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ME 274 Lecture 27

Rigid body kinetics: Newton/Euler – Part 1

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03/25/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 26 (4.U and 4.V) due today!!**

2. **Quiz 5 due tonight on gradescope**

3. Office hours are changing to ME2008B…
   - Second floor of renovated side of ME.

4. Exam 2 Information:
   - Thursday, April 2, 8:00-9:30 PM
   - BHEE129
   - Coverage: Lectures 11-26 (up through angular impulse/momentum for particles)

5. Exam 2 Review sessions both videos to be posted on website
   - Pi Tau Sigma: Tuesday, March 31, 6:30-7:30 PM, WTHR 104 (WL in-person and Indy online)
   - ME 274 Instructor, CK: Wednesday, April 1, 7:00 PM, live on Zoom for both WL and Indy:
     - https://purdue.edu.zoom.us/j/94496659802?pwd=VMHo3NfyaHO3HbbmmLForgikls3PL7.1

Chapter 5: Planar Rigid Body Kinetics

- **In today's lecture, we will introduce and develop the set of Newton-Euler (N/E) Equations used when dealing with planar motion of rigid bodies**

  - **Today's Equations**

$$\sum \vec{F} = m\vec{a}_G$$

$$\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$$

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2} \sum \vec{F}\,dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2} \sum \vec{M}_O\,dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2} \sum \vec{M}_A\,dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A\vec{\omega}$

[pg. 252 and 295 content]

Background for deriving the N/E Rigid Body Equation

1. Earlier in the course we used the following equations for the kinetics of a **single particle**

- (1) Newton's Second Law and (2) angular momentum equation.

(1) $$\vec{F}_i = m_i \vec{a}_i$$

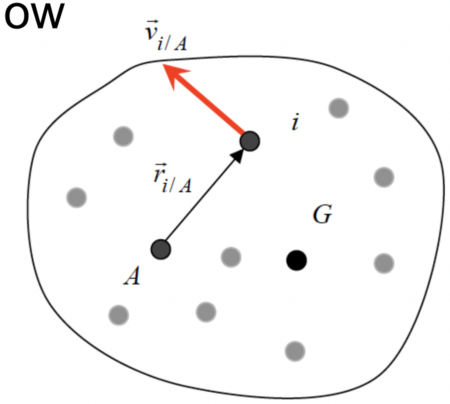
(2) $$\vec{M}_{Oi} = \frac{d}{dt}\left[\vec{r}_{i/O} \times (m_i \vec{v}_i)\right] \qquad ; \quad \text{O is a FIXED point}$$

2. We also saw that for a **system of particles** the above equations become the following below

- Where G is **center of mass** of the system and A is an **arbitrary** point in the system.

(3) $$\left(\sum \vec{F}\right)_{ext} = m\vec{a}_G$$

(4) $$\left(\sum \vec{M}_A\right)_{ext} = \frac{d}{dt}\sum_i \left[\vec{r}_{i/A} \times (m_i \vec{v}_{i/A})\right] + \vec{r}_{G/A} \times (m\vec{v}_A)$$



system of N particles

3. When extending this thought process to a continuous rigid body , we can replace the **summation for an integral:** $\sum_i (\bullet)\, m_i \rightarrow \int_{vol} (\bullet)\, dm$

[pg. 295 content]

Derivation of N/E Rigid Body Equation - Summary

1. Starting with the equations from previous slide for a **system of particles**:

(3) $\left(\sum \vec{F}\right)_{ext} = m\vec{a}_G$

(4) $\left(\sum \vec{M}_A\right)_{ext} = \frac{d}{dt}\sum_i \left[\vec{r}_{i/A} \times (m_i \vec{v}_{i/A})\right] + \vec{r}_{G/A} \times (m\vec{v}_A)$

2. We apply two things to get equivalent equations for a rigid body…
   a. Enforce a **rigid connection** between all points in the system: $\vec{v}_{i/A} = \vec{v}_i - \vec{v}_A = \vec{\omega} \times \vec{r}_{i/A}$

   b. Envision a rigid body as an ***infinite set of particles of infinitesimal size***: $\sum_i (\,\bullet\,)\; m_i \rightarrow \int_{vol} (\,\bullet\,)\; dm$

3. Newtons second law (3) **remains similar**.

4. Some math (p. 296-297) with **AIM Equation (4)** and we get:

$$\left(\sum \vec{M}_A\right)_{ext} = \frac{d}{dt}\left[\left(\sum m_i \left|\vec{r}_{i/A}\right|^2\right)\vec{\omega}\right] + \vec{r}_{G/A} \times (m\vec{a}_A)$$
$$= \frac{d}{dt}\left[I_A\vec{\omega}\right] + \vec{r}_{G/A} \times (m\vec{a}_A)$$
$$= I_A\vec{\alpha} + \vec{r}_{G/A} \times (m\vec{a}_A)$$

[pg. 296-297 content]

5. Where: $I_A = \sum m_i \left|\vec{r}_{i/A}\right|^2$

$$\rightarrow \int r^2 dm$$

- I is the **mass moment of inertia of a rigid body…**
- will speak more about in next lecture.

3 Special Forms of the Euler's Equation where $\vec{r}_{G/A} \times (m\vec{a}_A)$ = 0

$$\left(\sum \vec{M}_A\right)_{ext} = I_A\vec{\alpha} + \vec{r}_{G/A} \times (m\vec{a}_A)$$

1. If you choose A to be the **center of mass, G.** $\vec{r}_{G/A} = \vec{0}$.

$$\sum \vec{M}_G = I_G\vec{\alpha}$$

2. If you choose A to be a **fixed point on the body, O.** $\vec{a}_O = \vec{0}$.

$$\sum \vec{M}_O = I_O\vec{\alpha}$$

3. If you choose a point A which has an acceleration vector that is **parallel to** $\vec{r}_{G/A}$ **.** $\vec{r}_{G/A} \times \vec{a}_A = \vec{0}$,

$$\sum \vec{M}_A = I_A\vec{\alpha}$$

Aside/Note: You are free to choose any point A on the body you want. **But remember you must be consistent with it**, in your Euler's Equation.

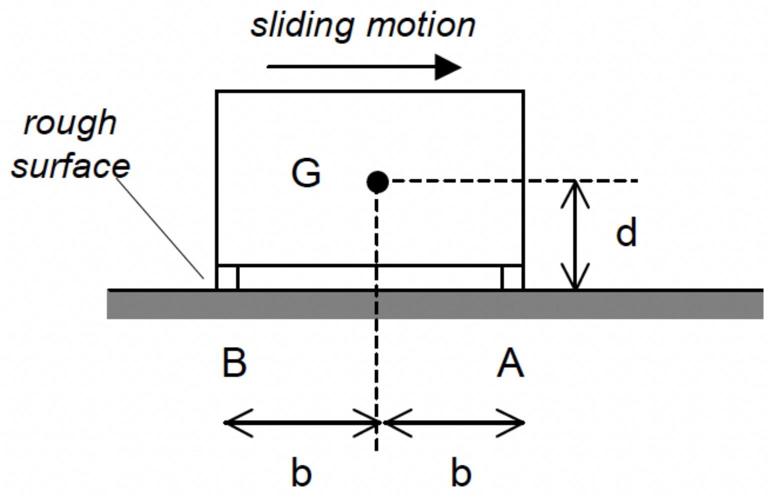
- This will show up more whenever we have to use actual equations for I, instead of making it 0.

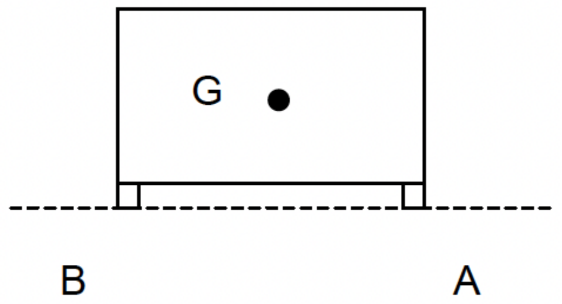
[pg. 298 content]

Example 5.A.1

Given: A crate of mass m slides to the right on a rough surface (with a kinetic coefficient of friction of μ_k).

Find: Find the reactions at contact points A and B on the crate.





[pg. 308]

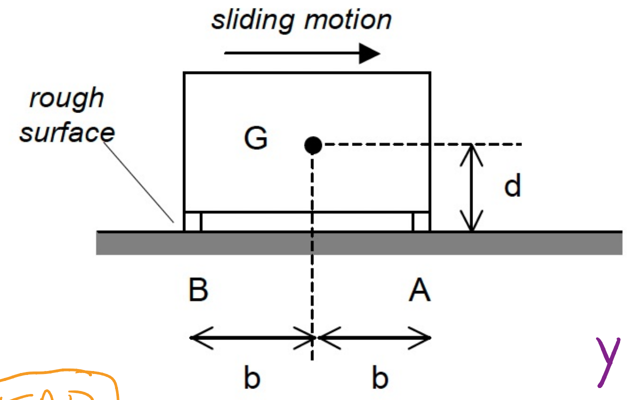
Example 5.A.1 p. 308 Look @ today's 'Friction dynamics' animation for video

Given: A crate of mass m slides to the right on a rough surface (with a kinetic coefficient of friction of μ_k).

Find: Find the reactions at contact points A and B on the crate.

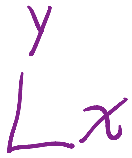
N_A & N_B?

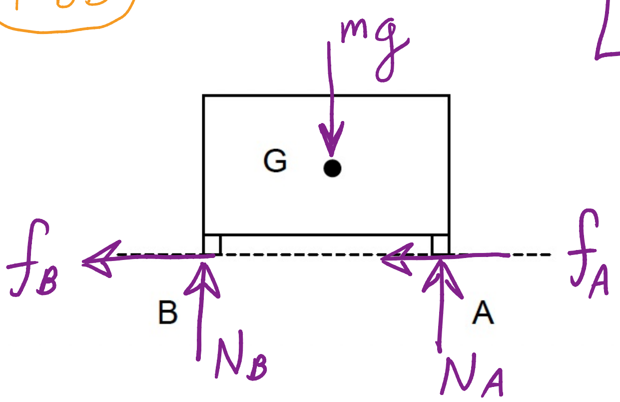
f_A & f_B?



⓪ Method: Newton/Euler
bc look @ instant

① FBD





Kinetics

② Sum of forces x

$\Sigma F_x = m a_{Gx}$

$\Rightarrow -f_B - f_A = m a_{Gx} \quad (1)$

③ Sum of forces y

$\Sigma F_y = 0$

$\Rightarrow N_B + N_A - mg = 0 \quad (2)$

④ Moment about G, no rotation

$\Sigma M_G = I_A \alpha = 0$ (α =0. (No tipping))

$\Rightarrow N_A b - N_B b - f_B d - f_A d = 0 \quad (3)$

4.5 Kinematics

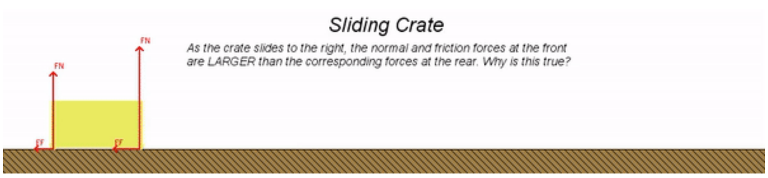
$\alpha = 0$ (no tipping)

⑤ Friction

$f_A = \mu_k N_A \quad (4)$

$f_B = \mu_k N_B \quad (5)$

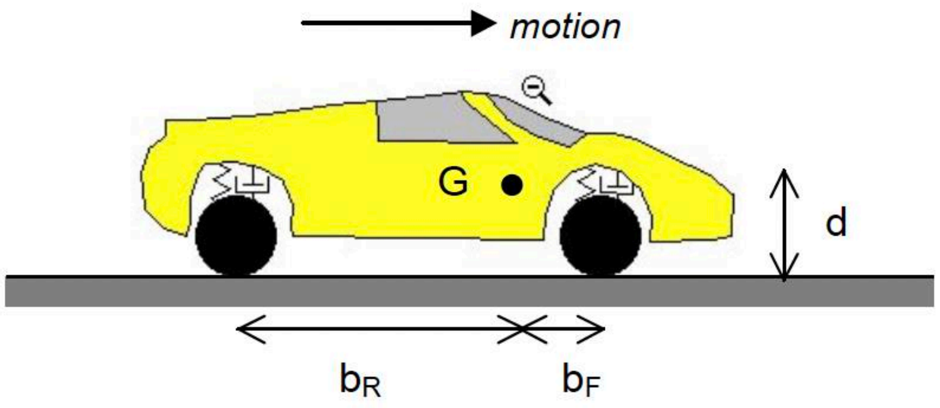
⑥ Solve 5 eqns 5 unkns

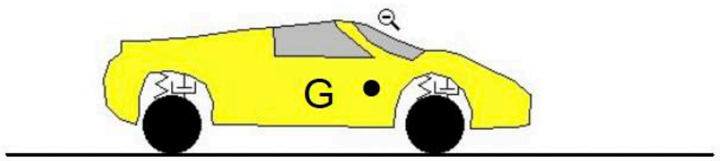


Example 5.A.2

Given: An automobile travels to the right as it brakes. The front of the car is known to *nose dive* during braking.

Find: Why does this phenomenon occur?



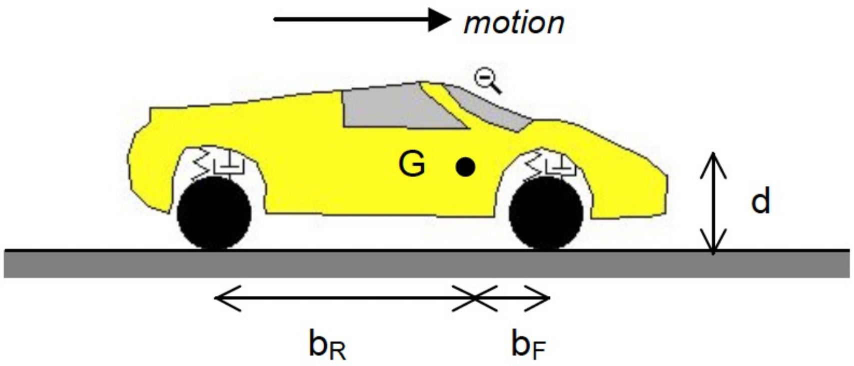


[pg. 309]

Example 5.A.2 p.309

Given: An automobile travels to the right as it brakes. The front of the car is known to *nose dive* during braking.

Find: Why does this phenomenon occur?



Kinetics

(2) x-dir

$\Sigma F_x = m a_x$

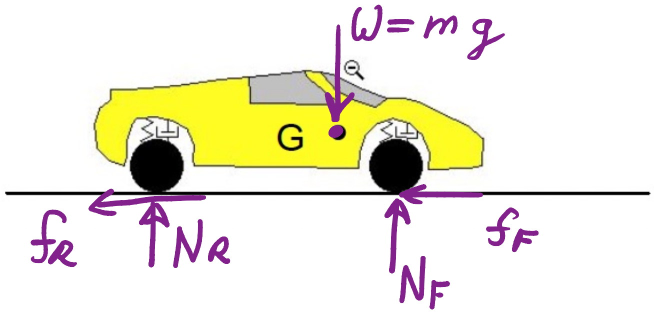
$\Rightarrow -f_R - f_F = m a_x \quad (1)$

(3) y-dir

$\Sigma F_y = 0$

$\Rightarrow N_R + N_F - mg = 0 \quad (2)$

(1) FBD



(4) Moment @ G. No rot.

$\circlearrowleft + \ \Sigma M_G = I_G \alpha = 0$

$\Rightarrow N_F b_F - N_R b_R - f_F d - f_R d = 0 \quad (3)$

(5) friction

$f_R = \mu_k N_R \quad (4)$

$f_F = \mu_k N_F \quad (5)$

(6) Solve

$(3) \Rightarrow N_F b_F - N_R b_R - \mu_k N_F d - \mu_k N_R d = 0$

$(2) \Rightarrow N_R = mg - N_F$

$\Rightarrow N_F b_F - (mg - N_F) b_R - \mu_k N_F d - \mu_k (mg - N_F) d = 0$; Plug in (2) in (3)

$\Rightarrow N_F b_F - mg b_R + N_F b_R - \mu_k N_F d - \mu_k mg d + \mu_k N_F d = 0$

(7) Solve for Front

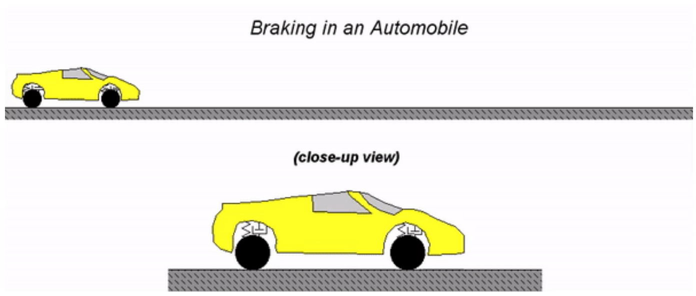
$N_F = \dfrac{mg(b_R + \mu_k d)}{b_F + b_R - \mu_k d + \mu_k d}$

$= \dfrac{mg(b_R + \mu_k d)}{b_F + b_R}$

(8) Solve for Rear

$N_R = mg - N_F$

$= mg - \dfrac{mg(b_R + \mu_k d)}{b_F + b_R}$

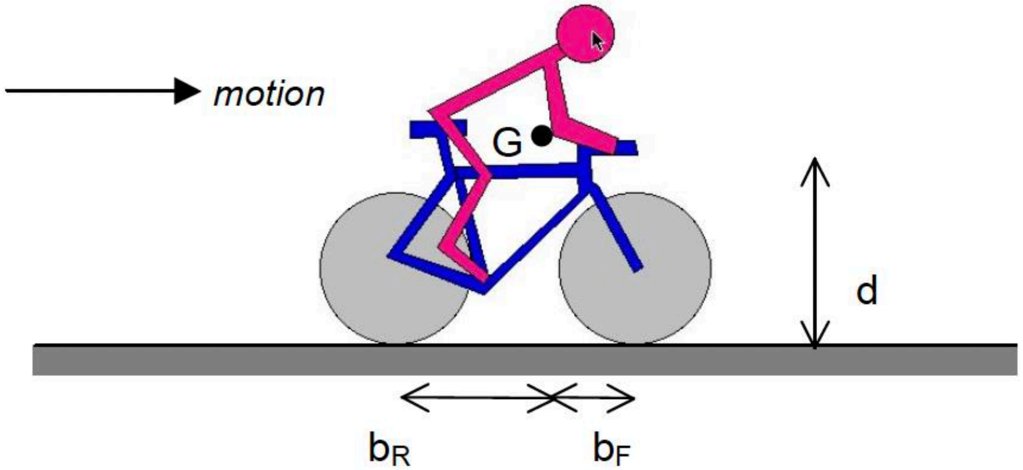


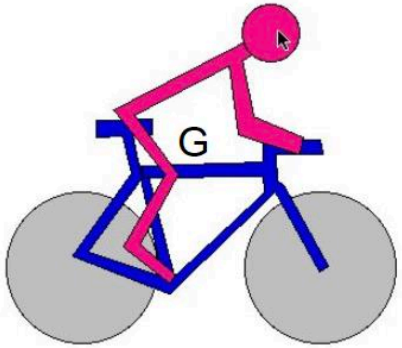
(9) if # are plugin. $N_R < N_F$ ∴ nose dive

Example 5.A.3

Given: A bicyclist travels to the right as she brakes. In one situation, she brakes with only the rear wheel. In another situation, she brakes with only the front wheel.

Find: Comment on the possible differences between rear-wheel and front-wheel braking (as viewed from a dynamics perspective)?



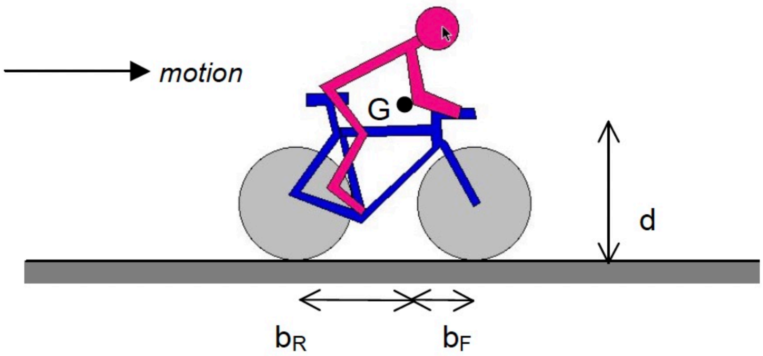


[pg. 310]

Example 5.A.3 P.316 Look @ today's 'friction dynamics' animation for video

Given: A bicyclist travels to the right as she brakes. In one situation, she brakes with only the rear wheel. In another situation, she brakes with only the front wheel.

Find: Comment on the possible differences between rear-wheel (r) and front-wheel (f) braking (as viewed from a dynamics perspective)? find a_{Gx} in front & rear scenarios



Kinetics

(2) x dir

$\Sigma F_x = m a_{Gx}$

$\Rightarrow -f_F - f_R = m a_{Gx}$ (1)

(3) y dir

$\Sigma F_y = 0$

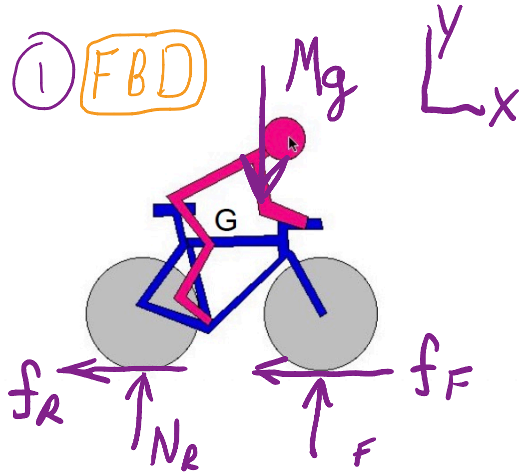
$\Rightarrow N_R + N_F - mg = 0$ (2)

(4) Moment @ G. No rot

$\Rightarrow \Sigma M_G = 0$

$N_F b_F - N_R b_R - f_R d - f_F d = 0$ (3)

(1) FBD



Solve

(5A) Scenario A: Rear break. Plug in values below

$f_R = \mu_K N_R$; $f_F = 0$

(1) $\Rightarrow -\mu_K N_R = m a_{Gx}$;

(2) $\Rightarrow N_R + N_F = mg$;

(3) $\Rightarrow N_F b_F - N_R b_R - \mu_K N_R d = 0$

$N_F = mg - N_R$; (2)

$\Longrightarrow mg b_F - N_R b_F - N_R b_R - \mu_K N_R d = 0$

(5B) Solve for N_R

$\Rightarrow N_R = \dfrac{mg b_F}{b_F + b_R + \mu_K d}$

(5C) Use (1). Solve for a_{Gx}.

$\Rightarrow a_{Gx} = \dfrac{-\mu_K m g b_F}{m(b_F + b_R + \mu_K d)}$

$= \dfrac{-\mu_K g b_F}{b_F + b_R + \mu_K d}$

(5B) Scenario B: Front break. Plug in values below

$f_R = 0$; $f_F = \mu_K N_F$

(1) $\Rightarrow -\mu_K N_F = m a_{Gx}$;

(2) $\Rightarrow N_R + N_F = mg$;

(3) $\Rightarrow N_F b_F - N_R b_R - \mu_K N_F d = 0$

$N_R = mg - N_F$; (2)

$\Rightarrow N_F b_F - mg b_R + N_F b_R - \mu_K N_F d = 0$

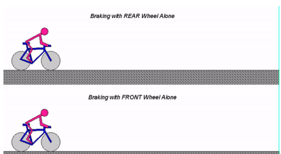
(6B) Solve for N_F

$N_F = \dfrac{mg b_R}{b_F + b_R - \mu_K d}$

(7B) Use (1). Solve for a_{Gx}.

$a_{Gx} = -\dfrac{\mu_K \cancel{m} g b_R}{\cancel{m}(b_F + b_R - \mu_K d)}$

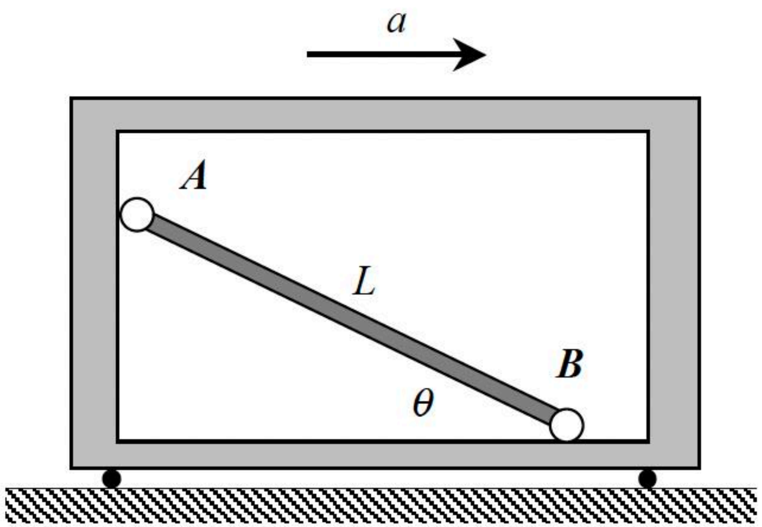
$= \dfrac{-\mu_K g b_R}{b_F + b_R - \mu_K d}$



Example 5.A.4

Given: The figure below.

Find: For what acceleration of the frame does the uniform slender rod maintain the orientation shown in the figure? Consider all surfaces to be smooth.



[pg. 311]

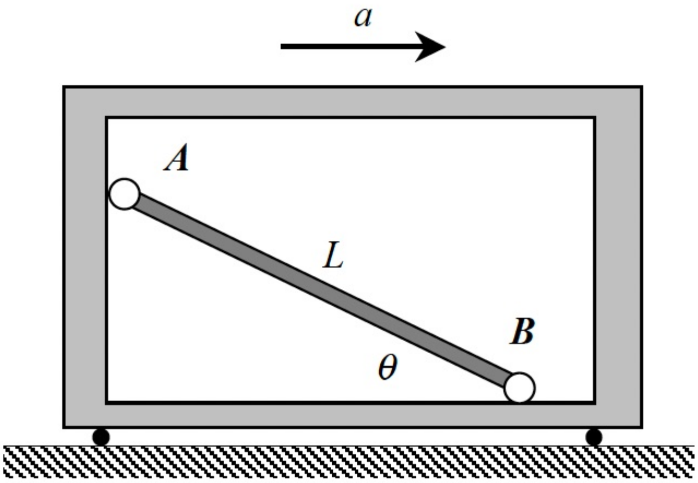
Example 5.A.4

p.311

Given: The figure below.

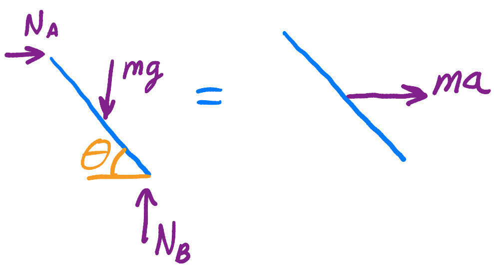
Find: For what acceleration of the frame does the uniform slender rod maintain the orientation shown in the figure? Consider all surfaces to be smooth.

a?



(0) Instant in time ∴ N/E Eqns

(1) FBD Rod



(2) x-dir Newton

$\Sigma F_x = ma$

$$\Rightarrow N_A = ma \quad (1)$$

(3) y-dir Newton

$\Sigma F_y = 0$

$$\Rightarrow N_B - mg = 0 \quad (2)$$

(4) Moment @ G. Euler. No rotation

$\circlearrowleft +$ $\Sigma M_G = 0$

$$\Rightarrow N_B\left(\frac{L}{2}\cos\theta\right) - N_A\left(\frac{L}{2}\sin\theta\right) = 0 \quad (3)$$

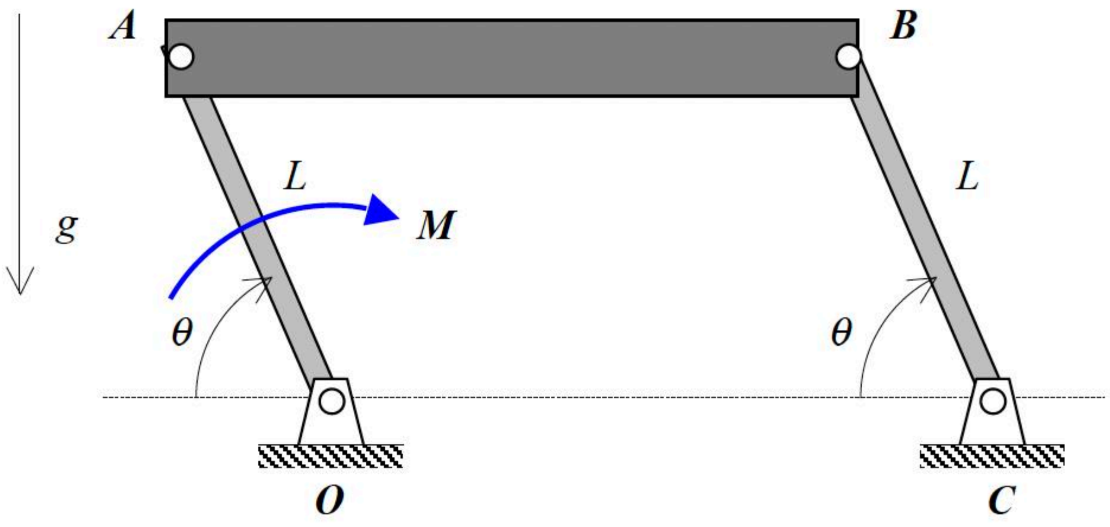
(5) Solve

$$\Rightarrow N_A, N_B, a$$

Example 5.A.5

Given: A uniform platform AB (mass of $m = 400$ kg) is raised by the application of a constant torque $M = 4000$ N-m which is applied to link OA. Links OA and BC each have a length of $L = 2$ m, with the mass of these links being small compared to the mass of the platform. At the position of $\theta = 53.13°$, links OA and BC are known to be rotating in the clockwise sense at a rate of $\omega = 3$ rad/s.

Find: Determine the magnitude of the load carried by pin A at this position.



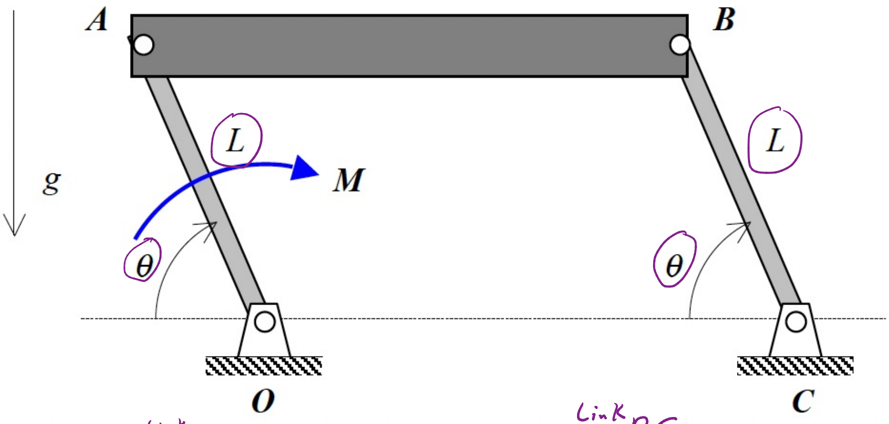
[pg. 312]

Example 5.A.5 p. 312

Given: A uniform platform AB (mass of $m = 400$ kg) is raised by the application of a constant torque $M = 4000$ N-m which is applied to link OA. Links OA and BC each have a length of $L = 2$ m, with the mass of these links being small compared to the mass of the platform. At the position of $\theta = 53.13°$, links OA and BC are known to be rotating in the clockwise sense at a rate of $\omega = 3$ rad/s.

Find: Determine the magnitude of the load carried by pin A at this position. $|A| = \sqrt{A_x^2 + A_y^2}$

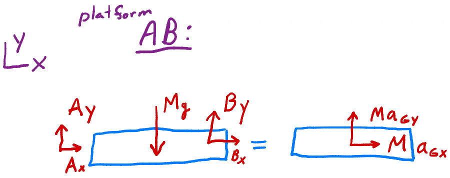
$m = 400$ kg $L = 2$ m
$M = 4000$ Nm $\theta = 53.13°$
$\omega = 3$ rad/s



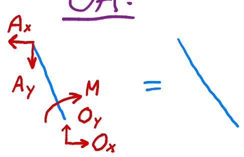
① Metod: N/E eqn
Instant in time.

② FBDs Indiv. FBDs

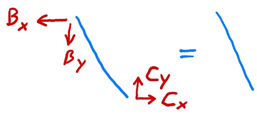
platform AB:



Link OA:



Link BC:



Kinetics

AB:

③ x dir
$\Sigma F_x = M a_{Gx}$
$\Rightarrow A_x + B_x = M a_{Gx}$ (1)

④ y dir
$\Sigma F_y = M a_{Gy}$
$\Rightarrow A_y + B_y - Mg = M a_{Gy}$ (2)

⑤ Moment @ G. No rot.
$\Sigma M_G = I_G \alpha = 0$
$\Rightarrow -A_y \frac{w}{2} + B_y \frac{w}{2} = 0$ (3)

OA:

⑥ Moment @ O. no rot
$\circlearrowleft \Sigma M_O = 0$
$\Rightarrow -M + A_y L\cos\theta + A_x L\sin\theta = 0$ (4)

BC:

⑦ Moment @ C. no rot
$\circlearrowleft \Sigma M_C = 0$
$\Rightarrow B_y L\cos\theta + B_x L\sin\theta = 0$ (5)

7,5 too many unKns. Need more eqns

⑧ Acceleration @ OA
$\vec{a}_A = \vec{a}_O + \vec{\alpha}_{AO} \times \vec{r}_{A/O} - \omega_{AO}^2 \vec{r}_{A/O}$

$a_{Ax}\hat{\imath} + a_{Ay}\hat{\jmath} = \vec{0} + \alpha_{AO}\hat{k} \times (-L\cos\theta\,\hat{\imath} + L\sin\theta\,\hat{\jmath}) - \omega_{AO}^2(-L\cos\theta\,\hat{\imath} + L\sin\theta\,\hat{\jmath})$

$a_{Gx}\hat{\imath} + a_{Gy}\hat{\jmath} = -\alpha_{AO}L\cos\theta\,\hat{\jmath} - \alpha_{AO}L\sin\theta\,\hat{\imath} + \omega_{AO}^2 L\cos\theta\,\hat{\imath} - \omega_{AO}^2 L\sin\theta\,\hat{\jmath}$

⑨
$\hat{\imath}: a_{Gx} = -\alpha_{AO}L\sin\theta + \omega_{AO}^2 L\cos\theta$ (6)
$\hat{\jmath}: a_{Gy} = -\alpha_{AO}L\cos\theta - \omega_{AO}^2 L\sin\theta$ (7)

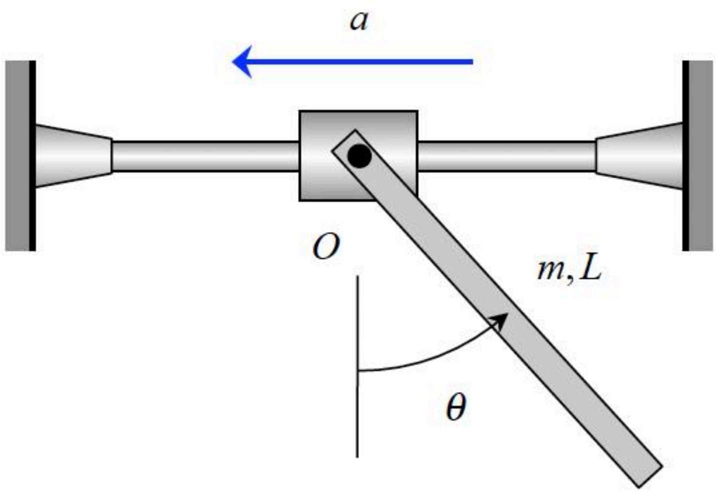
⑩ Solve 7 eqns 7 unKns

⑪ $|A| = \sqrt{A_x^2 + A_y^2}$. Mag of load @ A

Example 5.A.6

Given: The collar O has a constant acceleration of a to the left. A thin, homogeneous bar of length L and mass m is pinned to the collar.

Find: Find a value of a such that the bar is held at a constant angle of $\theta = 25°$.



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Example 5.A.6 p. 313 similar to hw problem due fri

Given: The collar O has a constant acceleration of a to the left. A thin, homogeneous bar of length L and mass m is pinned to the collar.

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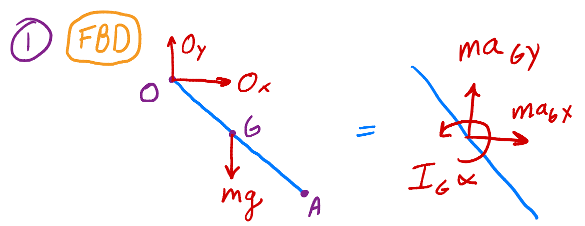
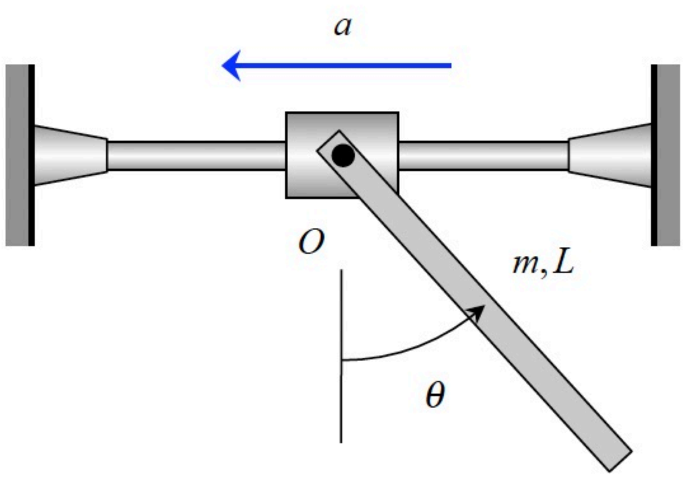
① FBD





Kinetics

② Newton on x

$\Sigma F_x = m a_{Gx}$

$O_x = m a_{Gx}$ (1)

③ Newton on y

$\Sigma F_y = m a_{Gy}$

$O_y - mg = m a_{Gy}$ (2)

④ Euler @ G

$\Sigma M_G = I_G \alpha$

$-O_y \frac{L}{2} \sin\theta - O_x \frac{L}{2} \cos\theta = I_G \alpha$ (3)

Kinematics

⑤ For no rotation:

→ $\alpha = 0$ (4)

→ $a_{Ox} = a_{Gx}$ (5)

→ $a_{Gy} = 0$; G moves on straight path x (6)

⑥ Solve Use (1-6) to find a

Summary: Newton/Euler Equations 1

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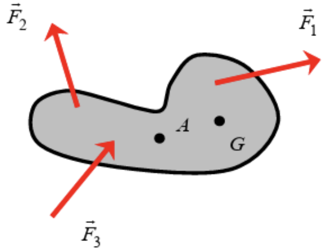

FUNDAMENTAL equations:

(1) $\sum \vec{F} = m\vec{a}_G$

(2) $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$

SAME point "A"!

CRITICAL ISSUES:

- For NEWTON (1): *G must be the center of mass* of the body
- For EULER (2): *A is ANY point on the body*. The same point "A" must be used across the board in the equation – you cannot mix and match points A.

SIMPLIFICATION: If A is*: EITHER the center of mass G OR a fixed point (zero acceleration) OR* $\vec{a}_A$ *is parallel to* $\vec{r}_{G/A}$, then the Euler equation (2) reduces to*:*

$$\sum \vec{M}_A = I_A\vec{\alpha}$$

We will use this form of the equation most of the time

TERMINOLOGY: I_A is known as the "*mass moment of inertia*" of the body about point A. The size of I_A is dependent on the location of A.

Lec 27 Short Feedback Form: