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ME 274 Lecture 23

Particle Kinetics – Central Impact

Eugenio “Henny” Frias-Miranda

03/09/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 22 (4.M and 4.N) due today!!**
2. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME.
3. Exam 2 equation sheet on course website

ME 274 – Exam No. 2 **Name** *(print)*________________________________

April 2, 2026 *(Last)* *(First)*

EQUATIONS

$$\begin{aligned}\vec{v} &= \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}\\ &= v\hat{e}_t\\ &= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta\end{aligned}$$

$$\begin{aligned}\vec{a} &= \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}\\ &= \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n\\ &= \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta\end{aligned}$$

$$\begin{aligned}\vec{v}_B &= \vec{v}_A + \vec{v}_{B/A}\\ &= \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}\\ &= \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}\end{aligned}$$

$$\begin{aligned}\vec{a}_B &= \vec{a}_A + \vec{a}_{B/A}\\ &= \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)\\ &= \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)\end{aligned}$$

$$\sum\vec{F} = m\vec{a}$$

$$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$$

$$\begin{aligned}V_{gr} &= mgh\\ V_{sp} &= \frac{1}{2}k\Delta^2\\ T &= \frac{1}{2}mv^2\\ U_{1\to2}^{(nc)} &= \int_1^2 \left(\vec{F} \cdot \hat{e}_t\right)ds\end{aligned}$$

$$\int_1^2 \sum\vec{F}_{ext}\,dt = \left(\sum_i m_i\vec{v}_i\right)_2 - \left(\sum_i m_i\vec{v}_i\right)_1$$

$$\int_1^2 \sum\vec{M}_O\,dt = \left(\sum_i \left(\vec{H}_O\right)_i\right)_2 - \left(\sum_i \left(\vec{H}_O\right)_i\right)_1$$

$$\left(\vec{H}_O\right)_i = \vec{r}_{i/O} \times \left(m_i\vec{v}_i\right)$$

Kinetics: Four-step problem solving method

1. **FBDs:**
   - Draw appropriate FBD**(s).**
   - Choose your coordinate system.
2. **Kinetics:**
   - Choose what solution method for the particular problem at hand **(we will go over these in the coming days…)**:
     - Newton/Euler (lectures 15-18) – **Analyzing an instant in time**
     - Work/Energy (lectures 19-20) – **Analyzing speed in terms of position**
     - Linear impulse/momentum (lectures 21-22) - **Analyzing change in velocity during a change in time**
       - **Central Impact (lecture 23)**
     - Angular impulse/momentum – (lectures 24-25) -
3. **Kinematics:**
   - Perform needed kinematic analysis (position/velocity/acceleration)
   - Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem
4. **Solve:**
   - Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
     a) Draw more FBDs
     b) You will need to do more kinematic analysis

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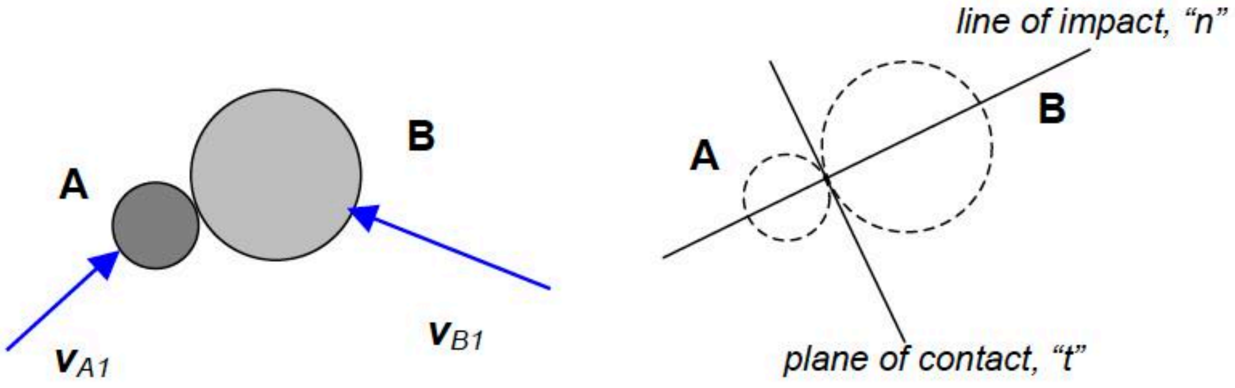
Central Impact of a Pair of Particles

- **Why/Motivation?** Helps us relate/find Velocities between two states. (Pre and Post collision)
- **Condition to use this concept:** *The line of impact passes through the centers of mass of each body involved in the impact. We are given a Coefficient of Restitution/'e' for the problem.*
- **Relevant/important definitions:**

1. Plane of contact A plane that is tangent to the contact surfaces of A and B. Denoted as 't'
2. Line of impact A line that is perpendicular to the plane of contact for A and B. Denoted as 'n'.
3. Central Impact An impact in which the line of impact passes through the centers of mass of each body involved in the impact.



(a) central impact (b) coordinate axes

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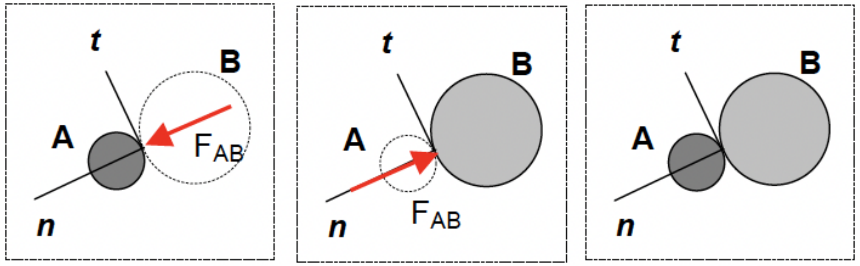
Central Impact Problems – How to/general outline

1. FBDs. Usually 3:



system A *system B* *system AB*

2. System A and System B, separately:

(1) $\sum F_t = 0 \quad \Rightarrow \quad m_A v_{At2} = m_A v_{At1} \quad \Rightarrow \quad \boxed{M_{At2} = v_{At1}}$

$\sum F_n = F_{AB} \neq 0 \quad \Rightarrow \quad$ momentum NOT conserved in "n" direction for A

$\sum F_t = 0 \quad \Rightarrow \quad m_B v_{Bt2} = m_B v_{Bt1} \quad \Rightarrow \quad \boxed{v_{Bt2} = v_{Bt1}}$

$\sum F_n = -F_{AB} \neq 0 \quad \Rightarrow \quad$ momentum NOT conserved in "n" direction for B

3. System A and B together

$\sum F_t = 0 \quad \Rightarrow \quad m_A v_{At2} + m_B v_{Bt2} = m_A v_{At1} + m_B v_{Bt1}$

Can't use. Same as eqns (1)+(2). Not an independent eqn

(3) $\sum F_n = 0 \quad \Rightarrow \quad \boxed{m_A v_{An2} + m_B v_{Bn2} = m_A v_{An1} + m_B v_{Bn1}}$

4. Coefficient of Restitution:

(4) $\boxed{e = \dfrac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}}$

Obtained empirically/experimentally.
Class demo...

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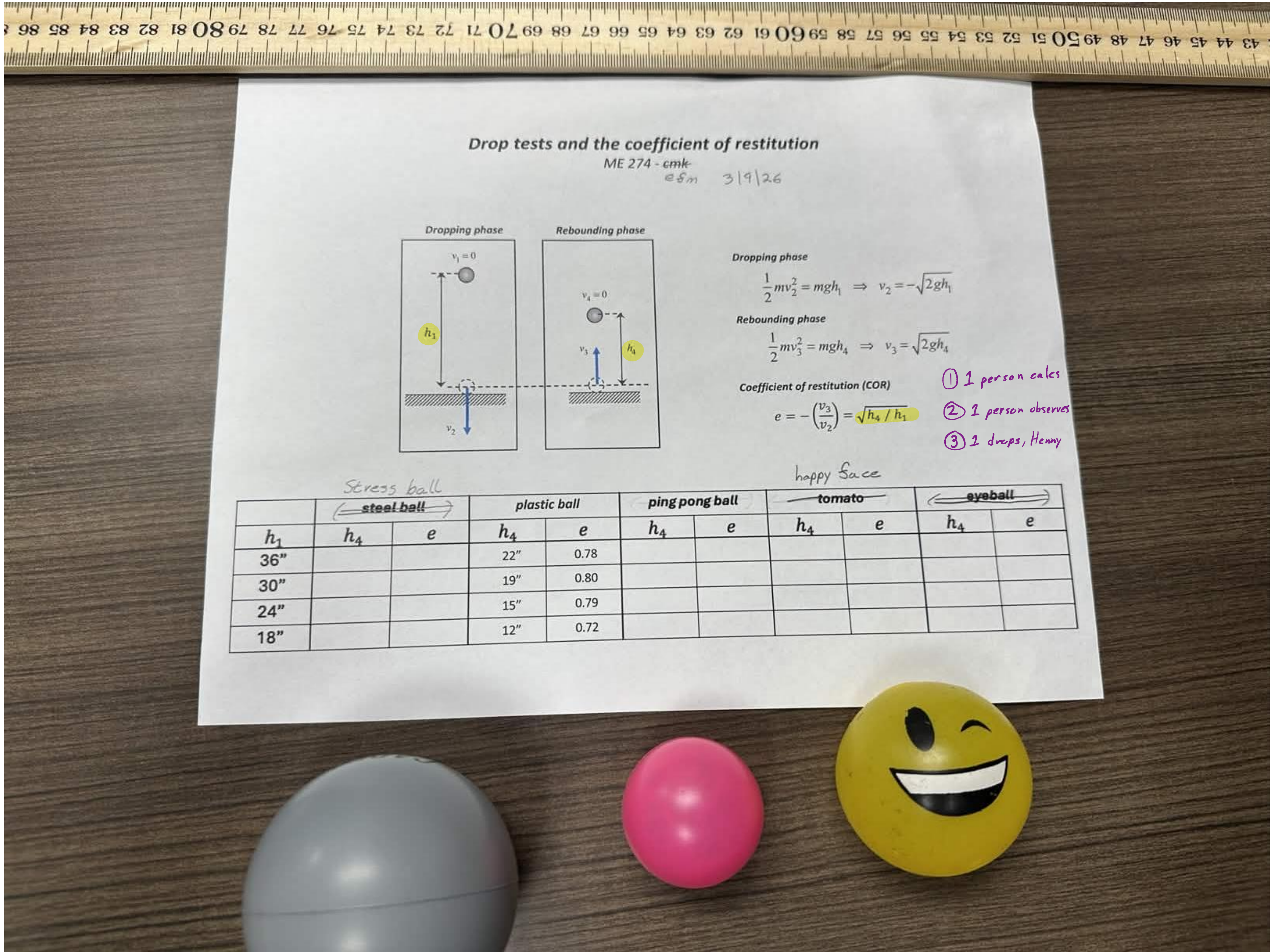
In class demo – calculating Coefficient Of Restitution (COR)



Drop tests and the coefficient of restitution

ME 274 - ~~cmk~~

efm 3/9/26

Dropping phase

Rebounding phase

$v_1 = 0$

h_1

v_2

$v_4 = 0$

h_4

v_3

Dropping phase

$$\frac{1}{2}mv_2^2 = mgh_1 \Rightarrow v_2 = -\sqrt{2gh_1}$$

Rebounding phase

$$\frac{1}{2}mv_3^2 = mgh_4 \Rightarrow v_3 = \sqrt{2gh_4}$$

Coefficient of restitution (COR)

$$e = -\left(\frac{v_3}{v_2}\right) = \sqrt{h_4 / h_1}$$

1 1 person calcs

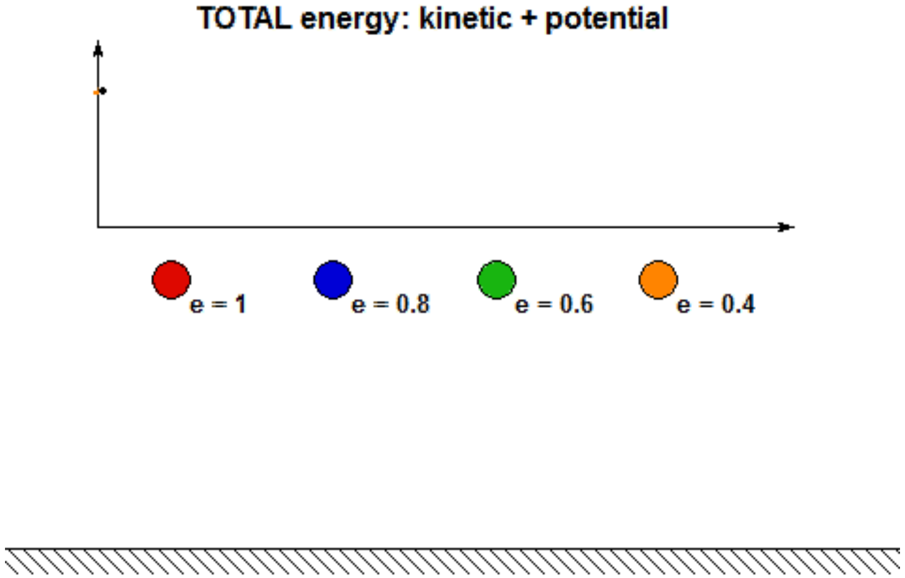
2 1 person observes

3 1 drops, Henny

	Stress ball ~~steel ball~~		plastic ball		ping pong ball		happy face ~~tomato~~		~~eyeball~~	
h_1	h_4	e	h_4	e	h_4	e	h_4	e	h_4	e
36”			22″	0.78						
30”			19″	0.80						
24”			15″	0.79						
18”			12″	0.72						

Central Impact Problems – Discussion

1. Equations Used in this type of problems:



$$
\begin{aligned}
&(1) \quad v_{At2} = v_{At1} \\
&(2) \quad v_{Bt2} = v_{Bt1} \\
&(3) \quad m_A v_{An2} + m_B v_{Bn2} = m_A v_{An1} + m_B v_{Bn1} \\
&(4) \quad e = -\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}
\end{aligned}
$$

2. Central impact = line of impact passes through centers of mass of bodies involved.

3. Reminder: The coefficient of restitution equation (COR) above (equation 4) is valid only for velocity components in the normal direction, n.

4. Mechanical energy in System AB is not conserved during impact. Except in special case of e = 1 (which is physically impossible). See figure in top left or link:

- https://www.purdue.edu/freeform/me274/course-material/animations/impacts-and-energy/

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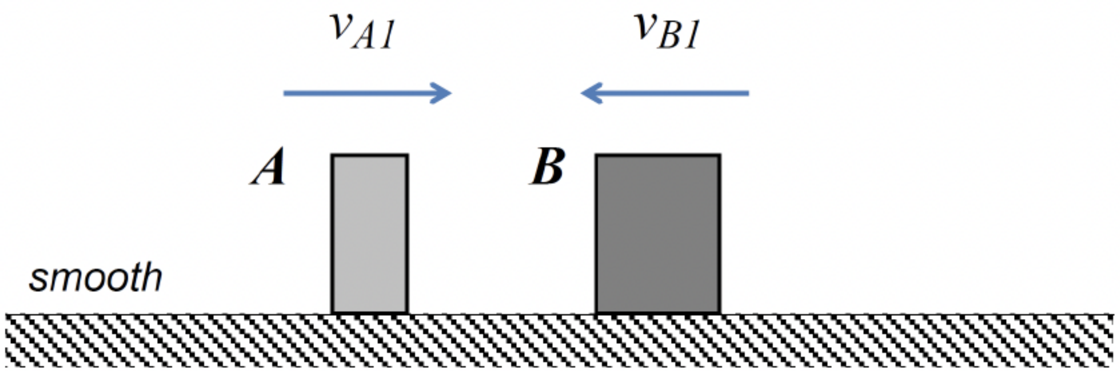
Example 4.C.10

Given: Blocks A and B (having masses of m_A and m_B, respectively) are initially moving to the right and left, respectively, on a smooth horizontal surface with speeds of v_{A1} and v_{B1}, respectively. At some instant in time, A strikes B. The coefficient of restitution of this impact is e. As a result of the impact, block B becomes stationary.

Find: Determine the initial speed v_{B1} of block B.

Use the following parameters in your analysis: $m_A = 2$ kg, $m_B = 3$ kg, $v_{A1} = 1.5$ m/s and $e = 0.4$.



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Example 4.C.10 p.245

Given: Blocks A and B (having masses of m_A and m_B, respectively) are initially moving to the right and left, respectively, on a smooth horizontal surface with speeds of v_{A1} and v_{B1}, respectively. At some instant in time, A strikes B. The coefficient of restitution of this impact is e. As a result of the impact, block B becomes stationary.

Find: Determine the initial speed v_{B1} of block B. $v_{B1} = ?$

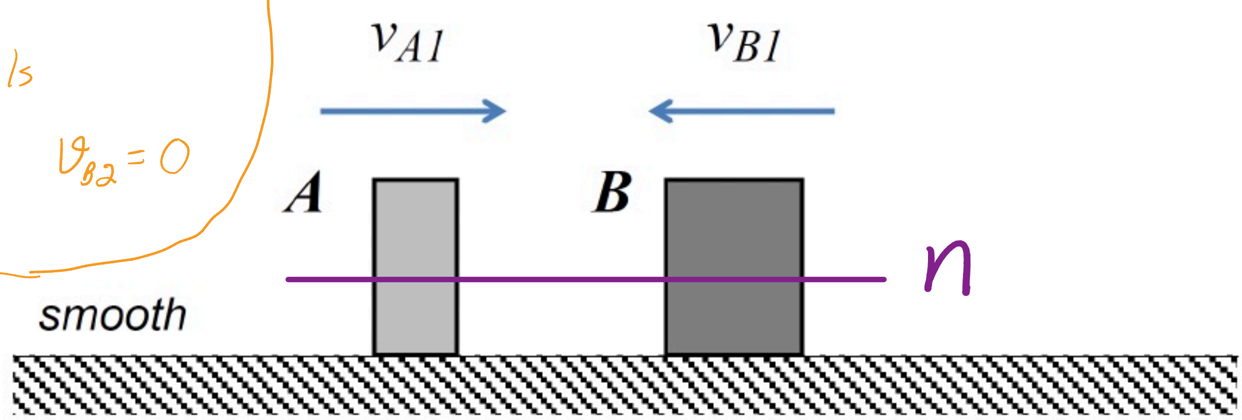
Use the following parameters in your analysis: $m_A = 2$ kg, $m_B = 3$ kg, $v_{A1} = 1.5$ m/s and $e = 0.4$.

$m_A = 2\,Kg$ $m_B = 3\,Kg$

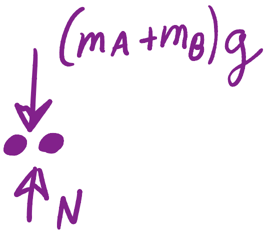
$v_{A1} = 1.5\,m/s$

$v_{B2} = 0$

$e = 0.4$



1 FBD



2 Kinetics

Lin Momentum is conserved in n-dir:

$$m_A v_{A1} + m_B(-v_{B1}) = \underset{?}{m_A v_{A2}} + m_B \cancel{v_{B2}}^{\text{given}} \quad (1)$$

3 Impact Rule (IR):

$$e = \frac{\cancel{v_{B2}}^{\text{given}} - v_{A2}}{v_{A1} - v_{B1}}$$

$$\Rightarrow -\overset{?}{v_{A2}} = e(v_{A1} - v_{B1}) \quad (2)$$

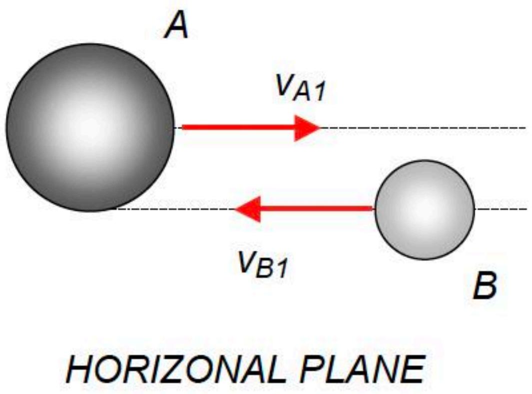
4 Solve Use (1) & (2). Solve for v_{A2} & v_{B1}

Example 4.C.11

Given: Sphere A has a mass of 20 kg and a radius of 75 mm, while B has a mass of 5 kg and a radius of 48 mm. The coefficient of restitution for the impact of A and B is known to be $e = 0.6$.

Find: Determine the velocities of the spheres immediately after impact.

Use the following parameters in your analysis: $v_A = 5$ m/s and $v_B = 15$ m/s.



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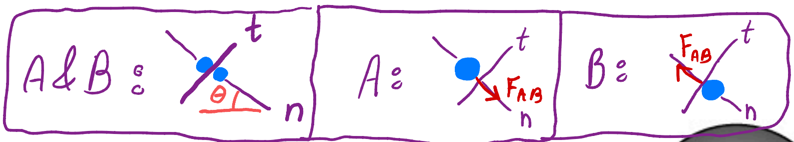
Example 4.C.11

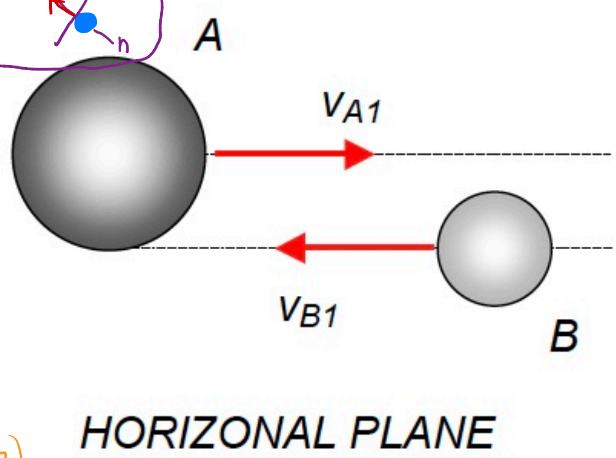
Given: Sphere A has a mass of 20 kg and a radius of 75 mm, while B has a mass of 5 kg and a radius of 48 mm. The coefficient of restitution for the impact of A and B is known to be $e = 0.6$.

Find: Determine the velocities of the spheres immediately after impact. $\vec{v}_{A,2}$? $\vec{v}_{B,2}$?

Use the following parameters in your analysis: $v_A = 5$ m/s and $v_B = 15$ m/s.

(1) FBD



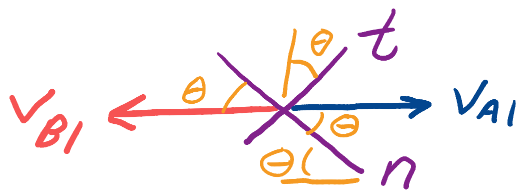


Find θ for solving later:

$$\theta = \sin^{-1}\left(\frac{r_A}{r_A + r_B}\right)$$

(2) Kinetics LM in t-dir is cons: (pg. 233)

Kinematics Projections



A: $\Sigma F_t = 0$

$$V_{A1t} = V_{A2t}$$

$$v_{A1} \sin\theta = v_{A2t} \quad (1)$$

B: $\Sigma F_t = 0$

$$v_{B1t} = v_{B2t}$$

$$-V_{B1} \sin\theta = V_{B2t} \quad (2)$$

(3) LM for syst is cons in n-dir A&B: $\Sigma F_n = 0$

$$m_A V_{A1N} + m_B V_{B1N} = m_A V_{A2N} + m_B V_{B2N}$$

$$\Rightarrow m_A (v_{A1} \cos\theta) + m_B (-v_{B1} \cos\theta) = m_A v_{A2N} + m_B v_{B2N} \quad (3)$$

(4) 3 eqns, 4 unkns, Need Impact rule/e eqn

IR: $$e = \frac{V_{B2N} - V_{A2N}}{V_{A1N} - V_{B1N}}$$

$$= \frac{v_{2BN} - v_{A2N}}{V_{A1} \cos\theta + v_{B1} \cos\theta} \quad (4)$$

(5) Solve Get $\vec{v}_{A2}$ & $\vec{v}_{B2}$ vectors. Using 4 eqns, 4 unkns

(1)–(4): $\vec{V}_{A2} = V_{A2t}\,\hat{e}_t + v_{A2N}\,\hat{e}_n$

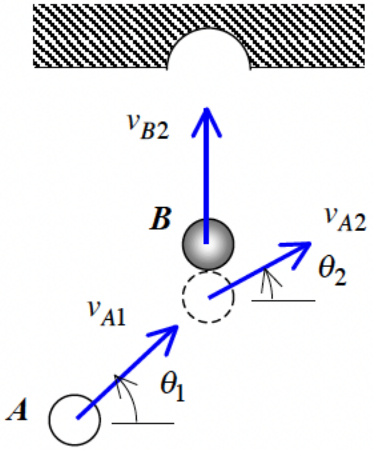
$\vec{v}_{B2} = V_{B2t}\,\hat{e}_t + v_{B2N}\,\hat{e}_n$

Example 4.C.12

Given: Cue ball A strikes a stationary object ball B, with a speed v_{A1} as shown in the figure below. The coefficient of restitution for this impact is e. After impact, A moves along a line defined by the angle θ_2, and B moves directly to the side pocket.

Find: Determine the numerical value of the rebound angle θ_2 of A, assuming the masses of A and B are the same.

Use the following parameters in your analysis: $\theta_1 = 45°$ and $e = 0.9$.



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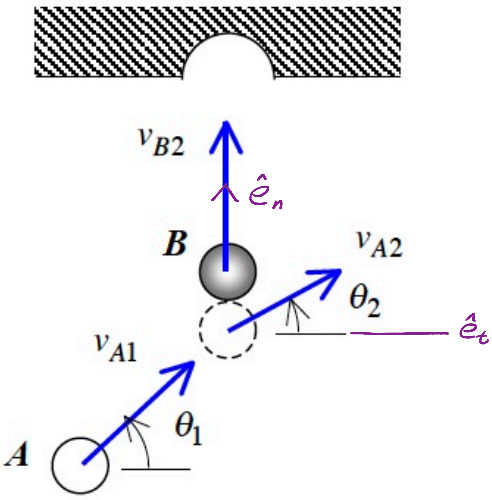
Example 4.C.12 Billiards problem

Given: Cue ball A strikes a stationary object ball B, with a speed v_{A1} as shown in the figure below. The coefficient of restitution for this impact is e. After impact, A moves along a line defined by the angle θ_2, and B moves directly to the side pocket.

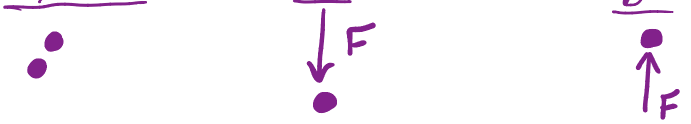
Find: Determine the numerical value of the rebound angle θ_2 of A, assuming the masses of A and B are the same. θ_2 ?

Use the following parameters in your analysis: $\theta_1 = 45°$ and $e = 0.9$.



① FBD

System: A: B:



② Cons of Lin Mom. n-dir of system

$$m v_{A1} \sin\theta_1 = v_{B2} m + m v_{A2} \sin\theta_2 \quad (1)$$

③ Cons of Lim Mom. t-dir of each particle

Particle A: $\cancel{m} v_{A1} \cos\theta_1 = \cancel{m} v_{A2} \cos\theta_2 \quad (2)$

Particle B: $0 = 0$

④ Impact Rule (I R):

$$e = \frac{v_{B2N} - v_{A2N}}{v_{A1N} - v_{B1N}} = \frac{v_{B2} - v_{A2}\sin\theta_2}{v_{A1}\sin\theta - 0} \quad (3)$$

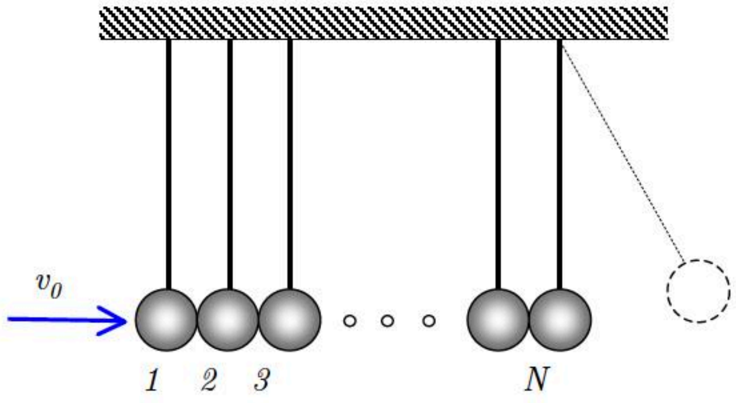
⑤ Solve 3 eqns 3 unkns.

$\theta_2 \Rightarrow$

Example 4.C.13

Given: A "Newton's Cradle" is made up of N identical pendulums. The particle for pendulum 1 has a horizontal velocity of v_0 when it strikes the particle for pendulum 2 (all pendulums except 1 are at rest prior to this impact). The coefficient of restitution for the impacts of all particles is e.

Find: Determine:
(a) The speed of particle N immediately after the final impact; and
(b) The maximum height h reached by particle N.



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Example 4.C.13 Newton's Cradle Problem

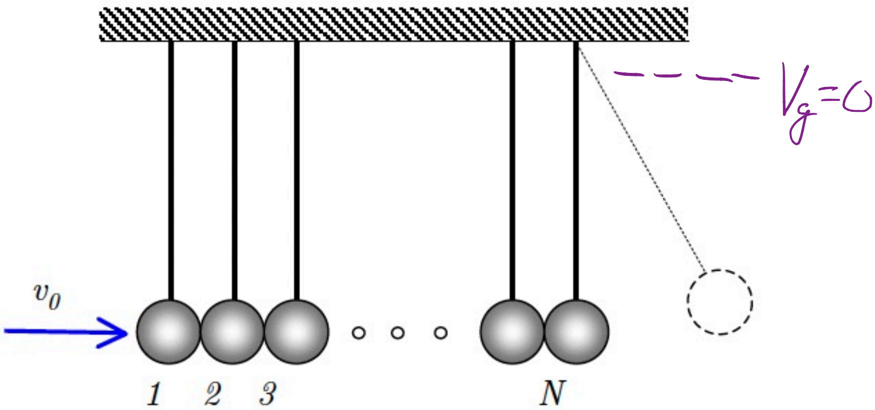
Given: A "Newton's Cradle" is made up of N identical pendulums. The particle for pendulum 1 has a horizontal velocity of v_0 when it strikes the particle for pendulum 2 (all pendulums except 1 are at rest prior to this impact). The coefficient of restitution for the impacts of all particles is e.

Find: Determine:

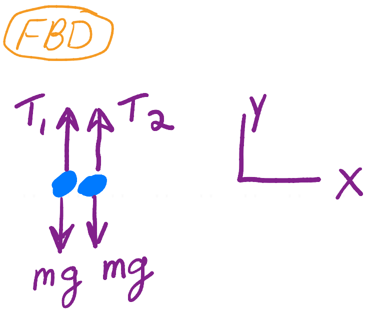
(a) The speed of particle N immediately after the final impact; and v_N' ?

(b) The maximum height h reached by particle N. h ?

prime(') = post_impact



① FBD



② Conservation of Linear Momentum in x-dir

$$\cancel{m}v_0 = \cancel{m}v_1' + \cancel{m}v_2'$$

$$\Rightarrow \underline{v_0 = v_1' + v_2'} \quad (1)$$

③ Impact Rule (IR):

$$e = \frac{v_2' - v_1'}{v_1 - v_2}$$

$$\Rightarrow e = \frac{v_2' - v_1'}{v_0}$$

$$\Rightarrow v_1' = v_2' - e v_0 \quad (2)$$

④ Plug (2) into (1)

$$v_0 = v_2' - e v_0 + v_2'$$

$$\Rightarrow v_2' = \frac{(1+e)}{2} v_0$$

⑤ Above eqn means...

$$v_N' = v_0 \left(\frac{1+e}{2}\right)^{N-1}$$

⑥ W/E

$$T_N + V_N + U^{NC}_{N\to final} = T_{final} + V_{final}$$

$$\begin{cases} T_N = \frac{1}{2} m \left[v_0 \left(\frac{1+e}{2}\right)^{N-1}\right]^2 \\ V_N = -mgL \\ U^{NC}_{N\to final} = 0 \\ T_{final} = 0 \\ V_{final} = -mgL\cos\theta_N \end{cases}$$

⑦ Use W/E to get θ_N

$$\Rightarrow \theta_N$$

Summary: Central Impact Problems

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FUNDAMENTAL equations: the linear impulse-momentum equations and coefficient of restitution (COR) equation

$$A: \quad \sum F_t = 0 \quad \Rightarrow \quad m_A v_{A1t} = m_A v_{A2t} \quad \Rightarrow \quad v_{A2t} = v_{A1t}$$

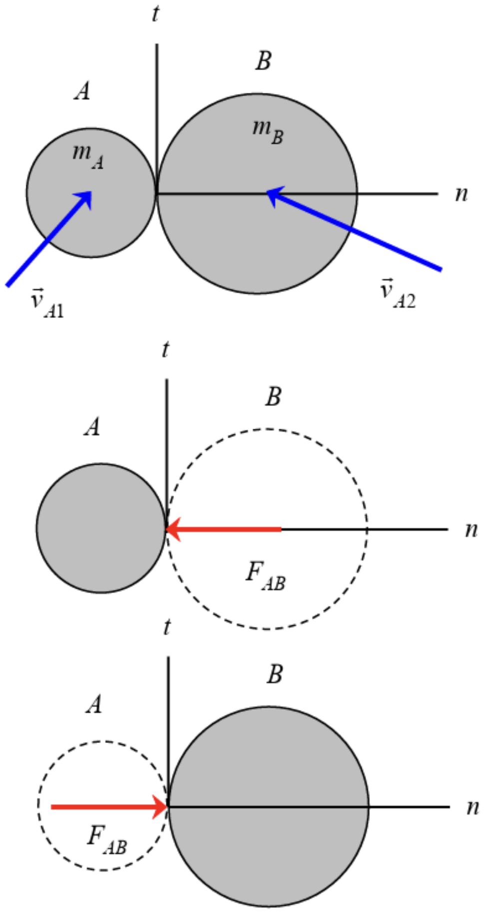
$$B: \quad \sum F_t = 0 \quad \Rightarrow \quad m_B v_{B1t} = m_B v_{B2t} \quad \Rightarrow \quad v_{B2t} = v_{B1t}$$

$$A+B: \quad \sum F_n = 0 \quad \Rightarrow \quad m_A v_{A1n} + m_B v_{B1n} = m_A v_{A2n} + m_B v_{B2n}$$

$$COR: \quad e = -\left(\frac{v_{B2n} - v_{A2n}}{v_{B1n} - v_{A1n}} \right)$$

COMMENTS:

- The COR equation is valid for ONLY the n-components of velocity.
- Energy is NOT conserved during impact, except for e = 1.



Lec 23 Short Feedback Form: