filled

# ME 274 Lecture 25

## Particle Kinetics – Angular Impulse Momentum – Part 2

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# Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 24 (4.Q and 4.~~S~~R) due today!!**
2. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME.
3. Bonus quiz grade at end of the semester if we get a good response rate to QR code surveys at the end of lecture.
    - If you are unable to attend lecture on that day/forget to fill it out:
        - Feel free to give feedback based on the content of that lecture's slides.
        - Way of you reviewing previous content and giving feedback to me.

# Kinetics: Four-step problem solving method

1. **FBDs:**
   - Draw appropriate FBD**(s).**
   - Choose your coordinate system.
2. **Kinetics:**
   - Choose what solution method for the particular problem at hand **(we will go over these in the coming days…)**:
     - Newton/Euler (lectures 15-18) – **Analyzing an instant in time**
     - Work/Energy (lectures 19-20) – **Analyzing speed in terms of position**
     - Linear impulse/momentum (lectures 21-22) - **Analyzing change in velocity during a change in time**
       - Central Impact (lecture 23) - **Analyze velocities between two states**
     - Angular impulse/momentum – (lectures 24-25) – **Analyze Angular Velocity during a change in time**
3. **Kinematics:**
   - Perform needed kinematic analysis (position/velocity/acceleration)
   - Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem
4. **Solve:**
   - Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
     a) Draw more FBDs
     b) You will need to do more kinematic analysis

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# Angular Impulse Momentum (AIM) - Overview

$$\boxed{\int_1^2 \Sigma \vec{M}_O \, dt = \left(\sum_i \left(\vec{H}_O\right)_i\right)_2 - \left(\sum_i \left(\vec{H}_O\right)_i\right)_1}$$

**Where:**

- $\boxed{\left(\vec{H}_O\right)_i = \vec{r}_{i/O} \times (m_i \vec{v}_i)}$ **is angular impulse momentum of particle P about a fixed-point O**

1. AIM equation relates the ***change in time to change in angular momentum***

2. For *central force problems* all forces on the particle pass **through it, M_O = 0.** Therefore, angular momentum **is conserved.**

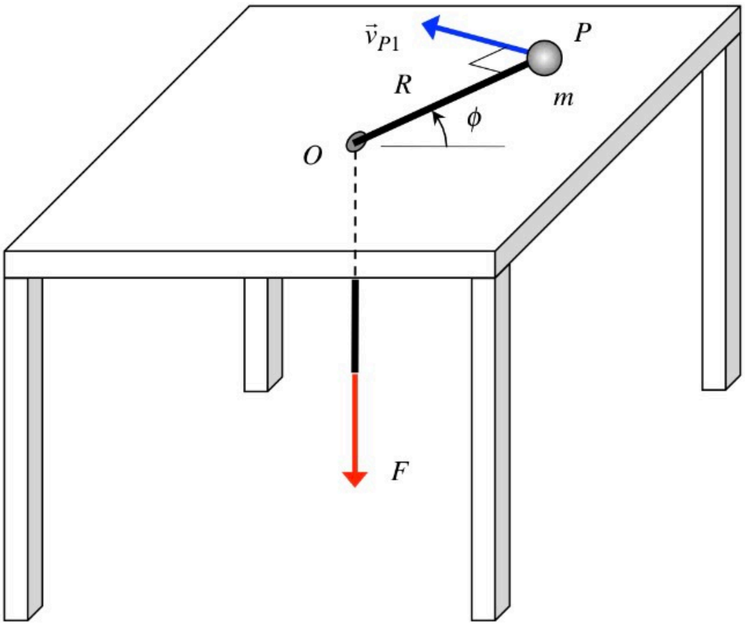
3. Angular momentum gives information of velocity in the **theta component.**
   - If we want the **entire velocity vector** (+ radial component of velocity) we usually use the **Work-Energy Equation** for this.

[pg. 252,254,255 content]

**Homework H4.R**

**Given:** Particle P, having a mass of $m$, is able to slide on the smooth, horizontal top of a table. A flexible cable is attached to P, with the cable being fed through a hole in the table at O. A constant force $F$ acts on the other end of the cable. The system is released with P being at a radial distance $R = R_1$ from O, and with P having a velocity perpendicular to OP with a speed of $v_{P1}$.

**Find:** Determine the numerical values for $\dot{R}$ and $\dot{\phi}$ when P has moved to a position for which $R = R_2$.



Use the following parameters in your analysis: $m = 3$ kg, $R_1 = 1.5$ m, $R_2 = 0.5$ m, $v_{P1} = 8$ m/s and $F = 2000$ N.

Similar to 4.D.3

① ∘ Use cons of Ang mom.

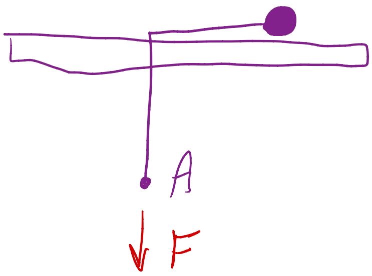
② ∘ W/E Eqn

∘ No change in potential. $V_1 = 0$ ; $V_2 = 0$

∘ Energy is not conserved $\left(U_{1-2}^{NC} \neq 0\right)$ ; Work is done.

$$U_{1-2}^{(F)} = F \Delta R$$

w/ $\Delta R = R_1 - R_2$

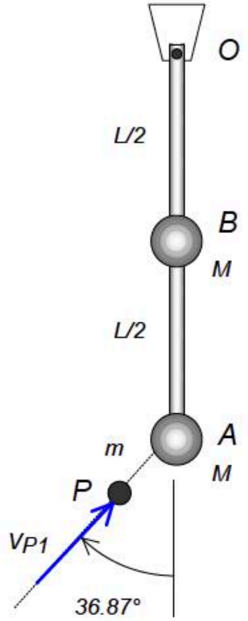


## Example 4.D.5

**Given:** Particles A and B (each having a mass of $M$) are attached to rigid bar OA (this bar has negligible mass). Bar OA is pinned to ground at end O. This system is at rest when A is struck by a bullet P (having a mass of $m$) with the bullet traveling in the direction shown with a speed of $v_{P1}$. Immediately upon impact, the bullet becomes embedded in particle A.

**Find:** Determine the angular velocity of bar OA immediately after the collision is completed.

Use the following parameters in your analysis: $M = 2$ kg, $m = 0.1$ kg, $L = 3$ m and $v_{P1} = 700$ m/s.



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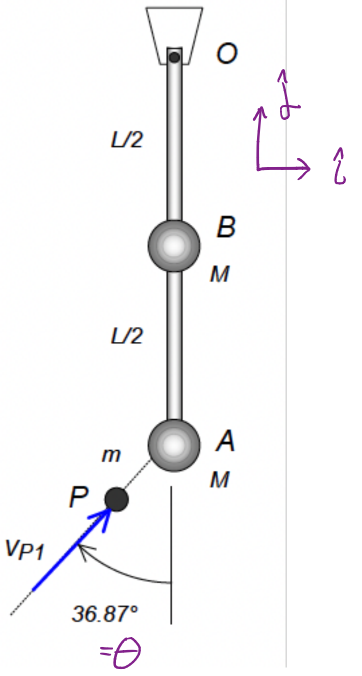
**Example 4.D.5** p.262 Similar to H.4.P

**Given:** Particles A and B (each having a mass of $M$) are attached to rigid bar OA (this bar has negligible mass). Bar OA is pinned to ground at end O. This system is at rest when A is struck by a bullet P (having a mass of $m$) with the bullet traveling in the direction shown with a speed of $v_{P1}$. Immediately upon impact, the bullet becomes embedded in particle A.

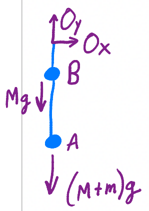
**Find:** Determine the angular velocity of bar OA immediately after the collision is completed. $\vec{W}_2$ ?

↳ Hint to use A.I.M. Eqn

Use the following parameters in your analysis: $M = 2$ kg, $m = 0.1$ kg, $L = 3$ m and $v_{P1} = 700$ m/s.



① FBD A+B+P+bar



② Kinetics

- Since looking for 'w', we need A.I.M. eqn
- Conservation of A.I.M.

$\vec{H}_{o2} = \vec{H}_{o2} + \int \vec{M}_o \, dt$

$\Rightarrow \Sigma M_o = 0$ ; all forces go thru point O

$\Rightarrow (\vec{H}_o)_1 = (\vec{H}_o)_2$

③ Angular Momentum @ 1

$$(\vec{H}_o)_1 = \vec{r}_1 \times m \vec{v}_1$$

$$= \underbrace{M \vec{r}_{A/O} \times \underset{rest}{\cancel{v_{A1}}}}_{A} + \underbrace{M r_{B/O} \times \underset{rest}{\cancel{v_{B1}}}}_{B} + \underbrace{m \, r_{P/O} \times v_{P1}}_{P}$$

$$= 0 + 0 - L\hat{j} \times m \underbrace{(v_{P1}\sin\theta \, \hat{i} + v_{P1}\cos\theta \, \hat{j})}$$

Kinematics Vector projection

$$= m L v_{P1} \sin\theta \, \hat{k} \quad (1)$$

④ Angular Momentum @ 2

$$(\vec{H}_o)_2 = \sum_i \vec{r}_{2i} \times m_i \vec{v}_{2i}$$

$$= M \vec{r}_{A/O} \times v_{A2} + M r_{B/O} \times v_{B2} + m \, r_{P/O} \times v_{P2}$$

Look@ step ④.5

$$= \underbrace{-L\hat{j} \times (m+M)(L\omega_2 \hat{i})}_{A+P} \underbrace{- \tfrac{L}{2}\hat{j} \times M(\tfrac{L}{2}\omega_2 \hat{i})}_{B}$$

$$= (m+M)L^2\omega_2 \hat{k} + M\frac{L^2}{4}\omega_2 \hat{k} \quad (2)$$

④.5 Kinematics Since point O is the IC of the bar...

$v_{A2} = L\omega_2$ $\quad v_{B2} = \frac{L}{2}\omega_2$ $\quad v_{P2} = v_{A2} = L\omega_2$

⑤ Equate (1) & (2)

$$\Longrightarrow m L v_{P1} \sin\theta \, \cancel{\hat{k}} = (m+M)L^2\omega_2 \cancel{\hat{k}} + \tfrac{1}{4} M L^2 \omega_2 \cancel{\hat{k}}$$

⑥ Solve for angular speed $\omega_2$

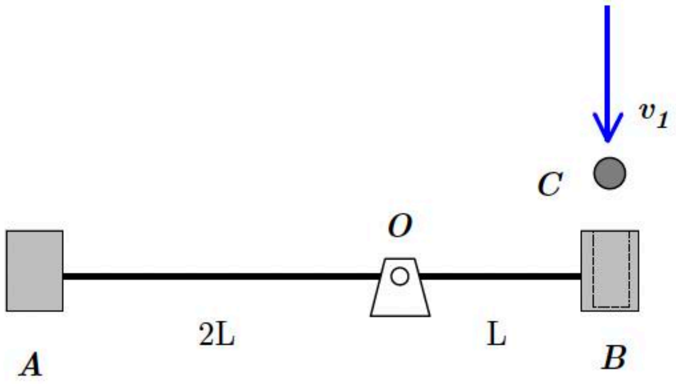
⑦ Express velocity in vector form

$$\Rightarrow \vec{\omega}_2 = \omega_2 \hat{k} = \left[\frac{m L V_{P1} \sin\theta}{ML^2 + \frac{1}{4}ML^2 + mL^2}\right]\hat{k}$$

## Example 4.D.6

**Given:** Particles A and B, of masses $4m$ and $2m$, respectively, are attached to the ends a stationary rigid rod of negligible mass. The rod is pinned to ground at O. A third particle C, of mass $m$, strikes particle B with a speed of $v_1$. On impact, C sticks to B. The system lies in the horizontal plane.

**Find:** Determine the angular speed of the bar immediately after the impact occurs.



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### Example 4.D.6

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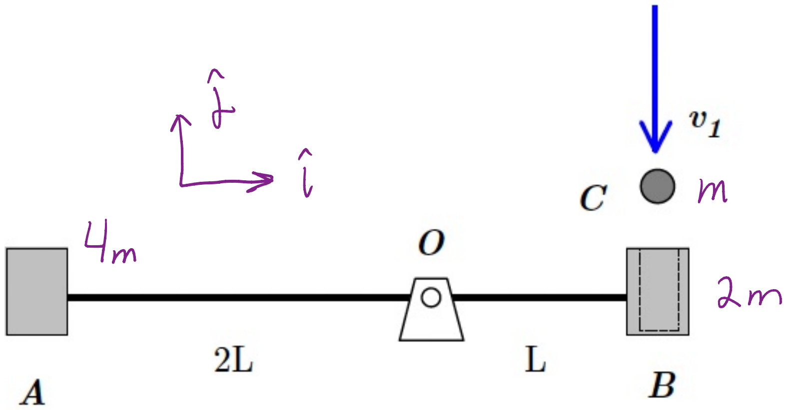
**Find:** Determine the angular speed of the bar immediately after the impact occurs.

$\omega_2$?

① FBD



Horizontal Plane

② Kinetics

A.I.M. Eqn.

Ang Mom is conserved.

$$\vec{H}_{O2} = \vec{H}_{O1} + \int_1^2 \vec{M}_O \, dt$$

$$\Rightarrow \Sigma M_O = 0$$

$$\Rightarrow (\vec{H}_O)_1 = (\vec{H}_O)_2$$

③ Angular Momentum @ 1

$$(\vec{H}_O)_1 = \sum_i \vec{r}_{i1} \times m_i \vec{v}_{i1}$$

$$= \cancel{\overbrace{-2L\hat{i} \times 4m(\vec{0})}^{A}} + \cancel{\overbrace{L\hat{i} \times 2m(\vec{0})}^{B}} + \overbrace{L\hat{i} \times m(-v_1\hat{j})}^{C}$$

$$= -mLv_1\hat{k}$$

④ Angular Momentum @ 2

$$(\vec{H}_O)_2 = \sum_i \vec{r}_{i2} \times m_i \vec{v}_{i2}$$

$$= \overbrace{-2L\hat{i} \times 4m(-L\omega_2\hat{j})}^{A} + \overbrace{L\hat{i} \times 2m(L\omega_2\hat{j})}^{B} + \overbrace{L\hat{i} \times m(L\omega_2\hat{j})}^{C}$$

$$= 8mL^2\omega_2\hat{k} + 2mL^2\omega_2\hat{k} + mL^2\omega_2\hat{k}$$

$$= 11mL^2\omega_2\hat{k}$$

⑤ Solve

Equate (1)&(2) Solve for $\omega_2$

$$-\cancel{m}\cancel{L}v_1\cancel{\hat{k}} = 11\cancel{m}L^{\cancel{2}}\omega_2\cancel{\hat{k}}$$

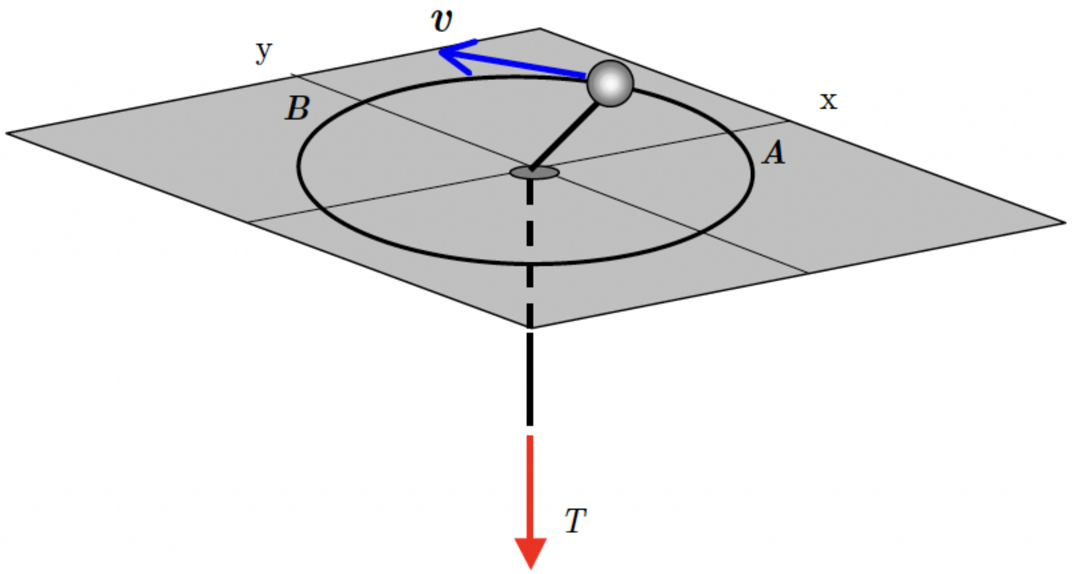
$$\Rightarrow \omega_2 = \frac{-v_1}{11L}$$

**Example 4.D.7**

**Given:** A particle of mass $m$ is attached to a cord with the particle being able to slide on a smooth horizontal table top. This cord is pulled through a hole in the table top at O with a force of $T$ being applied to the free end of the cord. The path of the particle is known to follow a path given by: $x^2/9 + y^2/36 = 1$. The speed of the particle at position A known to be $v_A$.

**Find:** Determine the force $T$ when the particle is at position B.

Use the following parameters in your analysis: $m = 2\text{kg}$ and $v_A = 10$ m/s.



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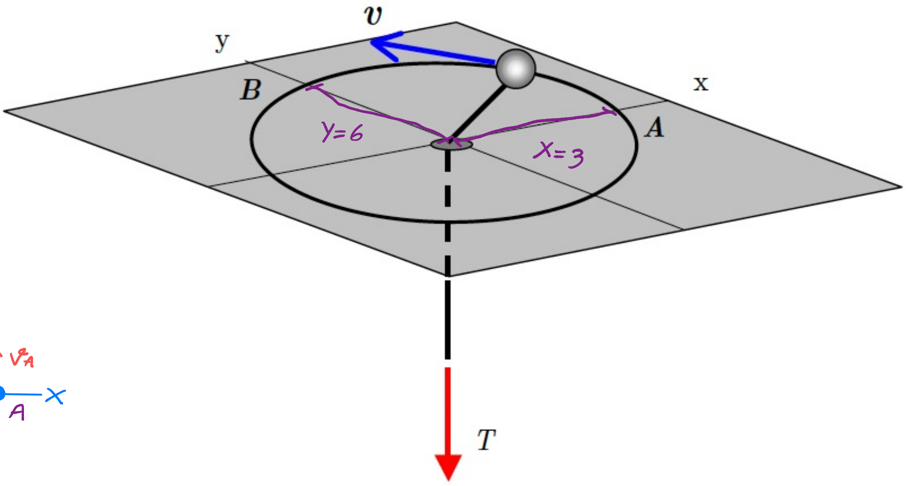
**Example 4.D.7**

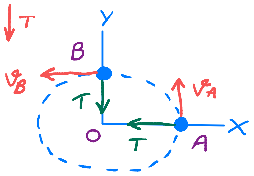
**Given:** A particle of mass $m$ is attached to a cord with the particle being able to slide on a smooth horizontal table top. This cord is pulled through a hole in the table top at O with a force of $T$ being applied to the free end of the cord. The path of the particle is known to follow a path given by: $x^2/9 + y^2/36 = 1$. The speed of the particle at position A known to be $v_A$.

**Find:** Determine the force $T$ when the particle is at position B. $T$?

Use the following parameters in your analysis: $m = 2$kg and $v_A = 10$ m/s.



① FBD



② Kinetics AIM

$\vec{H}_{o2} = \vec{H}_{o1} + \int_1^2 \vec{M}_o \, dt$

$\Rightarrow \Sigma M_o = 0$

$\Rightarrow (\vec{H}_o)_1 = (\vec{H}_o)_2$

③ Angular Momentum @ A

$(\vec{H}_o)_A = m \, \vec{r}_{A/o} \times \vec{v}_A$

$= m \, (3\hat{i}) \times (v_A \hat{j})$

$= 3 m v_A \hat{k}$ (1)

④ Angular Momentum @ B

$(\vec{H}_o)_B = m \, \vec{r}_{B/o} \times \vec{v}_B$

$= m \, (6\hat{j}) \times (-v_B \hat{i})$

$= 6 m v_B \hat{k}$ (2)

⑤ Cons of Ang Mom. Equate (1) & (2).

$(\vec{H}_o)_A = (\vec{H}_o)_B$

$\Rightarrow 3 m v_A = 6 m v_B$

$\Rightarrow v_B = \frac{1}{2} v_A$

⑥ Need to find Tension. Use Newton.

@ B: $\Sigma F_x = 0 = m\ddot{x}$ $\Rightarrow \ddot{x} = 0$

$\Sigma F_y = -T = m\ddot{y}$

⑦ Kinematics Use shape of path to get $\ddot{x}$ & $\ddot{y}$

$$\frac{x^2}{9} + \frac{y^2}{36} = 1$$

$$\Rightarrow 4 \frac{\cancel{2} x\dot{x}}{\cancel{9}} + \frac{\cancel{2} y\dot{y}}{\cancel{36}} = 0$$ ; implicit differentiation

$\Rightarrow \dot{y} = 0$ ; bc $(x=0)$ @ B

$$\Rightarrow 4[\dot{x}^2 + x\overset{0}{\cancel{\ddot{x}}}] + \overset{0}{\cancel{\dot{y}}}^2 + y\ddot{y} = 0$$ ; implicit differentiation again

⑧ Solve for $\ddot{y}$

$$\ddot{y} = -\frac{4\dot{x}^2}{y}$$

$$= -\frac{(4)\, v_B^2}{6}$$

$$= -\frac{2}{3}\left(\frac{v_A}{2}\right)^2$$

$$= -\frac{v_A^2}{6}$$

⑨ Use $\Sigma F_y$ to solve for $T$

$T = -m\ddot{y}$

$= \frac{m v_A^2}{6} \Leftarrow$ @ B

# *Summary*: Angular impulse/momentum equation 2

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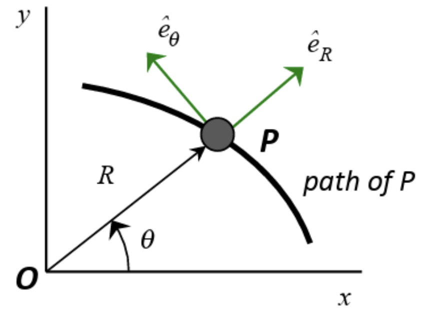
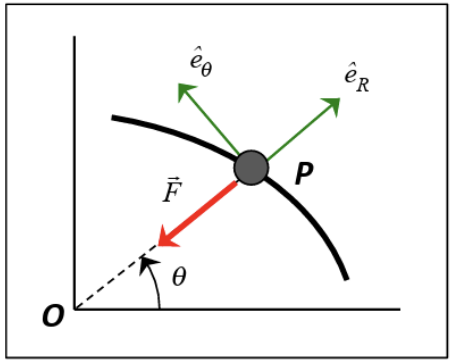
*FUNDAMENTAL* equation:

$$\left(\vec{H}_O\right)_2 = \left(\vec{H}_O\right)_1 + \int_1^2 \sum \vec{M}_O \, dt$$

where O is a FIXED point.



*WHEN* should I use this equation? Think *central-force problems*… When $\sum \vec{M}_O = \vec{0}$ , angular momentum about O is conserved.



***central force problem:***
***force F acts directly***
***toward point O***

*IMPORTANT*: This equation can NOT give information on the *radial* component of velocity for the particle. Why? Why is this important?

Look at the above equation for computing angular momentum. Typically, use work/energy for the additional equation.

Lec 25 Short Feedback Form: