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# ME 274 Lecture 26

**Particle Kinetics – Summary**

Eugenio "Henny" Frias-Miranda

03/23/26

# Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 25 (4.S and 4.T) due today!!**
2. Quiz 5 due next Wednesday
3. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME.
4. Bonus quiz grade at end of the semester if we get a good response rate to QR code surveys at the end of lecture.
    - If you are unable to attend lecture on that day/forget to fill it out:
        - Feel free to give feedback based on the content of that lecture’s slides.
        - Way of you reviewing previous content and giving feedback to me.

# Kinetics Summary Handout

particle = ch 4 ;
rigid body = will cover in ch 5

## Kinetics Table (page 352 of the lecture book)

when to use {

| Method | Body model | Fundamental equations |
|---|---|---|
| **Newton-Euler** (relating forces to accelerations) | **particle** | $\sum \vec{F} = m\vec{a}$ |
| | **rigid body** (G = c.m. and A = any point on body) | $\sum \vec{F} = m\vec{a}_G$<br>$\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$ |
| **Work-energy** (relating change in speed to change in position) | **particle** | $T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$<br>where $T = \frac{1}{2}mv^2$ |
| | **rigid body** (G = c.m. and A = any point on body) | $T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$<br>where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$ |
| **Linear impulse-momentum** (relating change in velocity to change in time) | **particle** | $\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_2 - m\vec{v}_1$ |
| | **rigid body** (G = c.m.) | $\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_{G2} - m\vec{v}_{G1}$ |
| **Angular impulse-momentum** (relating change in angular velocity to change in time) | **particle** (O = fixed point) | $\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_{O2} - \vec{H}_{O1}$<br>where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$ |
| | **rigid body** (A = fixed point or c.m.) | $\int_{t_1}^{t_2} \sum \vec{M}_A\, dt = \vec{H}_{A2} - \vec{H}_{A1}$<br>where $\vec{H}_A = I_A\vec{\omega}$ |

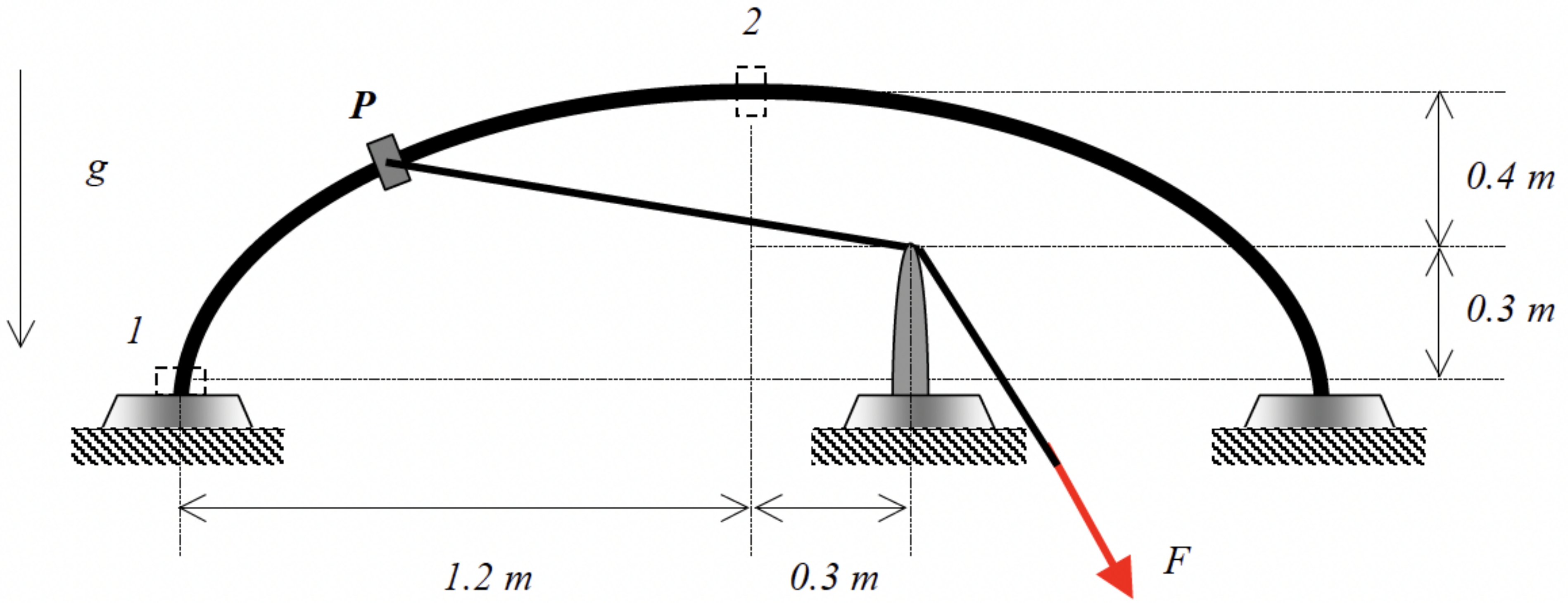
Notes on the *four-step* method:

1. ***FBD(s)***
   - Newton/Euler (*N/E*): typically individual FBDs
   - Work/energy (*W/E*), linear impulse momentum (*LIM*) and angular impulse/momentum (*AIM*): typically make it "BIG" (include all moving bodies)
2. ***Kinetics*** (see suggestions in the table to the right as to which method(s) to use)
   - Choose coordinate system(s) based on the *"Given"* and *"Find"*, including consideration of any motion constraints in the system.
   - *N/E*: point G in the Newton equation must be the center of mass for the body
   - *W/E*: mechanical energy is conserved if $U_{1\to2}^{(nc)} = 0$ (no non-conservative work being done on your system)
   - *LIM*: linear momentum in the *x*-direction is conserved if $\sum F_x = 0$ (no net force in the *x*-direction for your system)
   - *AIM*: angular momentum in the *z*-direction is conserved if $\sum M_{Oz} = 0$ (no net moment in the *z*-direction about point O for your system).For particles, O must be a fixed point. For rigid bodies, O can be either a fixed point or the center of mass.
3. ***Kinematics***
   - *N/E*: typically need *acceleration* kinematics
   - *W/E, LIM and AIM:* typically need *velocity* kinematics
4. ***Solve***

## Example 4.B.5

**Given:** Slider P, having a mass of $m = 0.6$ kg, moves freely along the fixed, smooth, curved rod from position A to position B in the vertical plane under the action of the constant $F = 20$ N tension in the cord.

**Find:** Determine the speed of the slider at position 2, if the slider is released from rest at position 1.



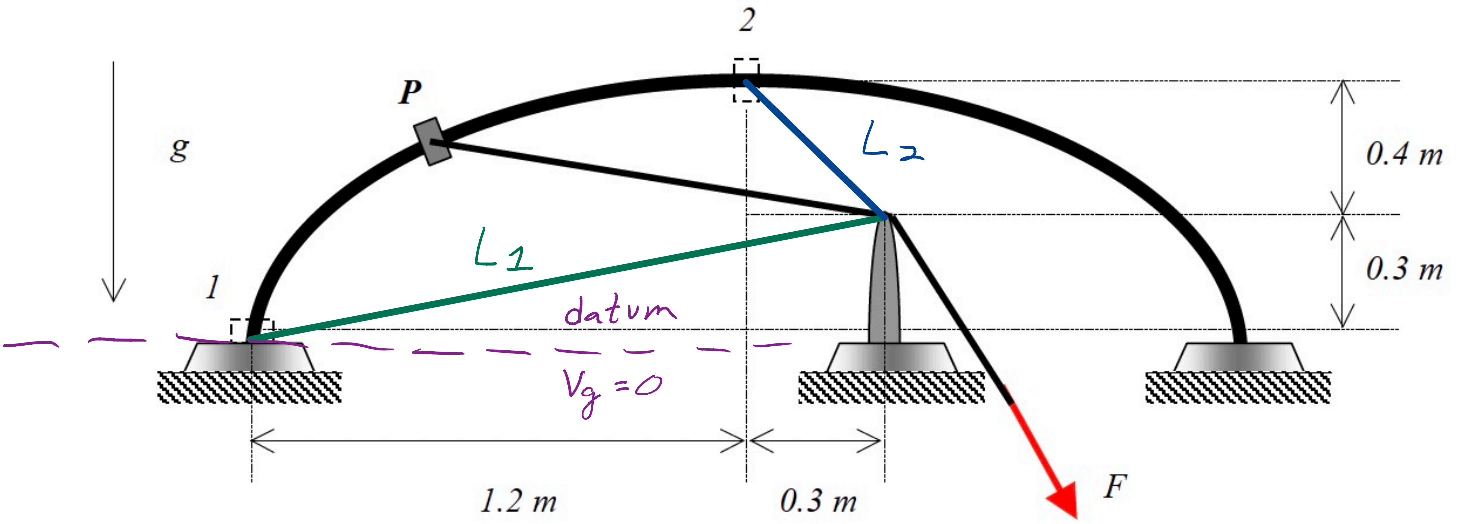

[pg. 223]

**Example 4.B.5** Similar to H.4.V pg. 223

**Given:** Slider P, having a mass of $m = 0.6$ kg, moves freely along the fixed, smooth, curved rod from position A to position B in the vertical plane under the action of the constant $F = 20$ N tension in the cord.

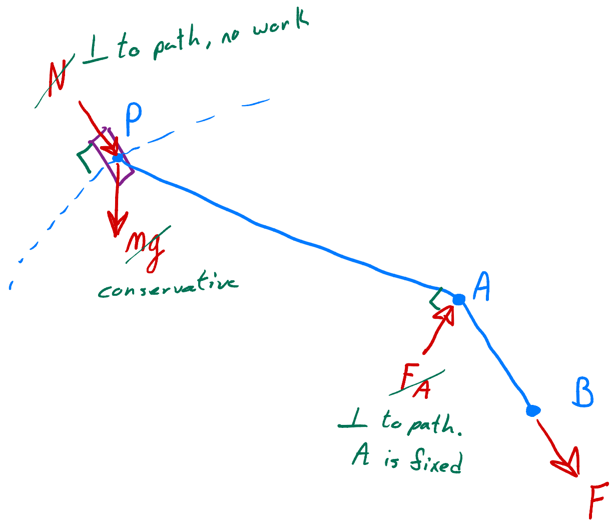
**Find:** Determine the speed of the slider at position 2, if the slider is released from rest at position 1.

$v_2 = ?$




⓪ Method: Relate change in spd to change in position → W/E

① FBD: Make it big (cable + P)




② Kinetics W/E

$$T_1 + V_1 + U_{1\to 2}^{(NC)} = T_2 + V_2$$

$T_1 = 0$ ; @ rest

$V_1 = 0$ ; datum

$$U_{1\to 2}^{(NC)} = \int_1^2 (\vec{F}\,\hat{e}_{tB})\,ds_B$$

$$= F\,\Delta s_B$$

$T_2 = \frac{1}{2} m v^2$

$V_2 = m g h_2$ ; $h_2 = 0.3 + 0.4 = 0.7\,m$

③ Plug-in

$$F \Delta s_B = \frac{1}{2} m v_2^2 + m g h_2 \quad (1)$$

④ Kinematics

$$\Delta s_B = L_1 - L_2$$

$$= \sqrt{1.5^2 + 0.3^2} - \sqrt{0.4^2 + 0.3^2}$$

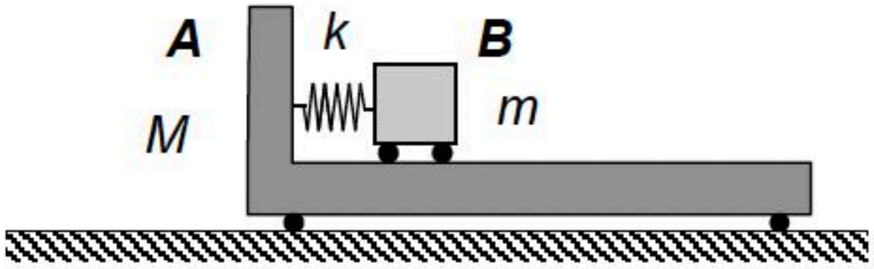
$$= \_\_\_\_$$

⑤ Solve

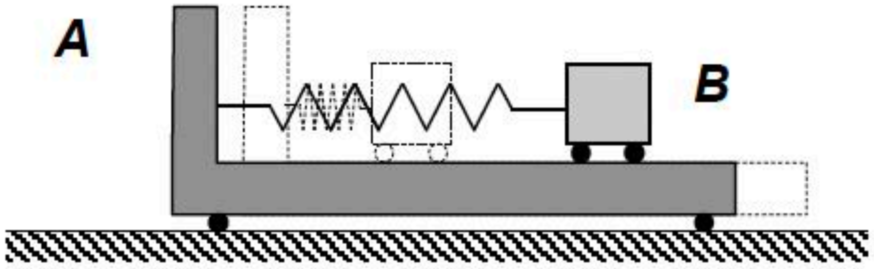
Use (1) for $v_2 = \_\_\_\_$

**Question C4.7**

Cart A and block B (having masses of $M = 4$ kg and $m = 2$ kg, respectively, are connected by a spring of stiffness $k = 300$ N/m. The system is released from rest with the spring being compressed 0.2 m (Position 1). Find the speed of the cart at Position 2 at the instant when the spring is uncompressed/unstretched. Consider all surfaces to be smooth.




***Position 1***
*(at rest)*




***Position 2***
*(both A and B moving)*

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**Question C4.7** p.274

Cart A and block B (having masses of $M = 4$ kg and $m = 2$ kg, respectively, are connected by a spring of stiffness $k = 300$ N/m. The system is released from rest with the spring being compressed 0.2 m (Position 1). Find the speed of the cart at Position 2 at the instant when the spring is uncompressed/unstretched. Consider all surfaces to be smooth.

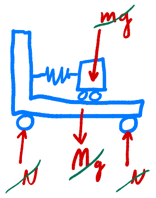
$\upsilon_{A,2}$ ?


***Position 1***
*(at rest)*

***Position 2***
*(both A and B moving)*

(0) Method. W/E : Change in spd → Change in position

(1) FBD : A + B + spring




all forces are ⊥ to path

(2) W/E

$T_1 + V_1 + U^{NC}_{1-2} = T_2 + V_2$

$T_1 = 0$ ; @ rest

$V_1 = \frac{1}{2} k \Delta_1^2$ ; $\Delta_1 = 0.2$ m

$U^{NC}_{1-2} = 0$

$T_2 = \frac{1}{2} M \upsilon_{A,2}^2 + \frac{1}{2} m \upsilon_{B,2}^2$

$V_2 = 0$ ; $\Delta_2 = 0$

(3) Plug in

$\frac{1}{2} M v_{A,2}^2 + \frac{1}{2} m v_{B,2}^2 = \frac{1}{2} k \Delta_1^2$ (1)

(4) 2 unkns 1 eqn

(6) LIM in x-dir

$\Sigma F_x = 0$

$m V_{B1} + M \upsilon_{A1} = m V_{B2} + M V_{A2}$

↳ $V_{B2} = -\frac{M}{m} V_{A2}$ (2)

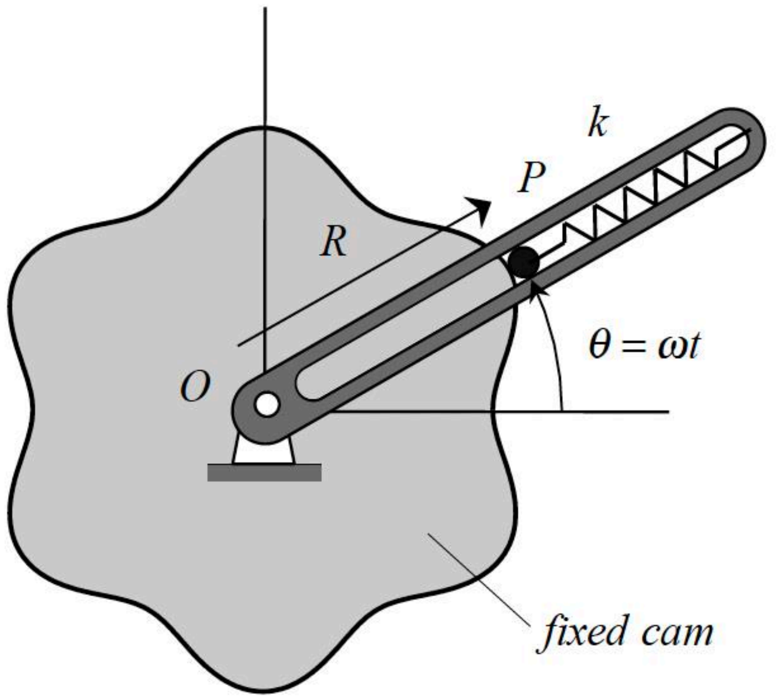
(7) $\boxed{\text{No Kinematics}}$

(8) $\boxed{\text{Solve}}$ 2 eqns 2 unkns

**Example 4.A.11**

**Given:** A slotted arm rotates about a vertical shaft passing through O that is at the center of a FIXED cam, as shown in the figure. A particle P, having a mass of $m = 0.2$ kg moves within the slot in the arm and remains in contact with the surface of the cam under the action of a spring attached between P and the outer end of the arm. The shape of the cam is such that the radial distance from O to P is given by the equation $R = R_0 - R_1 \cos(6\theta)$ where $R_0 = 0.5$ m and $R_0 = 0.1$ m. The spring has a stiffness of $k = 500$ N/m and is compressed by an amount of $\Delta = 0.2$ m when $\theta = 0$. The arm rotates at a constant rate of $\omega = 10$ rad/s.

**Find:** Determine the force acting on P by the cam when P passes over the top of the lobe in the position shown.




[pg. 204]

**Example 4.A.11** p.204 Lec18 problem

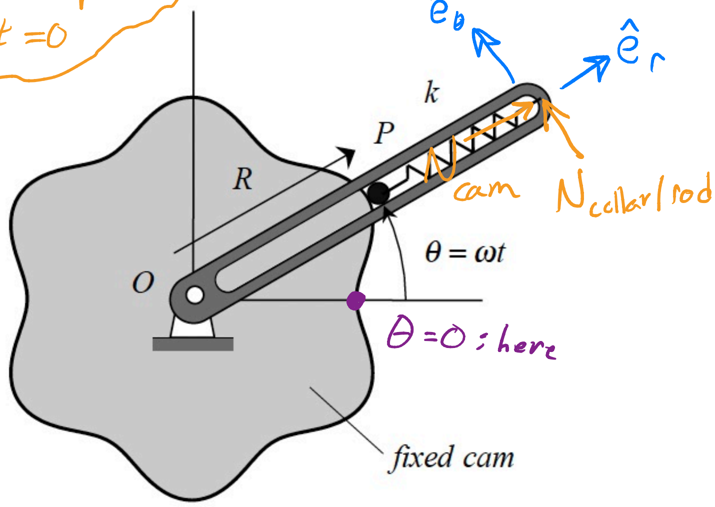
**Given:** A slotted arm rotates about a vertical shaft passing through O that is at the center of a FIXED cam, as shown in the figure. A particle P, having a mass of $m = 0.2$ kg moves within the slot in the arm and remains in contact with the surface of the cam under the action of a spring attached between P and the outer end of the arm. The shape of the cam is such that the radial distance from O to P is given by the equation $R = R_0 - R_1\cos(6\theta)$ where $R_0 = 0.5$ m and $R_1 = 0.1$ m. The spring has a stiffness of $k = 500$ N/m and is compressed by an amount of $\Delta = 0.2$ m when $\theta = 0$. The arm rotates at a constant rate of $\omega = 10$ rad/s.

**Find:** Determine the force acting on P by the cam when P passes over the top of the lobe in the position shown. $N$?

$m = 0.2$ kg
$R = R_0 - R_1\cos(6\theta)$
$R_0 = 0.5$ m
$R_1 = 0.1$ m
$K = 500$ N/m
$\Delta_0 = 0.2$ @ $\theta = 0$

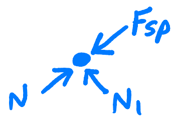
$\omega = 10$ rad/s ; $\dot{\omega} = 0$

horizontal plane
Weight = 0




Top View:

① FBD

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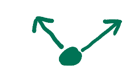
Kinetics Diagram:




Kinetics

② $\Sigma F$ in radial direction

$\Sigma F_r = ma_r$

$\Rightarrow N - F_{sp} = m(\ddot{r} - r\dot{\theta}^2)$

Kinematics

③ What's $F_{sp}$? Use Hook's law

$F_{sp} = k(\Delta)$ : Hook's Law

④ What's $\Delta$?

@ $\theta = 0$ : $\Delta_0$

@ $\theta$ : $\Delta_0 + 2R_1$

⑤ Solve

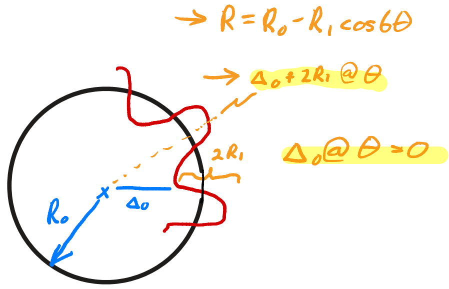
$N = k(\Delta_0 + 2R_1) + m(\ddot{r} - r\dot{\theta}^2)$ $\omega$

$r = R_0 - R_1\cos 6\theta$

$\dot{r} = (R_1 \sin 6\theta)\, 6\dot{\theta}$ $\omega$ ; $\frac{d}{dt}$

$\ddot{r} = R_1\cos 6\theta\,(6\dot{\theta})^2 + R_1\sin 6\theta\,(6\ddot{\theta})$ ; $\omega$ const

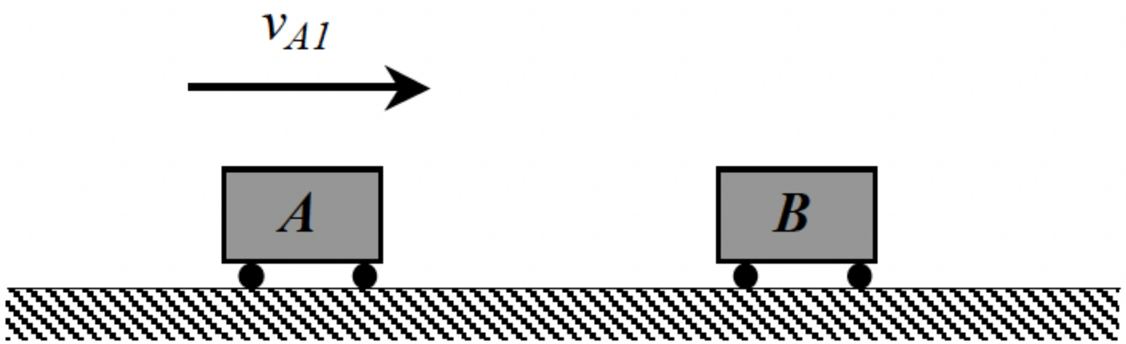
⑥ Solve for N



**Example 4.C.4**

**Given:** Car A (having mass of $m_A$) travels to the right with an initial speed of $v_{A1}$. Car A then impacts a stationary car B (having a mass of $m_B$). After impact, the two cars stick together.

**Find:** The change in kinetic energy for the system of A and B together after the impact and resulting coupling. What fraction is this change to the initial kinetic energy of the system of A and B?



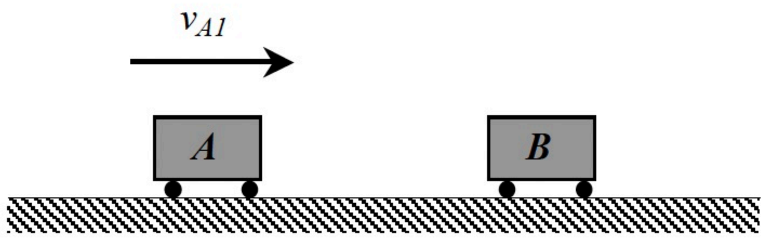

[pg. 239]

**Example 4.C.4** p. 239

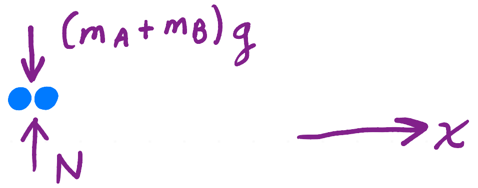
**Given:** Car A (having mass of $m_A$) travels to the right with an initial speed of $v_{A1}$. Car A then impacts a stationary car B (having a mass of $m_B$). After impact, the two cars stick together.

**Find:** (a) The change in kinetic energy for the system of A and B together after the impact and resulting coupling. (b) What fraction is this change to the initial kinetic energy of the system of A and B?

(a) $T_2 - T_1$ ? (b) $\dfrac{T_2 - T_1}{T_1}$




(1) FBD




Kinetics

(2) Use LIM bc relating change in vel to change in time

LIM x-dir: $\cancel{\int_{t_1}^{t_2} \sum F\,dt} = m\vec{v}_2 - m\vec{v}_1$

$$m_A v_{A1} + \cancel{m_B v_{B1}}^{\text{stationary}} = m_A v_{A2} + m_B v_{B2}$$

$$= (m_A + m_B) v_2 \quad ; \; v_{A2} = v_{B2}$$

Solve

$$v_2 = \frac{m_A}{m_A + m_B} v_{A1}$$

(3) Find $T_2 - T_1$, change in Kinetic Energy

$$T_2 - T_1 = \frac{1}{2}(m_A + m_B) v_2^2 - \frac{1}{2} m_A v_{A1}^2$$

$$= \frac{1}{2}(m_A + m_B)\left[\frac{m_A^2}{(m_A+m_B)^2}\right] v_{A1}^2 - \frac{1}{2} m_A v_{A1}^2$$

$$= \frac{1}{2} \frac{m_A^2}{m_A + m_B} v_{A1}^2 - \frac{1}{2} m_A v_{A1}^2$$

(4) Find $\dfrac{T_2 - T_1}{T_1}$

$$\frac{T_2 - T_1}{T_1} = \frac{\cancel{\frac{1}{2}} \frac{m_A^{\cancel{2}}}{m_A + m_B} \cancel{v_{A1}^2} - \cancel{\frac{1}{2}} \cancel{m_A} \cancel{v_{A1}^2}}{\cancel{\frac{1}{2}} \cancel{m_A} \cancel{v_{A1}^2}}$$

$$= \frac{m_A}{m_A + m_B} - 1$$

Aside, let give values to our m's:

(If $m_A = m_B = m \Rightarrow \dfrac{T_2 - T_1}{T_1} = -0.5$)

We lose 50% of energy due to impact

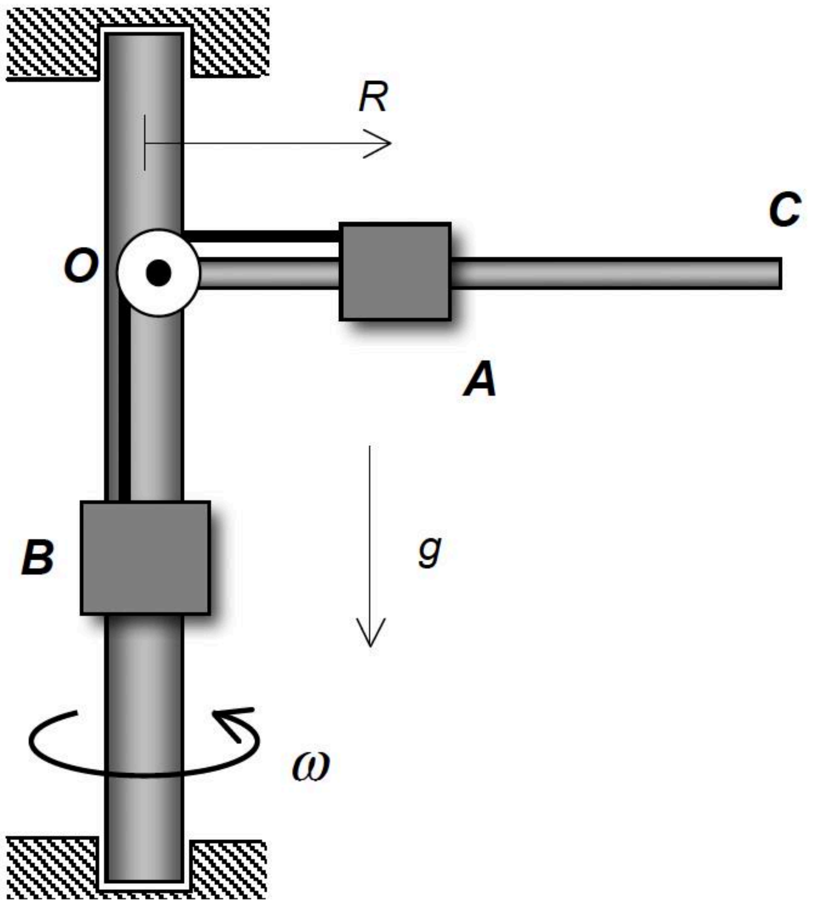
**Example 4.D.4**

**Given:** Particles A and B, whose masses are 0.5 kg each, are NOT sliding over their smooth guides at a position of $R = 0.8$ m with $\omega = 6$ rad/s. Assume the mass of the guides and pulley to the negligible and that the influences of friction are negligible everywhere.

**Find:** For the position where $R = 1.2$ m, find:

(a) The angular speed of arm OC; and

(b) The speed of particle B.




[pg. 261]

**Example 4.D.4** p.261

**Given:** Particles A and B, whose masses are 0.5 kg each, are NOT sliding over their smooth guides at a position of $R = 0.8$ m with $\omega = 6$ rad/s. Assume the mass of the guides and pulley to the negligible and that the influences of friction are negligible everywhere.

**Find:** For the position where $R = 1.2$ m, find:

(a) The angular speed of arm OC; and

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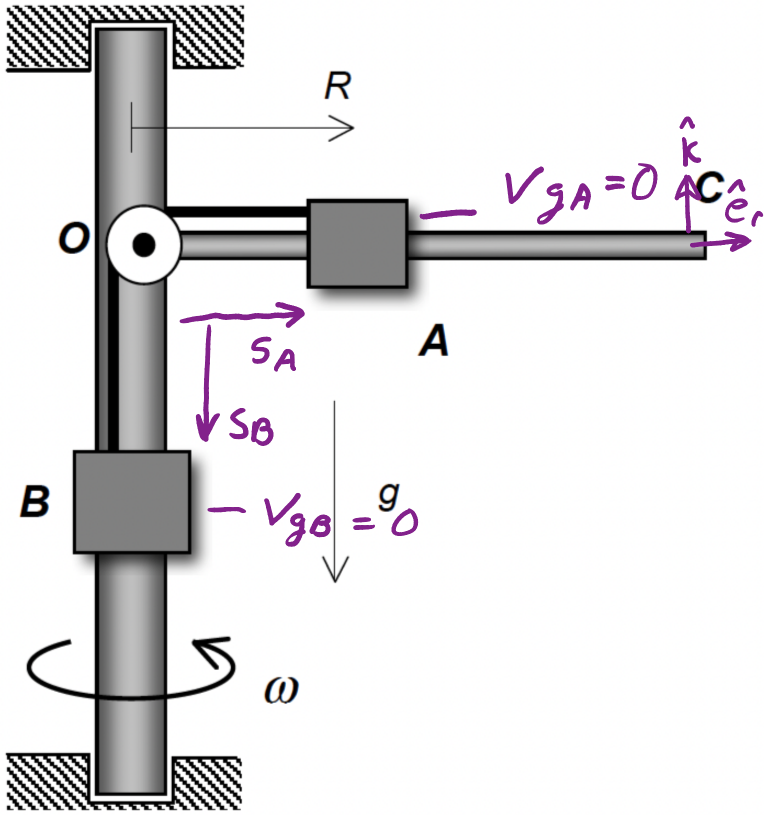
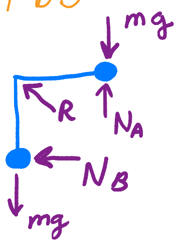



(1) FBD




(2) Kinetics: AIM

$\Sigma M_o = 0$

$(\vec{H}_o)_1 = (\vec{H}_o)_2$

(3)

$(\vec{H}_o)_1 = \vec{r}_1 \times m\vec{v}_1$

$= R_1 \hat{e}_r \times m(R_1 \omega \hat{e}_\theta)$

$= m R_1^2 \omega \hat{k}$

(4)

$(\vec{H}_o)_2 = R_2 \hat{e}_r \times m(\dot{R}_2 \hat{e}_r + R_2 \omega_2 \hat{e}_\theta)$

$= m R_2^2 \omega_2 \hat{k}$

(5) Equate

$\Rightarrow \cancel{m} R_1^2 \omega \cancel{\hat{k}} = \cancel{m} R_2^2 \omega_2 \cancel{\hat{k}}$

$\omega_2 = \left(\frac{R_1}{R_2}\right)^2 \omega$

(3.5) Kinematics

$s_A + s_B = C$

$\dot{s}_A + \dot{s}_B = 0$

$\dot{R}_{2A} + \dot{R}_{2B} = 0$

(6) W/E

$T_1 + V_1 + U^{NC}_{1\to 2} = T_2 + V_2$

$T_1 = \frac{1}{2} m (R_1 \omega)^2$ ;

$V_1 = 0$ ;

$U^{NC}_{1\to 2} = 0$

$T_2 = \frac{1}{2} m (\dot{R}_2^2 + R_2^2 \omega_2^2) + \frac{1}{2} m \dot{R}_2^2$

$V_2 = m g (R_2 - R_1)$

(7) Plug-in

$\frac{1}{2} m R_1^2 \omega^2 = m \dot{R}_2^2 + \frac{1}{2} m R_2^2 \omega_2^2 + mg(R_2 - R_1)$

$\Rightarrow \dot{R}_2$

# *Summary*: Particle Kinetics

[pg. 352]

*WHICH TOOL(s) TO USE*?

Put effort up front deciding on which method(s) to use: *Newton*, *work/energy*, *linear impulse momentum* or *angular impulse momentum*. Use the Kinetics Table in Section 5.D of the lecture book as a guide.

*THE FOUR-STEP PLAN*: Follow it…it is your friend!

*Kinetics Table*

| Method | Body model | Fundamental equations |
|---|---|---|
| ***Newton-Euler*** *(relating forces to accelerations)* | **particle** | $\sum \vec{F} = m\vec{a}$ |
| | **rigid body** *(G = c.m. and A = any point on body)* | $\sum \vec{F} = m\vec{a}_G$<br>$\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A} \times \vec{a}_A$ |
| ***Work-energy*** *(relating change in speed to change in position)* | **particle** | $T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$<br>where $T = \frac{1}{2}mv^2$ |
| | **rigid body** *(G = c.m. and A = any point on body)* | $T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$<br>where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \cdot (\vec{\omega} \times \vec{r}_{G/A})$ |
| ***Linear impulse-momentum*** *(relating change in velocity to change in time)* | **particle** | $\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_2 - m\vec{v}_1$ |
| | **rigid body** *(G = c.m.)* | $\int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{v}_{G2} - m\vec{v}_{G1}$ |
| ***Angular impulse-momentum*** *(relating change in angular velocity to change in time)* | **particle** *(O = fixed point)* | $\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_{O2} - \vec{H}_{O1}$<br>where $\vec{H}_O = m\vec{r}_{P/O} \times \vec{v}_P$ |
| | **rigid body** *(A = fixed point or c.m.)* | $\int_{t_1}^{t_2} \sum \vec{M}_A\, dt = \vec{H}_{A2} - \vec{H}_{A1}$<br>where $\vec{H}_A = I_A\vec{\omega}$ |

Lec 26 Short Feedback Form: