filled

ME 274 Lecture 22

Particle Kinetics – Linear Impulse Momentum Part 2

Eugenio “Henny” Frias-Miranda

03/06/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 21 (4.K and 4.L) due today!!**
2. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME.
3. Bonus quiz grade at end of the semester if we get a good response rate to QR code surveys at the end of lecture.
    - If you are unable to attend lecture on that day/forget to fill it out:
        - Feel free to give feedback based on the content of that lecture’s slides.
        - Way of you reviewing previous content and giving feedback to me.

Kinetics: Four-step problem solving method

1. **FBDs:**
   - Draw appropriate FBD**(s).**
   - Choose your coordinate system.
2. **Kinetics:**
   - Choose what solution method for the particular problem at hand **(we will go over these in the coming days…)**:
     - Newton/Euler (lectures 15-18) – **Analyzing an instant in time**
     - Work/Energy (lectures 19-20) – **Analyzing speed in terms of position**
     - Linear impulse/momentum (lectures 21-23) - **Analyzing change in velocity during a change in time**
     - Angular impulse/momentum -
3. **Kinematics:**
   - Perform needed kinematic analysis (position/velocity/acceleration)
   - Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem
4. **Solve:**
   - Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
     a) Draw more FBDs
     b) You will need to do more kinematic analysis

[pg. 184 content]

Linear Impulse-Momentum (LIM) Equation

$$m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \vec{R}\,dt$$

Where:

- $m\vec{v}$ is the **linear momentum** of the particle
- $\int_1^2 \vec{R}\,dt$ is the **impulse** of the net force acting on the particle

1. This equation relates the ***change in velocity to a change in time***

2. ***Conservation of momentum*** does NOT imply or result from ***conservation of energy***

3. Internal Forces cancel, and only the impulse due to the **External Forces** contributes to the total linear momentum of the chosen system/FBD

4. Make your choice of the system to be as large as possible, to make as many forces as possible *internal* to the system.

- However, remember that there might be cases where you are asked to **solve for a force that you made internal to your system.** In this case you may need to redraw your FBD.

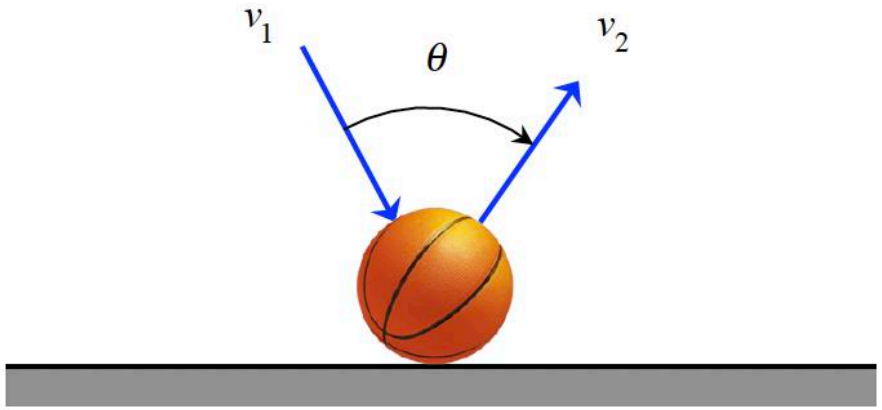
[pg. 229 and 232 content]

Example 4.C.5

Given: A basketball having a weight W strikes a floor with a speed of v_1. The impact with the floor has a duration of time Δt. Immediately after impact, the basketball has a speed of v_2 with the line of travel of the basketball being at an angle of θ relative to the initial line of travel prior to impact, as shown in the figure. Treat the basketball as a particle.

Find: Determine the average force vector acting on the basketball by the floor during impact. Compare the magnitude of this average impact force with the weight of the basketball.

Use the following parameters in your analysis: $W = 1.3$ lb, $v_1 = 20$ ft/s, $v_2 = 17$ ft/s, $\theta = 20°$ and $\Delta t = 0.01$ s.



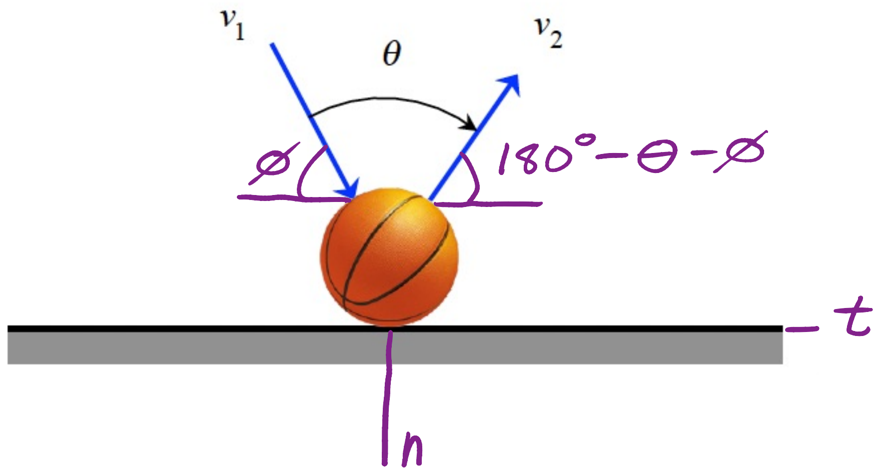
[pg. 240]

Example 4.C.5 p.240

Given: A basketball having a weight W strikes a floor with a speed of v_1. The impact with the floor has a duration of time Δt. Immediately after impact, the basketball has a speed of v_2 with the line of travel of the basketball being at an angle of θ relative to the initial line of travel prior to impact, as shown in the figure. Treat the basketball as a particle.

Find: Determine the average force vector acting on the basketball by the floor during impact. Compare the magnitude of this average impact force with the weight of the basketball.

Use the following parameters in your analysis: $W = 1.3$ lb, $v_1 = 20$ ft/s, $v_2 = 17$ ft/s, $\theta = 20°$ and $\Delta t = 0.01$ s.



Lin Mom in t-dir is Cons.

$$\frac{W}{g} v_1 \cos\theta = v_2 \cos(180° - \theta - \phi) \frac{W}{g}$$

$$\Rightarrow \phi$$

LIM in n-dir:

$$\frac{W}{g} v_1 \sin\phi + \underbrace{\int (W - F)\,dt}_{(W - F_{AV})\Delta t} = \frac{W}{g}(-v_2 \sin(180° - \theta - \phi))$$

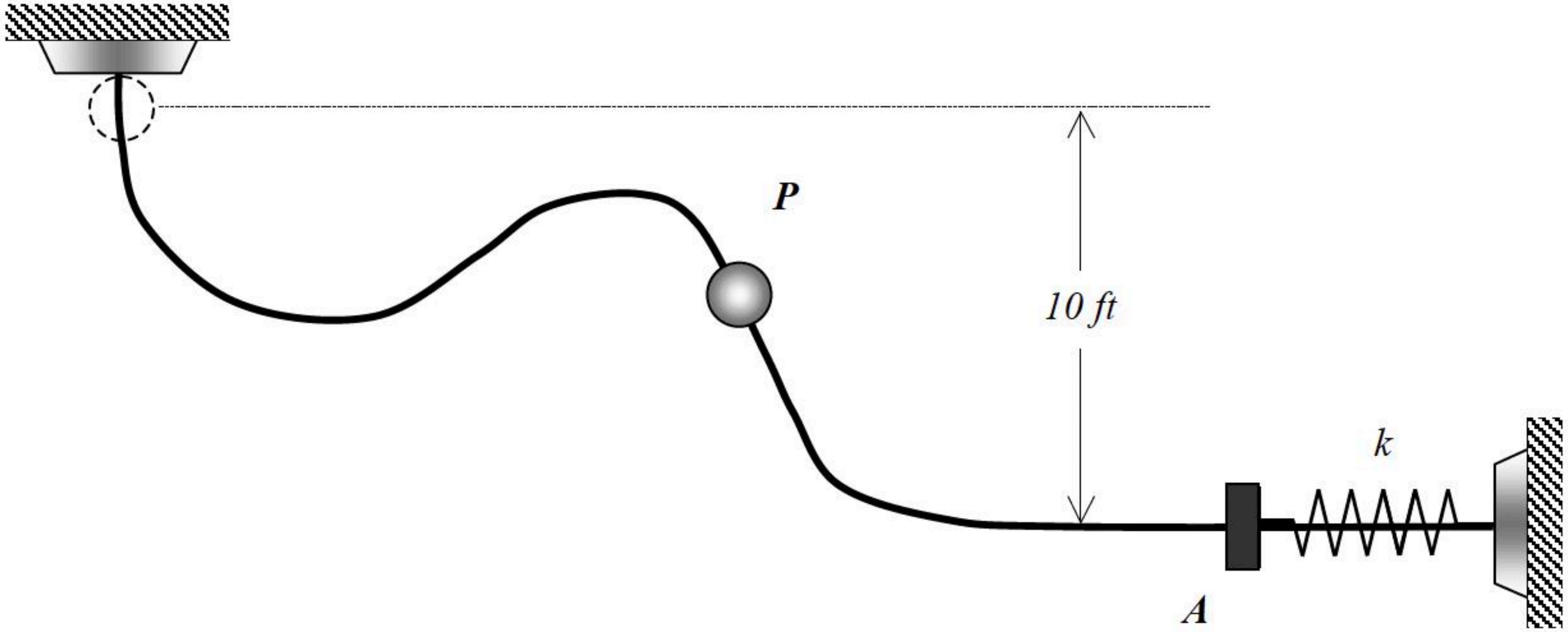
$$\frac{W}{g} v_1 \sin\phi + (W - F_{AV})\Delta t = \frac{-W}{g} v_2 \sin(180° - \theta - \phi)$$

$$\Rightarrow F_{AV}$$

Example 4.C.6

Given: Particle P (weighing 10 lb) is released from rest and slides down a smooth, curved rod and sticks to block A (weighing 5 lb).

Find: Determine the maximum deflection of the spring attached to A, if the spring has a stiffness of $k = 100$ lb/ft.



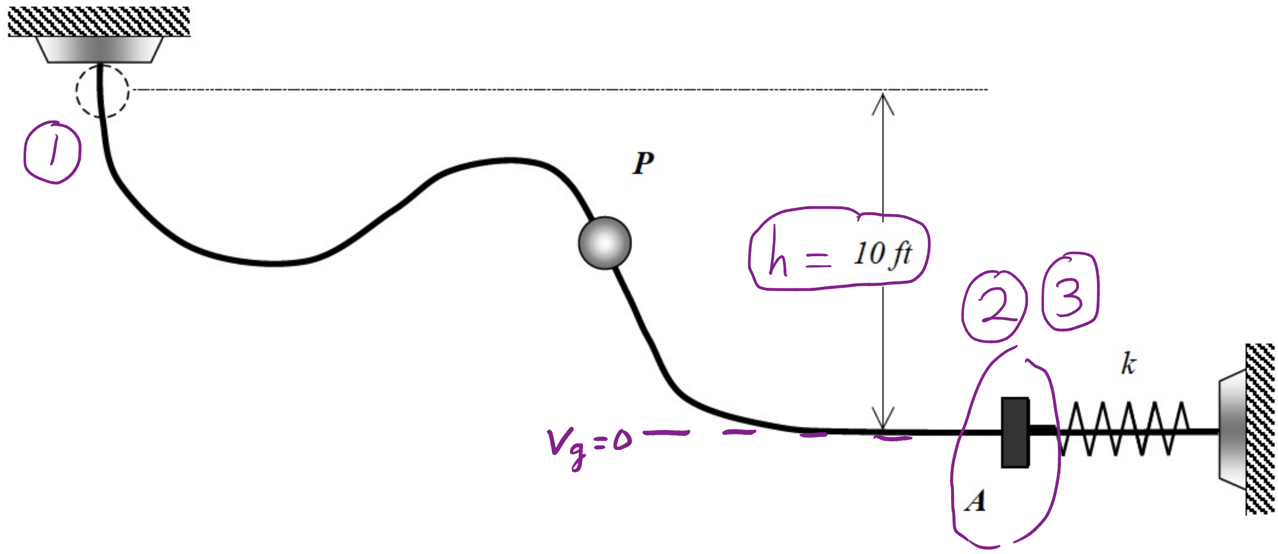
[pg. 241]

Example 4.C.6

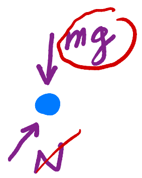
p.241

Given: Particle P (weighing 10 lb) is released from rest and slides down a smooth, curved rod and sticks to block A (weighing 5 lb).

Find: Determine the maximum deflection of the spring attached to A, if the spring has a stiffness of $k = 100$ lb/ft.



$1 \to 2$:

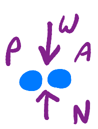


$$T_1 + V_1 + U_{1\to 2}^{NC} = T_2 + V_2$$

$$0 + W_p h + 0 = \frac{1}{2}\frac{W_p}{g} v_2^2 + 0$$

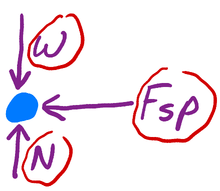
$$\Rightarrow v_2 = \sqrt{\frac{2 g W_p h}{W_p}} = \sqrt{2gh}$$

$2 \to 3$:



LIM x-dir : $\frac{W_P}{g} v_2 = \frac{(W_P + W_A)}{g} v_3$

$$\Rightarrow v_3 = \frac{W_P}{W_P + W_A} v_2 \quad (2)$$



$3 \to 4$: $T_3 + V_3 + U_{3\to 4}^{NC} = T_4 + V_4$

$$\frac{1}{2}\frac{(W_P + W_A)}{g} v_3^2 + 0 + 0 = 0 + \frac{1}{2} k \Delta^2 \quad (1)$$

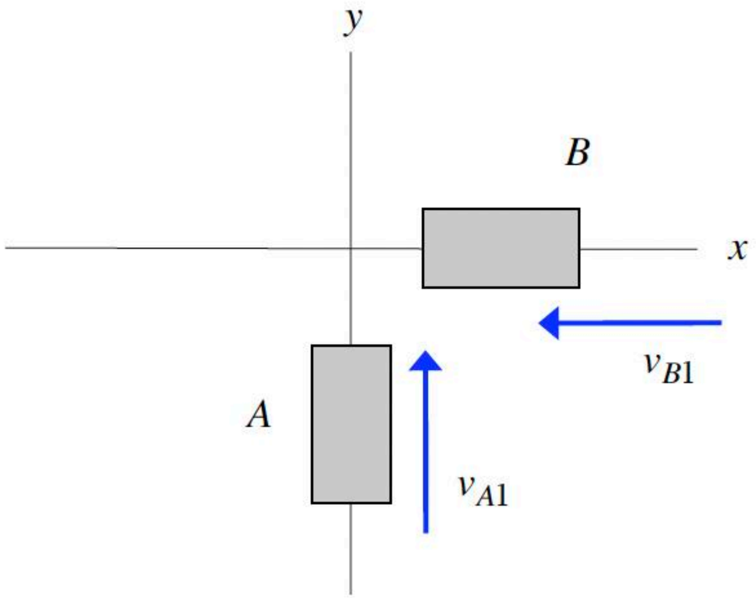
$$\Rightarrow \Delta$$

Example 4.C.7

Given: Cars A and B (having weights of W_A and W_B, respectively) strike each other with speeds of v_{A1} and v_{B1}. After a short collision time, the cars stick together.

Find: Determine the x and y components of velocity for the cars after the collision.

Use the following parameters in your analysis: $W_A = 3000$ lb, $W_B = 4000$ lb, $v_{A1} = 40$ mph and $v_{B1} = 25$ mph.



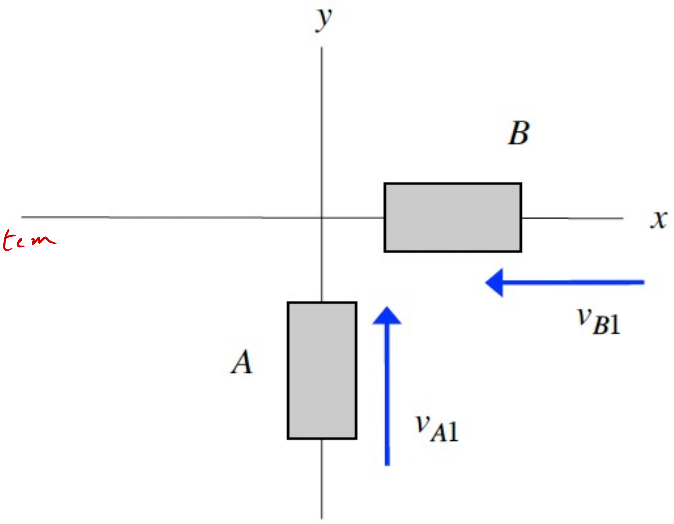
[pg. 242]

Example 4.C.7 p.242

Given: Cars A and B (having weights of W_A and W_B, respectively) strike each other with speeds of v_{A1} and v_{B1}. After a short collision time, the cars stick together.

Find: Determine the x and y components of velocity for the cars after the collision. v_{2x}? v_{2y}?

Use the following parameters in your analysis: $W_A = 3000$ lb, $W_B = 4000$ lb, $v_{A1} = 40$ mph and $v_{B1} = 25$ mph.



① FBD

A+B

② LIM x-dir:

no ext force in system

$$\sum m v_1 + \int (F)\,dt = \sum m v_2$$

$$-\frac{W_B}{g} v_{B1} = \frac{(W_A+W_B)}{g} v_{2x}$$

$$v_{2x} = \frac{-W_B}{W_A+W_B} v_{B1}$$

③ LIM y-dir

no ext force in system

$$\sum m v_1 + \int (F)\,dt = \sum m v_2$$

$$\frac{W_A}{g} v_{A1} = \frac{(W_A+W_B)}{g} v_{2y}$$

$$v_{2y} = \frac{W_A}{W_A+W_B} v_{A1}$$

---

④ Problem Doesn't ask but total vel of sys is:

$$\vec{v}_2 = \frac{-W_B}{W_A+W_B} v_{B1}\,\hat{i} + \frac{W_A}{W_A+W_B} v_{A1}\,\hat{j}$$

Example 4.C.8

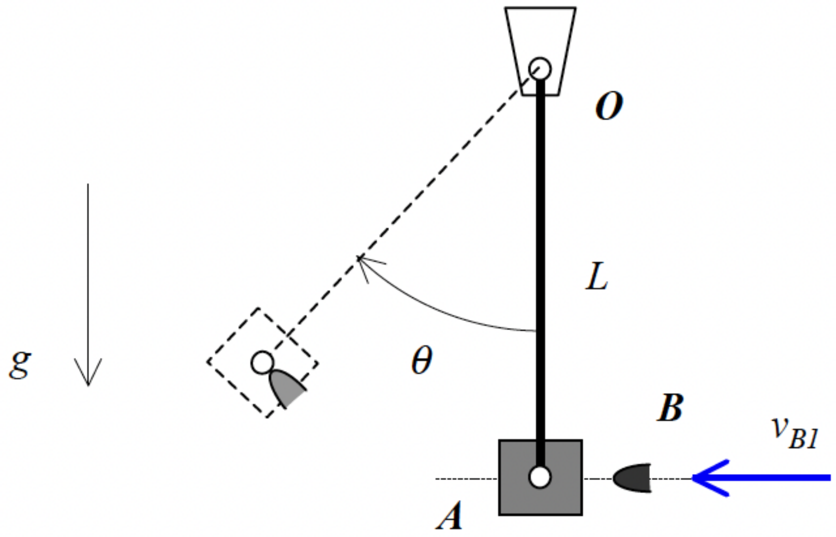
Given: Block A (having a weight of W) is suspended by a cord from fixed point O. Bullet B (having a weight of w) strikes the stationary block A with a speed of v_{B1}. On impact, the bullet sticks to block A.

Find: Determine:

(a) The maximum elevation angle θ of the cord after impact; and

(b) The energy lost during the impact of B with A.

Use the following parameters in your analysis: $w = 0.2$ lb, $W = 75$ lb, $L = 5$ ft and $v_{B1} = 1800$ ft/s.



[pg. 243]

Example 4.C.8 p.243

Given: Block A (having a weight of W) is suspended by a cord from fixed point O. Bullet B (having a weight of w) strikes the stationary block A with a speed of v_{B1}. On impact, the bullet sticks to block A.

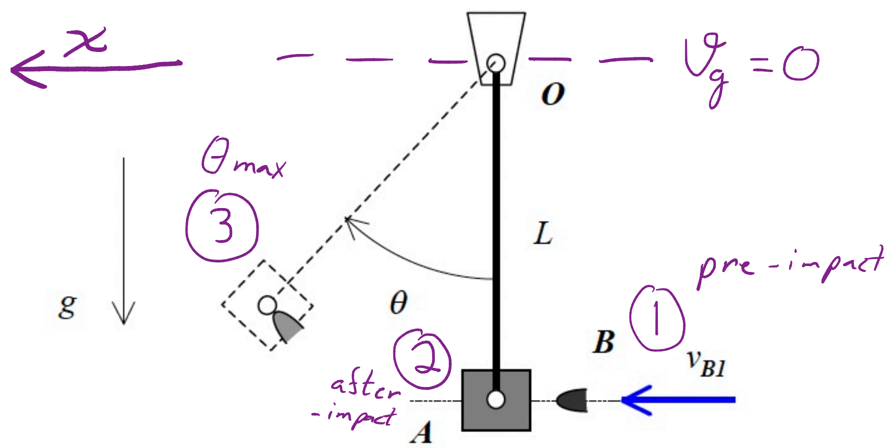
Find: Determine:

(a) The maximum elevation angle θ of the cord after impact; and θ_{max}?

(b) The energy lost during the impact of B with A. $T_2 - T_1$?

Use the following parameters in your analysis: $w = 0.2$ lb, $W = 75$ lb, $L = 5$ ft and $v_{B1} = 1800$ ft/s.

involves change in position W/E eqn. However, use LIM to acc for impact



State ① → ② during impact

① FBD

T

W+w

② Kinetics LIM x-dir

$\Sigma m v_1 + \int (\text{const.}\ T)\,dt = \Sigma m v_2$

$\frac{w}{g} v_{B1} = \frac{W+w}{g} v_2$

$v_2 = \frac{w}{W+w} v_{B1}$ (1)

State ② → ③ during motion of pendulum

③ FBD

T — no work, ⊥ to path

W+w — conservative

④ Kinetics W/E. datum

$T_2 + V_2 + U^{NC}_{2-3} = T_3 + V_3$

$T_2 = \frac{1}{2} \frac{W+w}{g} v_2^2$

$V_2 = -(W+w)L$

$U^{NC}_{2-3} = 0$; mech energy is conserved

$T_3 = 0$

$V_3 = -(W+w)L\cos\theta_{max}$; below datum ("−")

Kinematics

⑤ Solve Use (1) & (2). Solve θ_{max}

$\frac{1}{2}\frac{W+w}{g} v_2^2 + -(W+w)L + 0 = 0 - (W+w)L\cos\theta_{max}$ (2)

$\Rightarrow \theta_{max}$ (a)

⑥ No energy lost in State ②→③. to find energy lost in impact. Subtract $T_2 - T_1$

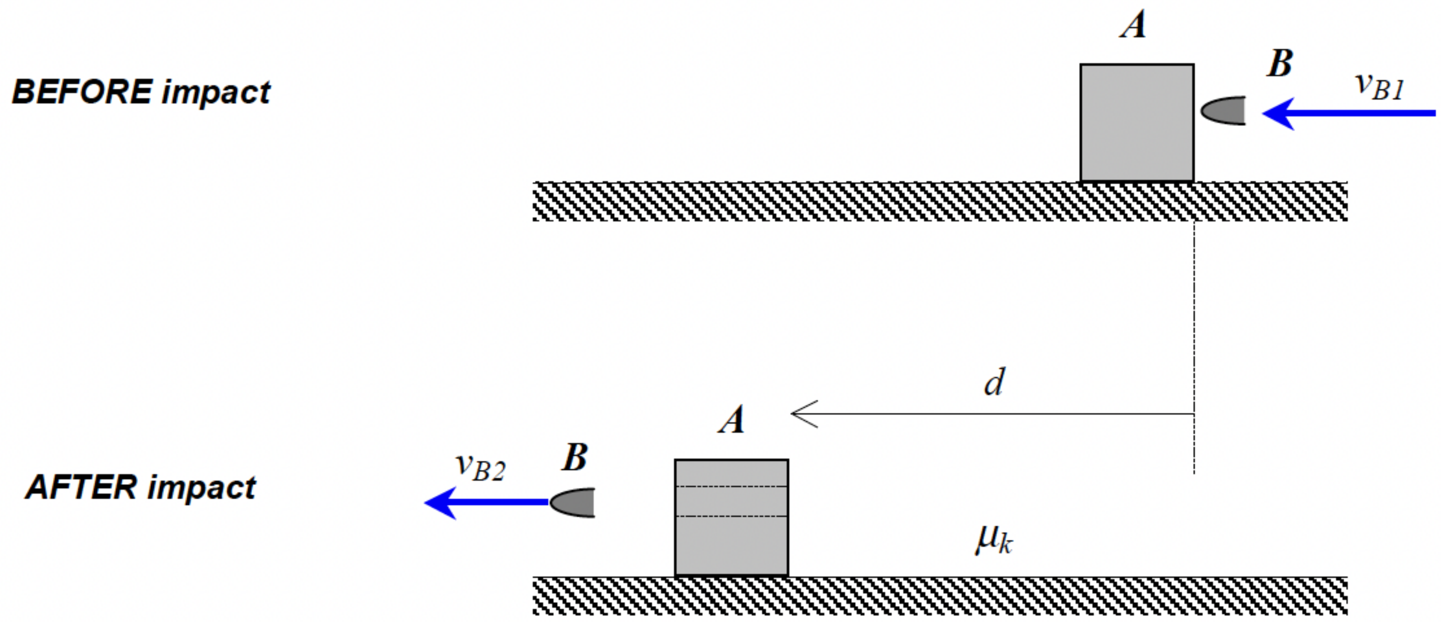
$T_2 - T_1 = \frac{1}{2}\left(\frac{W+w}{g}\right) v_2^2 - \frac{1}{2}\frac{w}{g} v_{B1}^2$

Example 4.C.9

Given: A stationary block A (having a mass of m_A) is struck by bullet B (having a mass of m_B). B has a speed of v_{B1} before impact. During impact, the bullet B passes through block A. B has a known speed of v_{B2} after exiting the left side of A. After impact, the speed of A is unknown. Block A continues to slide to the left on a rough horizontal surface (having a coefficient of kinetic friction of μ_k). Block A comes to rest after sliding through a distance of d.

Find: Determine the distance d traveled by block A after impact.

Use the following parameters in your analysis: $m_A = 5$ kg, $m_B = 0.1$ kg, $v_{B1} = 800$ m/s, $v_{B2} = 350$ m/s and $\mu_k = 0.1$.



[pg. 244]

Example 4.C.9

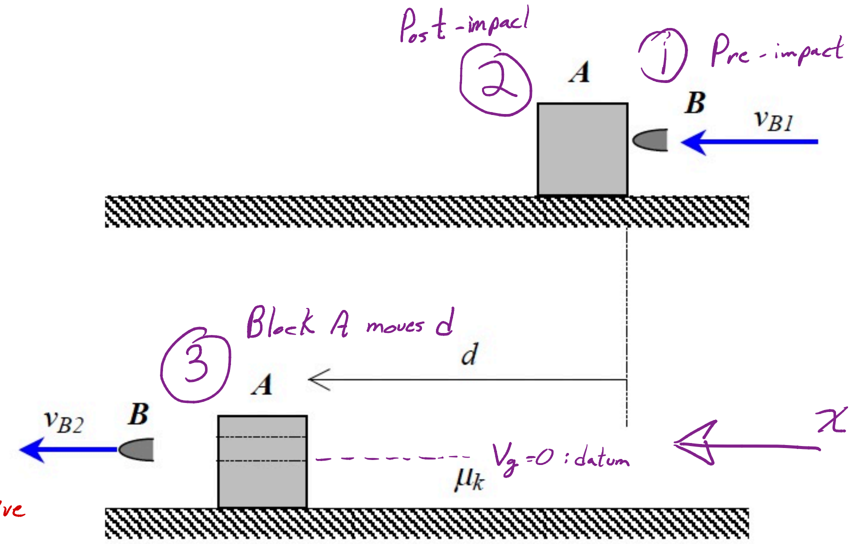
p. 244

Given: A stationary block A (having a mass of m_A) is struck by bullet B (having a mass of m_B). B has a speed of v_{B1} before impact. During impact, the bullet B passes through block A. B has a known speed of v_{B2} after exiting the left side of A. After impact, the speed of A is unknown. Block A continues to slide to the left on a rough horizontal surface (having a coefficient of kinetic friction of μ_k). Block A comes to rest after sliding through a distance of d.

Find: Determine the distance d traveled by block A after impact.

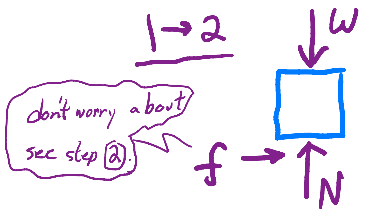
Use the following parameters in your analysis: $m_A = 5$ kg, $m_B = 0.1$ kg, $v_{B1} = 800$ m/s, $v_{B2} = 350$ m/s and $\mu_k = 0.1$.

① FBD



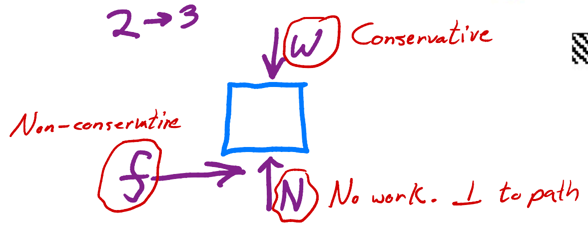
BEFORE impact

1→2



AFTER impact

2→3



② States (1→2) Assume: friction does not cause an impulsive load (here)

③ Kinetics L/M x-dir: (step 2)

$$\sum m v_i + \int (\sum F) dt = \sum m v_2$$

$$m_B v_{B1} = m_B v_{B2} + m_A v_{A2}$$

$$\Rightarrow v_{A2} = \frac{m_B (v_{B1} - v_{B2})}{m_A}$$

④ W/E & datum

$$T_2 + V_2 + U_{2\to3}^{NC} = T_3 + V_3$$

$$T_2 = \frac{1}{2} m_A v_{A2}^2$$

$V_2 = 0$; datum

$$U_{2\to3}^{NC} = \int_2^3 -f\hat{i} \cdot dx\hat{i} = -fd$$

$V_3 = 0$; datum

$T_3 = 0$

$$\frac{1}{2} m_A v_{A2}^2 - fd = 0$$

$$\Rightarrow d = \frac{m_A v_{A2}^2}{2f}$$

⑤ Kinematics $\sum F$ in y to find f

$$\sum F_y = 0 \Rightarrow N = W = m_A g \Rightarrow f = \mu_k N = \mu_k m_A g$$

⑥ Solve Plug-in

$$d = \frac{m_A v_{A2}^2}{2 \mu_k m_A g} = \frac{v_{A2}^2}{2 \mu_k g}$$

Question C4.9

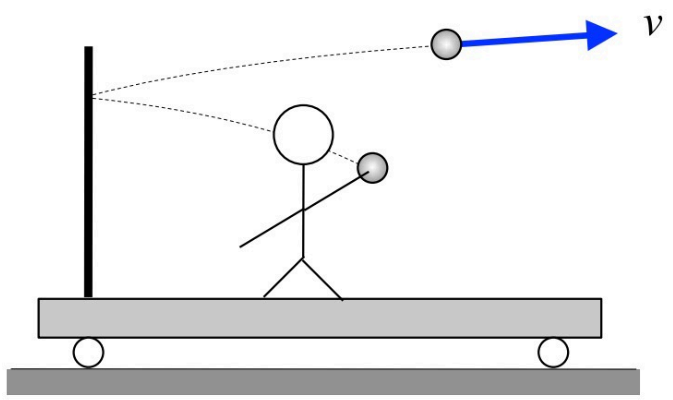
You are on a cart that is initially at rest on a smooth track. You throw a ball at a partition that is rigidly mounted on the cart. If the ball bounces off the partition as shown in the figure, then at the instant shown in the figure:

(a) The cart is moving to the right

(b) The cart is stationary

(c) The cart is moving to the left

(d) More information is needed about the impact of the ball with the partition in order to answer this question



pg. 276

Question C4.9 p.276

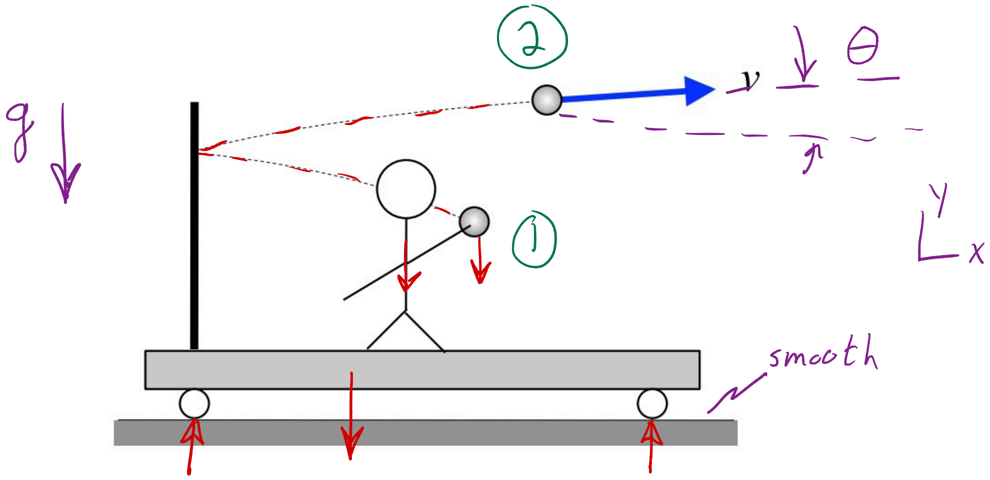
You are on a cart that is initially at rest on a smooth track. You throw a ball at a partition that is rigidly mounted on the cart. If the ball bounces off the partition as shown in the figure, then at the instant shown in the figure:

(a) The cart is moving to the right

(b) The cart is stationary

(c) The cart is moving to the left

(d) More information is needed about the impact of the ball with the partition in order to answer this question



V_{b2x} = x-comp of ball's vel
$= v\cos\theta$

m = mass of ball

V_{c2x} = vel. of person/cart

M = mass of person & cart

State ①: at rest

State ②: at position shown (you don't move w.r.t. the cart)

① FBD person, cart, ball impact force is internal

② Kinetics
$\sum F_x = 0 \Rightarrow$ lin. momentum in x-direction is conserved

~~$\sum m v_1 + \int(\sum F)dt$~~ $= \sum m v_2$

$0 + 0 = m v_{b2x} + M v_{c2x}$

③ Solve

$v_{c2x} = -\frac{m}{M} v\cos\theta$

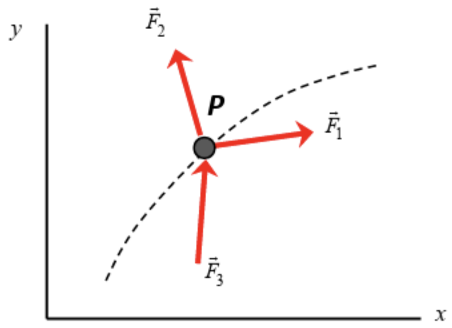
↑ cart/person moving to left

Summary: Linear Impulse-Momentum Equation 2

[pg. 229]

FUNDAMENTAL equation: the linear impulse-momentum equation:

$$m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \left(\sum \vec{F}\right) dt$$



Lec 22 Short Feedback Form:

SOLUTION PROCESS:

1. Draw free body diagram (FBD) for system of your choice. As mentioned before, make your system as "large" as possible.
2. Write down the impulse-momentum equation.
3. Write down the appropriate kinematics (velocity) equations for the problem.
4. If you have enough equations, solve for the desired unknowns. If you do not have enough equations, then you have probably missed some information from kinematics.