rightarrow f + F \cos \theta = ma_{Gx} \quad (1)$$

③ Newton on y dir

$$\sum F_y = 0$$

$$\Rightarrow N + F \sin \theta - mg = 0 \quad (2)$$

④ Euler on G

$$\sum M_G = I_G \alpha$$

$$\Rightarrow fR + Fr = mk_G^2 \alpha \quad (3)$$

⑤ 4 unknowns 3 eqns

Kinematics

⑥ GC

$$\vec{a}_G = \vec{a}_C + \vec{\alpha} \times \vec{r}_{G/C} - \omega^2 \vec{r}_{G/C}$$

$$\Rightarrow a_{Gx} \hat{i} = a_C \hat{j} + \alpha \hat{k} \times R \hat{j} - \omega^2 R \hat{j}$$

$$= a_C \hat{j} - \alpha R \hat{i} - \omega^2 R \hat{j}$$

$$\hat{i}: a_{Gx} = -\alpha R \quad (4)$$

$$\hat{j}: 0 = a_C - \omega^2 R$$

$$\Rightarrow a_C = \omega^2 R$$

Solve

⑦ Use (1) & (4).

$$(1) \Rightarrow f + F \cos \theta = -m \alpha R$$

$$\Rightarrow f = -F \cos \theta - m \alpha R$$

⑧ Use (3) & f.

$$(3) \Rightarrow -F \cos \theta R - m \alpha R^2 + Fr = mk_G^2 \alpha$$

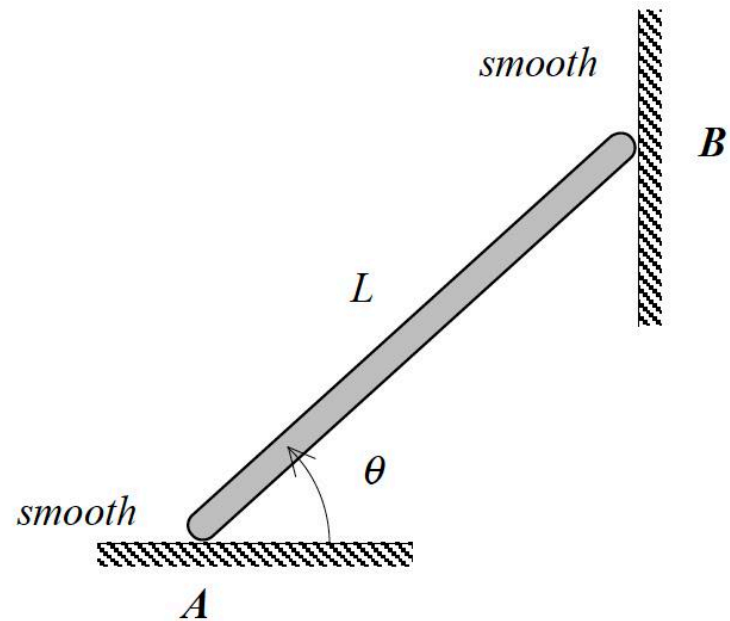
$$\Rightarrow Fr - F \cos \theta R = m(R^2 + k_G^2) \alpha$$

$$\Rightarrow \vec{\alpha} = \frac{F(r - R \cos \theta)}{m(R^2 + k_G^2)} \hat{k}$$

Example 5.A.13

Given: The thin, homogeneous bar AB (having a mass of $m = 30$ kg and length $L = 4$ m) is released from rest at the position shown where $\theta = 36.87^\circ$. Assume all surfaces to be smooth.

Find: Determine the angular acceleration of the bar on release.



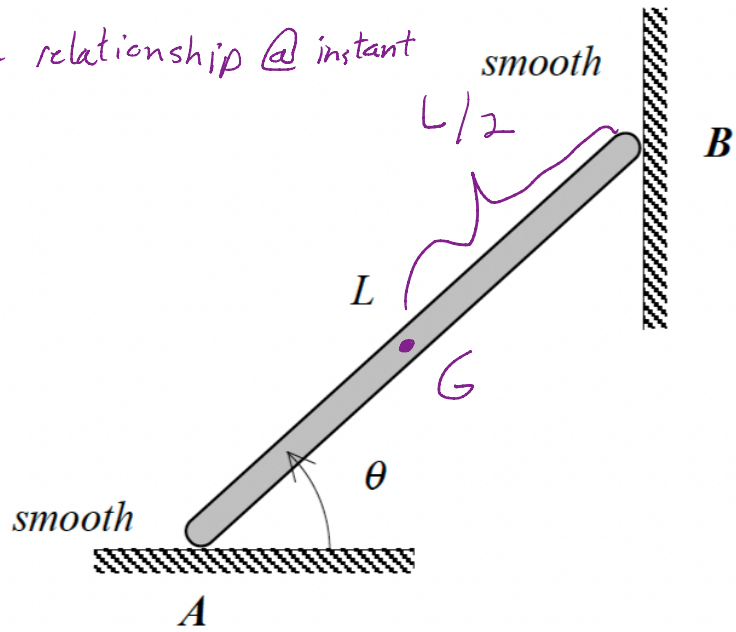
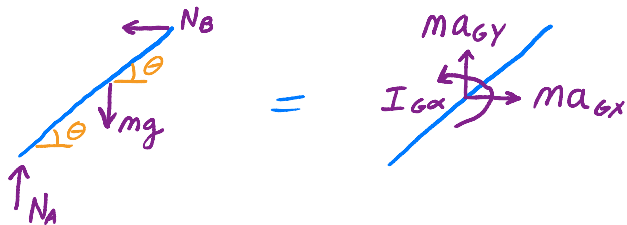
Given: The thin, homogeneous bar AB (having a mass of $m = 30 \text{ kg}$ and length $L = 4 \text{ m}$) is released from rest at the position shown where $\theta = 36.87^\circ$. Assume all surfaces to be smooth.

RFR $\rightarrow v_A, v_B, \omega$ are all 0

Find: Determine the angular acceleration of the bar on release. $\alpha?$

⑥ looking for Force - acceleration relationship @ instant
 \rightarrow N/E Eqn

① FBD



Kinetics

② Newton on x dir

$$\sum F_x = m a_{Gx}$$

$$\Rightarrow -N_B = m a_{Gx} \quad (1)$$

③ Newton on y dir

$$\sum F_y = m a_{Gy}$$

$$\Rightarrow N_A - mg = m a_{Gy} \quad (2)$$

④ Euler on G

$$\sum \vec{M}_G = I_G \vec{\alpha}$$

$$I_G = \frac{1}{12} mL^2$$

from table

$$\Rightarrow N_B \frac{L}{2} \sin \theta - N_A \frac{L}{2} \cos \theta = \frac{1}{12} mL^2 \alpha \quad (3)$$

⑤ 5 unkns 3 eqns

Kinematics

⑥ bar AB. Use to get a_B

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A} \quad \text{R.F.R.}$$

$$\Rightarrow a_B \hat{j} = a_A \hat{i} + \alpha \hat{k} \times (L \cos \theta \hat{i} + L \sin \theta \hat{j})$$

$$= a_A \hat{i} + \alpha L \cos \theta \hat{j} - \alpha L \sin \theta \hat{i}$$

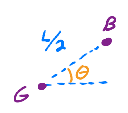
$$\hat{i}: 0 = a_A - \alpha L \sin \theta$$

$$\Rightarrow a_A = \alpha L \sin \theta \quad (4)$$

$$\hat{j}: a_B = \alpha L \cos \theta \quad (5)$$

⑦ GB

$$\vec{a}_G = \vec{a}_B + \vec{\alpha} \times \vec{r}_{G/B} - \omega^2 \vec{r}_{G/B} \quad \text{R.F.R.}$$



$$\Rightarrow a_{Gx} \hat{i} + a_{Gy} \hat{j} = a_B \hat{j} + \alpha \hat{k} \times \left(-\frac{L}{2} \cos \theta \hat{i} - \frac{L}{2} \sin \theta \hat{j} \right)$$

$$= a_B \hat{j} - \frac{\alpha L}{2} \cos \theta \hat{j} + \frac{\alpha L}{2} \sin \theta \hat{i}$$

$$\hat{i}: a_{Gx} = \frac{\alpha L}{2} \sin \theta \quad (6)$$

$$\hat{j}: a_{Gy} = a_B - \frac{\alpha L}{2} \cos \theta \quad (7)$$

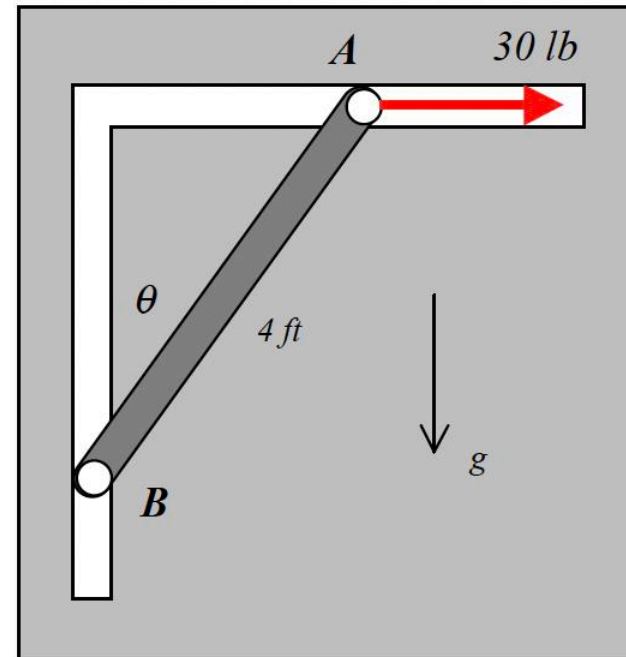
⑧ Solve 7 eqns 7 unkns

Example 5.A.14

Given: The thin, homogeneous bar AB (having a weight of $W = 60$ lb) is released from rest at the position shown where $\theta = 30^\circ$. Assume all surfaces to be smooth.

Find: Determine:

- The angular acceleration of the bar on release; and
- The reaction forces on the bar at ends A and B on release.



Example 5.A.14

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Given: The thin, homogeneous bar AB (having a weight of $W = 60 \text{ lb}$) is released from rest at the position shown where $\theta = 30^\circ$. Assume all surfaces to be smooth.

RFR

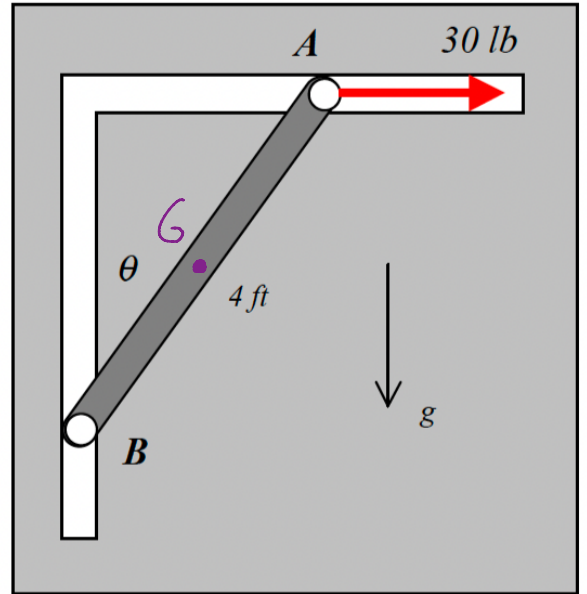
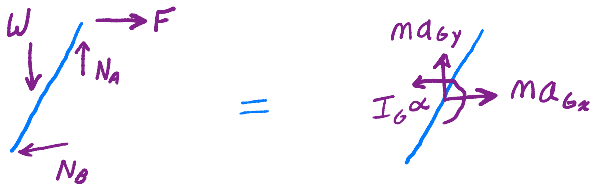
Find: Determine:

- (a) The angular acceleration of the bar on release; and $\alpha?$
- (b) The reaction forces on the bar at ends A and B on release.

$N_A, N_B?$



① FBD



Kinematics

② Bar AB

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B} \quad \text{RFR}$$

$$a_A \hat{i} = a_B \hat{j} + \alpha \hat{k} \times (L \sin \theta \hat{i} + L \cos \theta \hat{j})$$

$$= a_B \hat{j} + \alpha L \sin \theta \hat{j} - \alpha L \cos \theta \hat{i}$$

$$\hat{i}: a_A = -\alpha L \cos \theta \quad (4)$$

$$\hat{j}: 0 = a_B + \alpha L \sin \theta \Rightarrow a_B = -\alpha L \sin \theta \quad (5)$$

③ G

$$\vec{a}_G = \vec{a}_B + \vec{\alpha} \times \vec{r}_{G/B} - \omega^2 \vec{r}_{G/B} \quad \text{RFR}$$

$$a_{Gx} \hat{i} + a_{Gy} \hat{j} = a_B \hat{j} + \alpha \hat{k} \times \left(\frac{L}{2} \sin \theta \hat{i} + \frac{L}{2} \cos \theta \hat{j} \right)$$

$$= a_B \hat{j} + \frac{\alpha L}{2} \sin \theta \hat{j} - \frac{\alpha L}{2} \cos \theta \hat{i}$$

$$\hat{i}: a_{Gx} = -\frac{\alpha L}{2} \cos \theta \quad (6)$$

$$\hat{j}: a_{Gy} = a_B + \frac{\alpha L}{2} \sin \theta \quad (7)$$

④ Solve 7 eqns 7 unkns

Kinetics

② Newton on x dir

$$\sum F_x = m a_{Gx}$$

$$\Rightarrow F - N_B = m a_{Gx} \quad (1)$$

③ Newton on y dir

$$\sum F_y = m a_{Gy}$$

$$\Rightarrow N_A - W = m a_{Gy} \quad (2)$$

④ Euler on G

$$\sum M_G = I_G \alpha$$

$$\Rightarrow -N_B \frac{L}{2} \cos \theta + N_A \frac{L}{2} \sin \theta - F \frac{L}{2} \cos \theta \quad (3)$$

$$= \frac{1}{12} m L^2 \alpha$$

⑤ $\underline{\sum}$ unkns $\underline{3}$ eqns

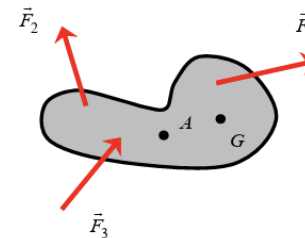
Summary: Newton/Euler Equations 3

FUNDAMENTAL equations:

$$(1) \quad \sum \vec{F} = m\vec{a}_G$$

$$(2) \quad \sum \vec{M}_A = I_A \vec{\alpha} \quad ; \quad A = \text{c.m. OR fixed point OR } \vec{r}_{G/A} \parallel \vec{a}_A$$

SAME point "A"!



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SOLUTION METHOD: the four-step plan

Kinetics: Four-Step Problem Solving Method

The suggested plan of action for solving kinetics problems:

1. **Free body diagram(s).** Draw appropriate free body diagrams (FBDs) for the problem. Your choice of FBDs is problem dependent. For some problems, you will draw an FBD for each body; for others, you will draw an FBD for the entire system. An integral part of your FBDs is your choice of coordinate systems. For each FBD, draw the unit vectors corresponding to your coordinate choice.
2. **Kinetics equations.** At this point, you will need to choose what solution method(s) that you will need to use for the particular problem at hand. In this section of the course we will study four basic methods: Newton/Euler, work/energy, linear impulse/momentum and angular impulse/momentum. Based on your choice of method(s), write down the appropriate equations from your FBD(s) from Step 1.
3. **Kinematics.** Perform the needed kinematic analysis. A study of the equations in Step 2 above will guide you in deciding what kinematics are needed to find a solution to the problem.
4. **Solve.** Count the number of unknowns and the number of equations from above. If you do not have enough equations to solve for your unknowns, then you either: (i) need to draw more FBDs, OR (ii) need to do more kinematic analysis. When you have sufficient equations for the number of unknowns, solve for the desired unknowns from the above equations.

Draw INDIVIDUAL free-body diagrams for Newton/Euler.

Be sure to use the correct mass moment of inertia for your choice of point "A". Use PAT if necessary.

Typically the most difficult step. Recall the rigid body kinematics from Chapter 2.

If you are short equations, go back to Step 3 – Kinematics.

Lec 28 Short Feedback Form:

