Filled

ME 274 Lecture 24

Particle Kinetics – Angular Impulse Momentum – Part 1

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03/11/26

Today:

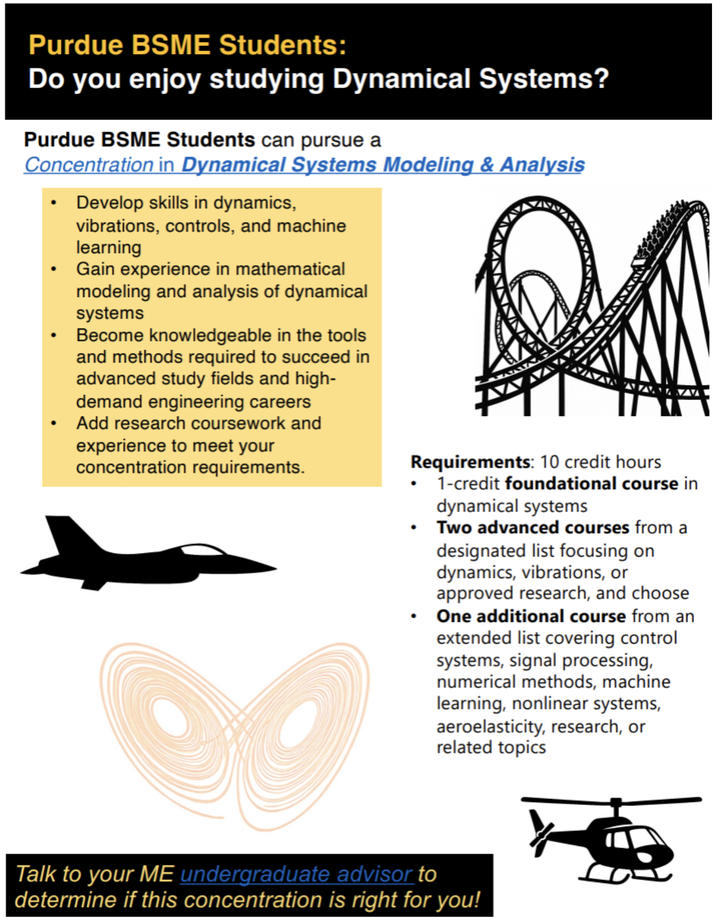
4D1

4D3

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 23 (4.O and 4.P) due today!!**
2. Office hours are changing to ME2008B…
   - Second floor of renovated side of ME.
3. Bonus quiz grade at end of the semester if we get a good response rate to QR code surveys at the end of lecture.
   - If you are unable to attend lecture on that day/forget to fill it out:
     - Feel free to give feedback based on the content of that lecture's slides.
     - Way of you reviewing previous content and giving feedback to me.
4. Ugrad research and Dynamical Systems Concentration
5. Bonus points if you attend this Friday's lecture (the lecture before spring break)



Kinetics: Four-step problem solving method

1. **FBDs:**
   - Draw appropriate FBD**(s).**
   - Choose your coordinate system.
2. **Kinetics:**
   - Choose what solution method for the particular problem at hand **(we will go over these in the coming days…)**:
     - Newton/Euler (lectures 15-18) – **Analyzing an instant in time**
     - Work/Energy (lectures 19-20) – **Analyzing speed in terms of position**
     - Linear impulse/momentum (lectures 21-22) - **Analyzing change in velocity during a change in time**
       - Central Impact (lecture 23) - **Analyze velocities between two states**
     - Angular impulse/momentum – (lectures 24-25) – **Analyze Angular Velocity during a change in time**
3. **Kinematics:**
   - Perform needed kinematic analysis (position/velocity/acceleration)
   - Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem
4. **Solve:**
   - Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
     a) Draw more FBDs
     b) You will need to do more kinematic analysis

[pg. 184 content]

Angular Impulse Momentum - Introduction

Objectives of this section/lecture:

- Develop the Angular Impulse-Momentum (AIM) equation for analyzing ***change in angular velocity during a change in time***
- Extend this idea for ***systems of particles***
- Concept of ***conservation of angular momentum***

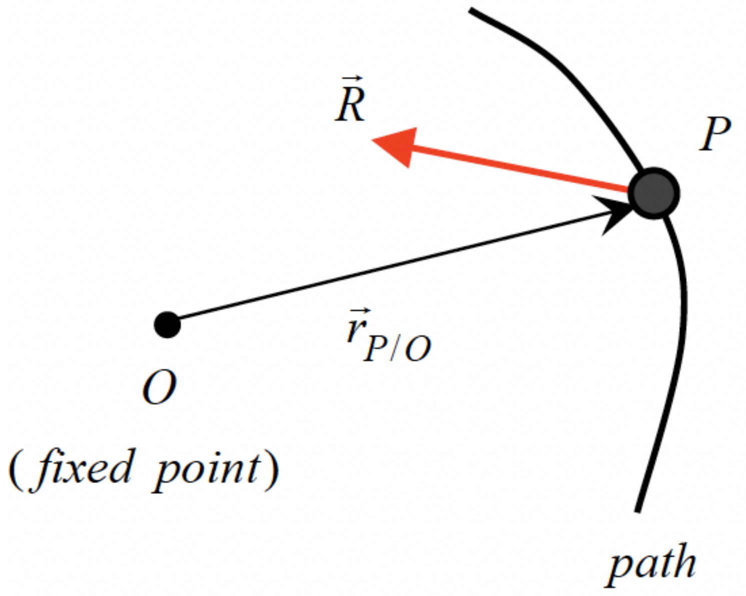
$$\boxed{\begin{aligned}\int_1^2 \vec{M}_o\, dt &= \int_1^2 d\vec{H}_o \\ &= \vec{H}_{o2} - \vec{H}_{o1}\end{aligned}}$$

Where:

- $\vec{H}_O = \vec{r}_{P/O} \times m\vec{v}_P$ **is angular impulse momentum of particle P about a fixed-point O**

[pg. 249 content]

Angular Impulse Momentum - Derivation



1. Newton's Second Law

$$\vec{R} = \sum \vec{F} = m\vec{a}$$; Resultant force

2. Moment Equation - manipulation

$$\vec{M}_O = \vec{r}_{P/O} \times \vec{R}$$; $M = d \cdot F$

$$\vec{M}_O = \vec{r}_{P/O} \times \left(m\frac{d\vec{v}_P}{dt}\right)$$; $F = ma$

$$\vec{a} \times \frac{d\vec{b}}{dt} = \frac{d}{dt}(\vec{a} \times \vec{b}) - \frac{d\vec{a}}{dt} \times \vec{b}$$; prod rule / cross product "magic" / manipulation

$$\vec{M}_O = \frac{d}{dt}[\vec{r}_{P/O} \times m\vec{v}] - \frac{d\vec{r}_{P/O}}{dt} \times (m\vec{v})$$; $\frac{dr_{P/O}}{dt} = \vec{v}$ bc O is a fixed pt

$$= \frac{d}{dt}[\vec{r}_{P/O} \times m\vec{v}] - \cancel{\vec{v} \times (m\vec{v})}^{\,0}$$; $\vec{v} \times \vec{v} = 0$

$$\vec{M}_o = \frac{d}{dt}[\vec{r}_{P/O} \times m\vec{v}]$$ ⟵ A.L.M. eqn

$$= \frac{d\vec{H}_O}{dt}$$ ⟶ separation of vars ⟶

$$\int_1^2 \vec{M}_o \, dt = \int_1^2 d\vec{H}_o = \vec{H}_{o2} - \vec{H}_{o1}$$

[pg. 250-251 content]

Angular Impulse Momentum - Discussion

$$\boxed{\int_1^2 \vec{M}_O \, dt = \int_1^2 d\vec{H}_O = \vec{H}_{O2} - \vec{H}_{O1}}$$

1. AIM equation relates the ***change in*** time ***to change in*** angular momentum

2. For *central force problems* all forces on the particle pass through it, $\vec{M}_o = 0$
Therefore, angular momentum is conserved

3. Angular momentum **can** be written in terms of polar coordinates $\vec{H}_O = m\left(\underline{r^2\dot{\theta}}\right)\hat{k}$

4. When Angular momentum is conserved, we have: $\vec{H}_{O2} = \vec{H}_{O1} \quad \Rightarrow \quad \dot{\theta}_2 = \frac{r_1^2}{r_2^2}\dot{\theta}_1$

- Angular velocity must decrease as radial distance, r, increases; in order for angular momentum to be conserved. Or vice versa.

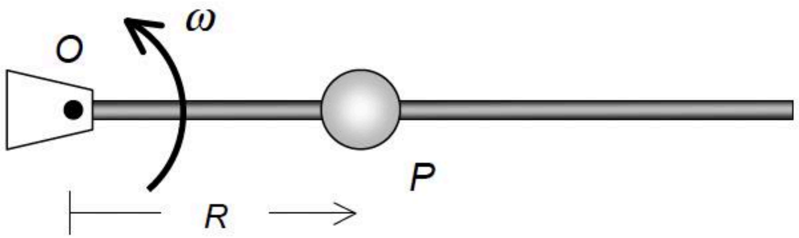
5. Angular momentum gives information of velocity in the θ component
- If we want the entire velocity vector, $\vec{v}$ (+ radial component of velocity) we usually use the Work-Energy Eqn for this. <u>*We will see this in example 4.D.3 today.*</u>

[pg. 252,254,255 content]

Example 4.D.1

Given: Particle P (weighing 2 lb) is able to slide on a smooth, lightweight horizontal arm that is rotating about a vertical axis. Initially, P is stationary relative to the arm when the arm is rotating at a rate of $\omega_1 = 20$ rad/s and P is $R = 3$ in from the rotation axis of the arm.

Find: Determine the angular speed of the arm when the P has moved outward to a position of $R = 24$ in from the axis of the shaft.



HORIZONAL PLANE

[pg. 258]

p. 258

Example 4.D.1

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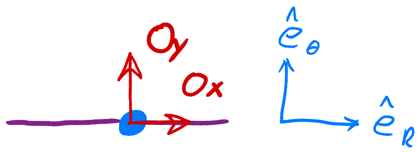
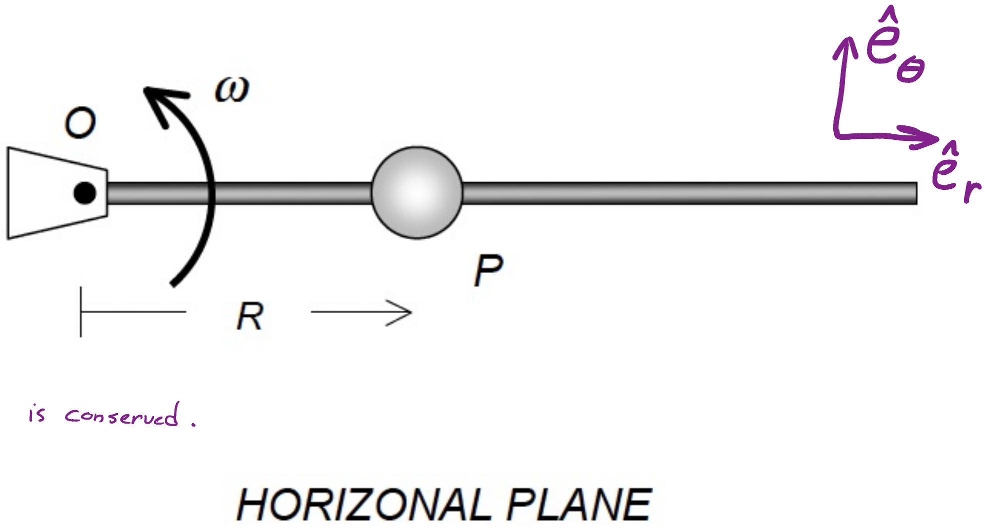
Find: Determine the angular speed of the arm when the P has moved outward to a position of $R = 24$ in from the axis of the shaft. ω_2?

① FBD Particle & Rod





② Kinetics Angular Momentum about O is conserved.

Define coordinates $\quad \vec{H}_o = \text{const}$ or

$$\Sigma M_o = 0 \Rightarrow (\vec{H}_o)_1 = (\vec{H}_o)_2$$

③ $\vec{H}_o$ @ state 1

$$(\vec{H}_o)_1 = \vec{r}_1 \times m\vec{v}_1$$

$$= R_1 \hat{e}_R \times \frac{W}{g}\underbrace{(\dot{R}_1 \hat{e}_R + R_1 \omega_1 \hat{e}_\theta)}_{\text{polar eqn}}$$

$$= \frac{W}{g} R_1^2 \omega_1 \hat{k} \quad (1)$$

②.⑤ Aside: Kinematics

$$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

$$m = \frac{W}{g}$$

$$\hat{e}_R \times \hat{e}_\theta = \hat{k}$$

$$\hat{e}_R \times \hat{e}_R = 0$$

④ $\vec{H}_o$ @ state 2

$$(\vec{H}_o)_2 = \vec{r}_2 \times m\vec{v}_2$$

$$= R_2 \hat{e}_R \times \frac{W}{g}\underbrace{(\dot{R}_2 \hat{e}_R + R_2 \omega_2 \hat{e}_\theta)}_{\text{polar eqn}}$$

$$= \frac{W}{g} R_2^2 \omega_2 \hat{k} \quad (2)$$

⑤ Solve Equate (1) & (2). Solve for angular spd ω_2

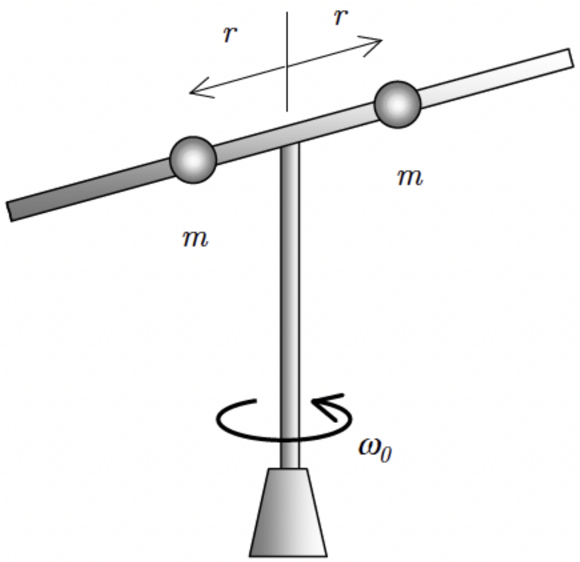
$$\Rightarrow \cancel{\frac{W}{g}} R_1^2 \omega_1 \cancel{\hat{k}} = \cancel{\frac{W}{g}} R_2^2 \omega_2 \cancel{\hat{k}}$$

$$\omega_2 = \left(\frac{R_1}{R_2}\right)^2 \omega_1$$

Example 4.D.2

Given: The assembly shown is rotating with a speed of ω_0 with two particles held fixed relative to the rotating arm at a distance of r from the axis of rotation. The particles are then released and slide outward on the arm to a distance of $2r$.

Find: Determine the angular speed of the arm at this new position.



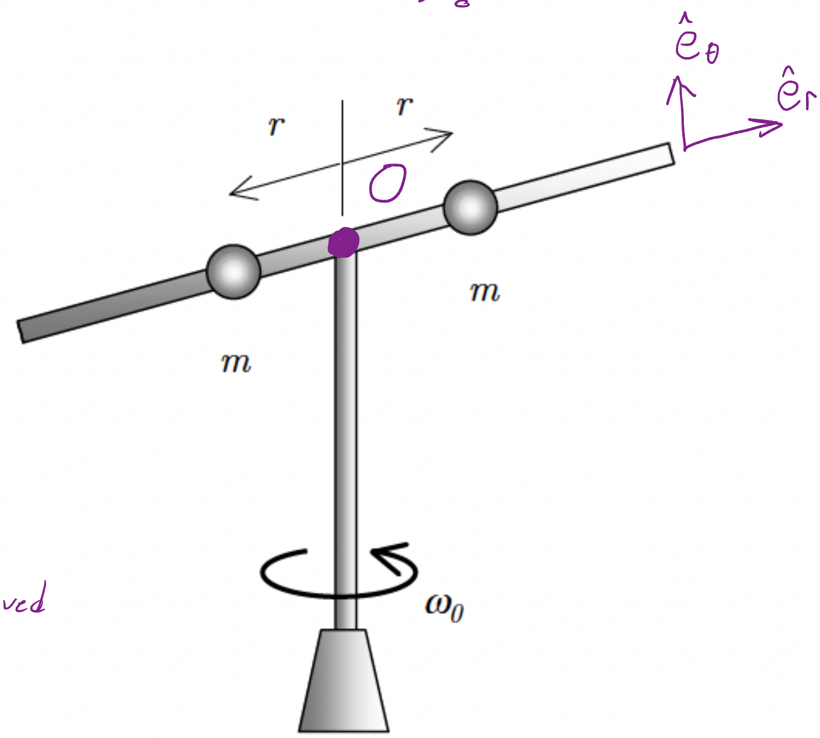
[pg. 259]

p. 259

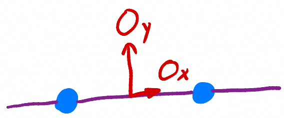
Example 4.D.2

Given: The assembly shown is rotating with a speed of ω_0 with two particles held fixed relative to the rotating arm at a distance of r from the axis of rotation. The particles are then released and slide outward on the arm to a distance of $2r$.

Find: Determine the angular speed of the arm at this new position. ω_2?



① FBD Particles as one system. Define coords.



② Kinetics Angular Momentum is conserved

$\Sigma M_o = 0 \Rightarrow (\vec{H}_o)_1 = (\vec{H}_o)_2$

③ $\vec{H}_o$ @ state 1

$$(\vec{H}_o)_1 = \Sigma\, \vec{r}_{i1} \times m_i \vec{v}_{i1}$$

$$= \left[r\hat{e}_r \times m\,\underbrace{(r\omega_0\hat{e}_\theta)}_{\text{polar eqn}}\right] - \left[r\hat{e}_r \times m\,\underbrace{(-r\omega_0\hat{e}_\theta)}_{\text{polar eqn}}\right]$$

$$= 2mr^2\omega_0\hat{k} \quad (1)$$

2.5 Aside: Kinematics

$$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

④ $\vec{H}_o$ @ state 2

$$(\vec{H}_o)_2 = \Sigma\, \vec{r}_{i2} \times m_i \vec{v}_{i2}$$

$$= \left[2r\hat{e}_r \times m\,\underbrace{(2r\omega_2\hat{e}_\theta)}_{\text{polar eqn}}\right] - \left[2r\hat{e}_r \times m\,\underbrace{(-2r\omega_2\hat{e}_\theta)}_{\text{polar eqn}}\right]$$

$$= 8mr^2\omega_2\hat{k} \quad (2)$$

⑤ Solve Equate (1) & (2). Solve for angular spd ω_2

$$\Rightarrow 2\cancel{mr^2}\,\omega_0\cancel{\hat{k}} = 8\cancel{mr^2}\,\omega_2\cancel{\hat{k}}$$

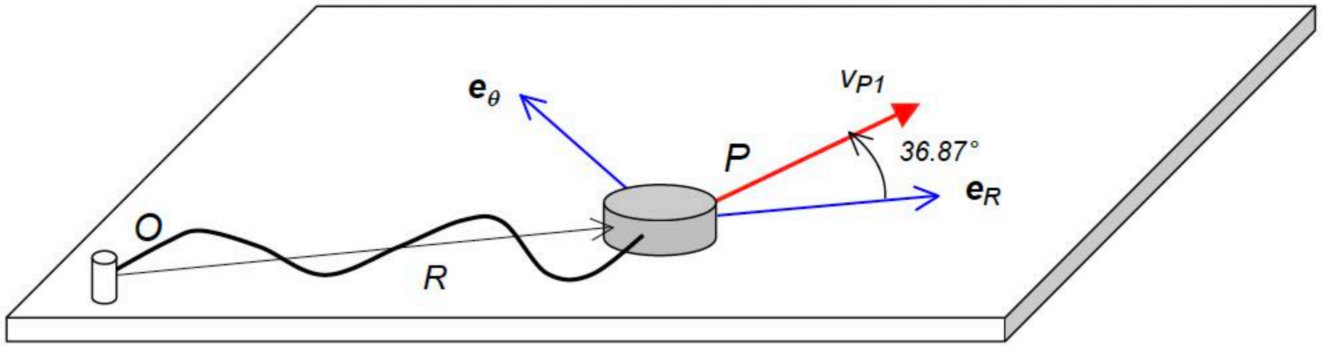
$$\omega_2 = \frac{2}{8}\omega_0$$

$$= \frac{1}{4}\omega_0$$

Example 4.D.3

Given: Disk P having a mass of 0.2 kg is able to slide on a smooth, horizontal surface. A rubber band (having a stiffness of $k = 10$ N/m and unstretched length of 0.6 m) attaches B to a fixed peg at O. Disk P is set into motion with a speed of $v_{P1} = 15$ m/s in the direction shown with $R = 0.4$ m.

Find: Determine the polar coordinates of the velocity of P when P is a distance 1.5 m from O.



[pg. 260]

p.260

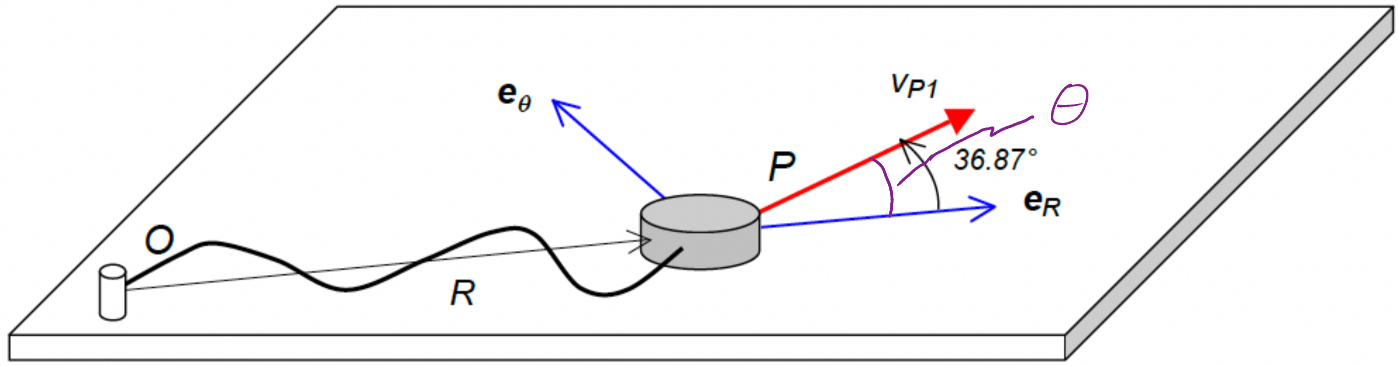
• Same as 3/11/26 animation

Example 4.D.3

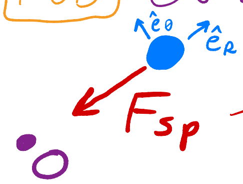
• H.4.R

Given: Disk P having a mass of 0.2 kg is able to slide on a smooth, horizontal surface. A rubber band (having a stiffness of $k = 10$ N/m and unstretched length of 0.6 m) attaches B to a fixed peg at O. Disk P is set into motion with a speed of $v_{P1} = 15$ m/s in the direction shown with $R = 0.4$ m.

Find: Determine the polar coordinates of the velocity of P when P is a distance 1.5 m from O. $\vec{v}_2$?



① FBD Of system in top view



conservative force

② Kinetics Conservation of Angular Momentum

$\Sigma M_O = 0 \Rightarrow (\vec{H}_O)_1 = (\vec{H}_O)_2$

↳ i.e. H_O = const.

③ $\vec{H}_O$ @ state 1

$(\vec{H}_O)_1 = \vec{r}_1 \times m\vec{v}_1$

$= R_1 \hat{e}_R \times m\,(\underbrace{v_{P1}\cos\theta\,\hat{e}_R + v_{P1}\sin\theta\,\hat{e}_\theta}_{\text{projection}})$

$= m R_1 v_{P1} \sin\theta\,\hat{k}$ (1)

④ $\vec{H}_O$ @ state 2

$(\vec{H}_O)_2 = \vec{r}_2 \times m\vec{v}_2$

$= R_2 \hat{e}_R \times m\,(\underbrace{v_{2r}\hat{e}_R + v_{2\theta}\hat{e}_\theta}_{\text{projection}})$

$= m R_2 v_{2\theta}\,\hat{k}$ (2)

⑤ Equate (1) & (2). Get velocity in θ-dir

$\Rightarrow \cancel{m} R_1 v_{P1} \sin\theta\,\cancel{\hat{k}} = \cancel{m} R_2 v_{2\theta}\,\cancel{\hat{k}}$

$v_{2\theta} = \frac{R_1}{R_2} v_{P1} \sin\theta$

5.5 Look @ the find... we need r-dir too... for $\vec{v}$

⑥ Bc we looking at change in position... W/E Eqn. To get $\vec{v}$ in r-dir

$T_1 + V_1 + U^{NC}_{1\to2} = T_2 + V_2$

$T_1 = \frac{1}{2} m v_{P1}^2$;

$V_1 = 0$; elastic band. No elastic potential @ state 1

$U^{NC}_{1\to2} = 0$; no non-conservative forces. Mech Energy Cons.

$T_2 = \frac{1}{2} m\,(v_{2r}^2 + v_{2\theta}^2)$

$V_2 = \frac{1}{2} k (R_2 - R_0)^2$; $\frac{1}{2}\cdot k\cdot$ stretch in spring

$\Rightarrow \frac{1}{2} m v_{P1}^2 = \frac{1}{2} m v_{2r}^2 + \frac{1}{2} m v_{2\theta}^2 + \frac{1}{2} k (R_2 - R_0)^2$

⑦ Solve for v_{2r}

$\Rightarrow v_{2r}$

⑧ Get velocity vector

$\Rightarrow \vec{v}_2 = v_{2r}\hat{e}_r + v_{2\theta}\hat{e}_\theta$

Summary: Angular Impulse/Momentum Equation 1

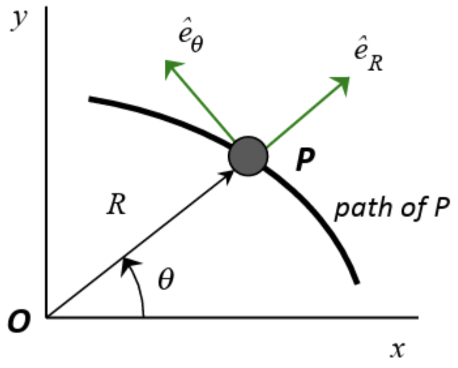
[pg. 255]

FUNDAMENTAL equation:

$$\left(\vec{H}_O\right)_2 = \left(\vec{H}_O\right)_1 + \int_1^2 \sum \vec{M}_O \, dt$$

COMPUTING angular momentum:

$$\begin{aligned} \vec{H}_O &= m\vec{r}_{P/O} \times \vec{v}_P && ;\ \textit{general} \\ &= m\left(x\hat{i} + y\hat{j}\right) \times \left(\dot{x}\hat{i} + \dot{y}\hat{j}\right) = m\left(x\dot{y} - y\dot{x}\right)\hat{k} && ;\ \textit{Cartesian} \\ &= m\left(r\hat{e}_r\right) \times \left(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta\right) = mr^2\dot{\theta}\hat{k} && ;\ \textit{polar} \end{aligned}$$



NOTE: O must be a <u>FIXED</u> point! Why?

Lec 24 Short Feedback Form: