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ME 274 Lecture 21

Particle Kinetics – Linear Impulse Momentum Part 1

Eugenio “Henny” Frias-Miranda

03/04/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 20 (4.I and 4.J) due today!!**
2. Office hours are changing to ME2008B…
   - Second floor of renovated side of ME.
3. Bonus quiz grade at end of the semester if we get a good response rate to QR code surveys at the end of lecture.
   - If you are unable to attend lecture on that day/forget to fill it out:
     - Feel free to give feedback based on the content of that lecture's slides.
     - Way of you reviewing previous content and giving feedback to me.

- Todays Examples:
  - 4.C.1 p. 236
  - 4.C.3 p. 238

4 Quiz 04 - due on Fri, 03/06. Gradescope

Kinetics: Four-step problem solving method

1. **FBDs:**
   - Draw appropriate FBD**(s).**
   - Choose your coordinate system.
2. **Kinetics:**
   - Choose what solution method for the particular problem at hand **(we will go over these in the coming days…)**:
     - Newton/Euler (lectures 15-18) – **Analyzing an instant in time**
     - Work/Energy (lectures 19-20) – **Analyzing speed in terms of position**
     - Linear impulse/momentum (lectures 21-23) - **Analyzing change in velocity during a change in time**
     - Angular impulse/momentum -
3. **Kinematics:**
   - Perform needed kinematic analysis (position/velocity/acceleration)
   - Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem
4. **Solve:**
   - Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
     a) Draw more FBDs
     b) You will need to do more kinematic analysis

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Linear Impulse-Momentum (LIM) Equation

Objectives for today's lecture/section:

1. Develop *linear impulse-momentum equation*, used for:
   - Analyzing change in velocity during a change in time
2. Extend idea to study *systems of particles* (like we did for W/E Equation)
3. Concept of *conservation of linear momentum*

Linear Impulse-momentum Equation:

$$\boxed{m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \vec{R}\,dt}$$

Where:

- $m\vec{v}$ is the linear momentum of the particle
- $\int_1^2 \vec{R}\,dt$ is the impulse of the net force acting on the particle

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Derivation - Linear Impulse-Momentum (LIM) Equation

1. Using Newton's Second law we have…

$$\vec{R} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

2. Integrate both sides

$$\int_1^2 \vec{R}\,dt = m\int_1^2 d\vec{v}$$
$$= m\vec{v}_2 - m\vec{v}_1$$

3. Rearrange and we get the LIM Equation

$$\boxed{m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \vec{R}\,dt}$$

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Discussion - Linear Impulse-Momentum (LIM) Equation

1. This equation relates the ***change in velocity to a change in time***

2. LIM Equation is a vector equation, unlike the W/E equation (from last lectures), **it can be resolved into components**

$$mv_{x2} = mv_{x1} + \int_1^2 R_x \, dt$$

$$mv_{y2} = mv_{y1} + \int_1^2 R_y \, dt$$

3. If $\int_1^2 \vec{R}\, dt$ is zero, then linear momentum is conserved *in that* ***direction.***

4. ***Conservation of momentum*** does NOT imply or result from ***conservation of energy***

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System of Particles – LIM Equation

We can extend the LIM Equation to a System of Particles:

$$\boxed{\sum m_j \vec{v}_{j2} = \sum m_j \vec{v}_{j1} + \int_1^2 \left[\sum \vec{F}_j^{ext}\right] dt}$$

1. $\boxed{\text{Internal Forces}}$ cancel, and only the impulse due to the **External Forces** contributes to the total linear momentum of the chosen system/FBD

2. Make your choice of the system to be as $\boxed{\text{large as possible}}$ to make as many forces as possible *internal* to the system.

- However, remember that there might be cases where you are asked to **solve for a force that you made internal to your system.**
  - In this case you may need to redraw your FBD. This is better seen in Example 4.C.3 Part (b). Where you redraw your FBD to solve for tension.

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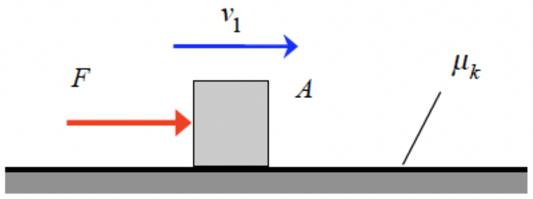
Example 4.C.1

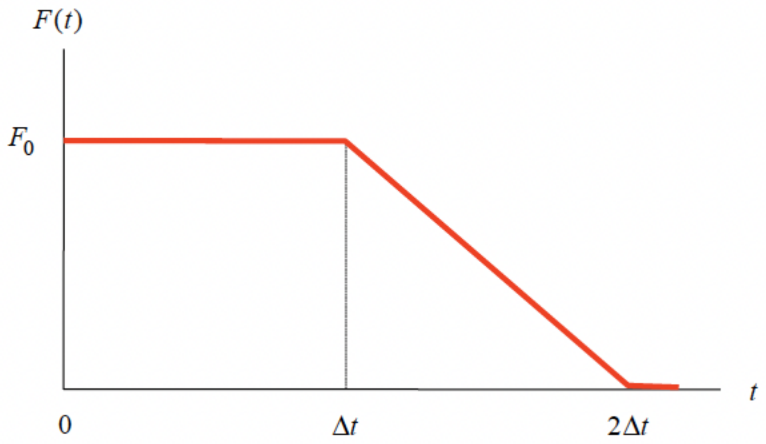
Given: Block A, having a mass of m, has an initial speed of v_1 as it moves to the right on a rough, horizontal surface (with a coefficient of kinetic friction of μ_k). As the block slides, a force $F(t)$ acts to the right on the block, with the time history for $F(t)$ shown below.

Find: Determine:

(a) The speed of the block at $t = \Delta t$; and
(b) The speed of the block at $t = 2\Delta t$.

Use the following parameters in your analysis: $m = 8$ kg, $F_0 = 20$ N, $v_1 = 15$ m/s, $\mu_k = 0.15$ and $\Delta t = 0.2$ s.





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Example 4.C.1 p.236

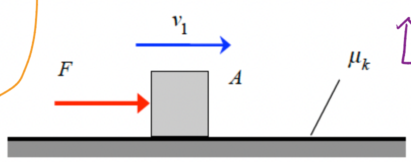
Given: Block A, having a mass of m, has an initial speed of v_1 as it moves to the right on a rough, horizontal surface (with a coefficient of kinetic friction of μ_k). As the block slides, a force $F(t)$ acts to the right on the block, with the time history for $F(t)$ shown below.

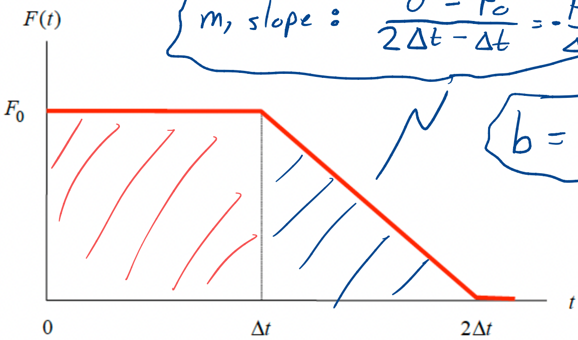
Find: Determine:

(a) The speed of the block at $t = \Delta t$; and v_2 @ Δt ?
(b) The speed of the block at $t = 2\Delta t$. v_2 @ $2\Delta t$?

Use the following parameters in your analysis: $m = 8$ kg, $F_0 = 20$ N, $v_1 = 15$ m/s, $\mu_k = 0.15$ and $\Delta t = 0.2$ s.

$m = 8\,kg$; $F_0 = 20\,N$
$v_1 = 15\,m/s$; $\mu_k = 0.15$
$\Delta t = 0.2s$

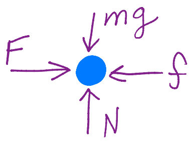




m, slope : $\dfrac{0 - F_0}{2\Delta t - \Delta t} = -\dfrac{F_0}{\Delta t}$

$b = 2F_0$

① (FBD)



② (Kinetics) Find f

$\Sigma F_y = 0 = N - mg$

$\Rightarrow N = mg$

$f = \mu_k N$
$= \mu_k mg$

③ Apply Linear Impulse Momentum Eqn (LIM) in x-dir

$m v_1 + \int (F - f)\,dt = m v_2$

④ Part (a) 0 to Δt

$m v_1 + \int_0^{\Delta t} (F_0 - \mu_k mg)\,dt = m v_2$

$\Rightarrow m v_1 + (F_0 - \mu_k mg)\Delta t = m v_2$

$\Rightarrow v_2$: 1 unkn 1 eqn. Solve

$\Rightarrow \vec{v}_2 = v_2 \hat{i}$

⑤ Part (b) 0 to $2\Delta t$ Approach 1: Area under curve

$m v_1 + \int_0^{2\Delta t} (F - f)\,dt = m v_2$

$m v_1 + \int_0^{\Delta t} (F_0 - \mu_k mg)\,dt + \int_{\Delta t}^{2\Delta t} (\underbrace{2F_0}_{b} - \underbrace{\tfrac{F_0}{\Delta t}}_{m} t - \mu_k mg)\,dt = m v_2$; split into $0 \to \Delta t$ & $\Delta t \to 2\Delta t$

$m v_1 + F_0 \Delta t - \mu_k mg\,\Delta t + \frac{1}{2} F_0 \Delta t - \mu_k mg\,\Delta t = m v_2$; area under curve.

$\Rightarrow v_2$; Solve for v_2

(5) Part (b) 0 to $2\Delta t$ Approach 2: Formal integration

$$mv_1 + \int_0^{2\Delta t}(F-f)\,dt = mv_2$$

$$mv_1 + \int_0^{\Delta t}(F_0-\mu_K mg)\,dt + \int_{\Delta t}^{2\Delta t}\left(2F_0-\frac{F_0}{\Delta t}t-\mu_K mg\right)dt = mv_2$$; split into $0\to\Delta t$ & $\Delta t\to 2\Delta t$

$$\Rightarrow \quad mv_1 + (F_0-\mu_K mg)\Delta t + \left[2F_0 t - \frac{F_0 t^2}{2\Delta t} - \mu_K mg t\right]_{\Delta t}^{2\Delta t} = mv_2$$

$$\Rightarrow mv_1 + F_0\Delta t - \mu_K mg\Delta t + 4F_0\Delta t - \frac{4F_0\Delta t^2}{2\Delta t} - 2\mu_K mg\Delta t - \left[2F_0\Delta t - \frac{F_0\Delta t^2}{2\Delta t} - \mu_K mg\Delta t\right] = mv_2$$

$$\Rightarrow \quad mv_1 + 5F_0\Delta t - \cancel{\mu_K mg\Delta t} - 2F_0\Delta t - 2\mu_K mg\Delta t - 2F_0\Delta t + \frac{F_0\Delta t}{2} + \cancel{\mu_K mg\Delta t} = mv_2$$

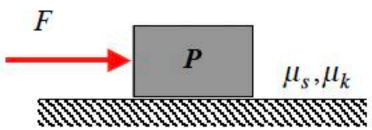
$$\Rightarrow \quad mv_1 + \frac{3F_0\Delta t}{2} - 2\mu_K mg\Delta t = mv_2$$

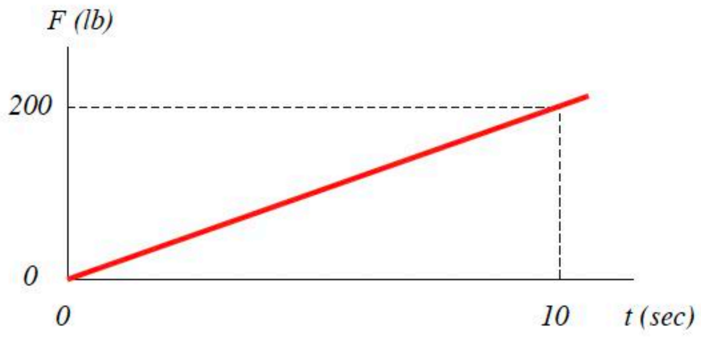
$\Rightarrow v_2$; Solve for v_2

Example 4.C.2

Given: Block P, weighing 100 lb, is initially stationary when a force $F(t)$ begins acting on it, with $F(t)$ given in the graph shown. The coefficients of static and kinetic coefficients of friction between the block and the horizontal surface on which it moves are known to be $\mu_s = 0.5$ and $\mu_k = 0.2$, respectively.

Find: Determine the speed of P at $t = 10$ s.





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Example 4.C.2 p.237

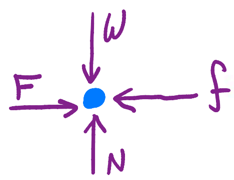
Given: Block P, weighing 100 lb, $= mg$ is initially stationary when a force $F(t)$ begins acting on it, with $F(t)$ given in the graph shown. The coefficients of static and kinetic coefficients of friction between the block and the horizontal surface on which it moves are known to be $\mu_s = 0.5$ and $\mu_k = 0.2$, respectively.

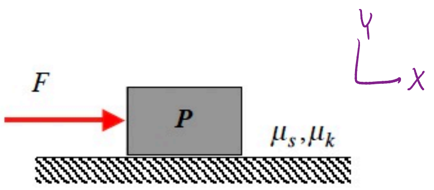
$W = 100\,lb$; $\mu_s = 0.5$; $\mu_k = 0.2$; $t = 10s$

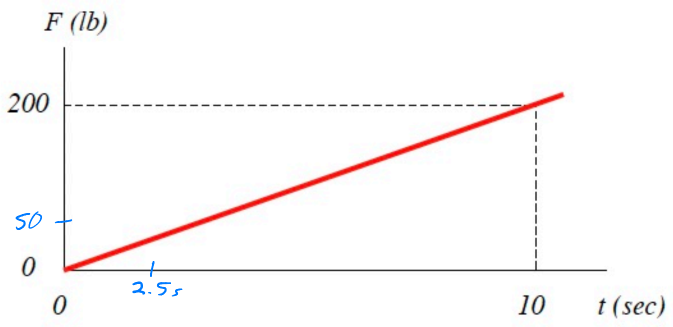
Find: Determine the speed of P at $t = 10$ s.

v_2 ?

① FBD







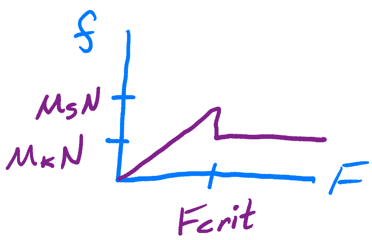
② Kinetics LIM x-dir:

initially @ rest

$$\cancel{mv_0} + \int (F - f)\,dt = mv_2 \quad (1)$$

② Determine Net impulse, $\int (F - f)\,dt$

③ f is a fcn w/ F. Net implse is 0 until f_{max} reached.



$f_{max} = \mu_s N$; So this

④ Kinematics Use kinematics to get f_{max}

$\Sigma F_y = 0$

$\Rightarrow N = W$

⑤ Solve Plug-in

$f_{max} = \mu_s W$

$= 50\,lb$

⑥ W/ 50lb det $t_1 = 2.5s$. Net impulse $\neq 0$ $t_1 \to t_2$.

$F = 20t$ (slope of line)

$$(1) \Rightarrow \int_{2.5}^{10} [20t - \mu_k W]\,dt = \frac{W}{g} v_2$$

$\Rightarrow v_2$

Example 4.C.3

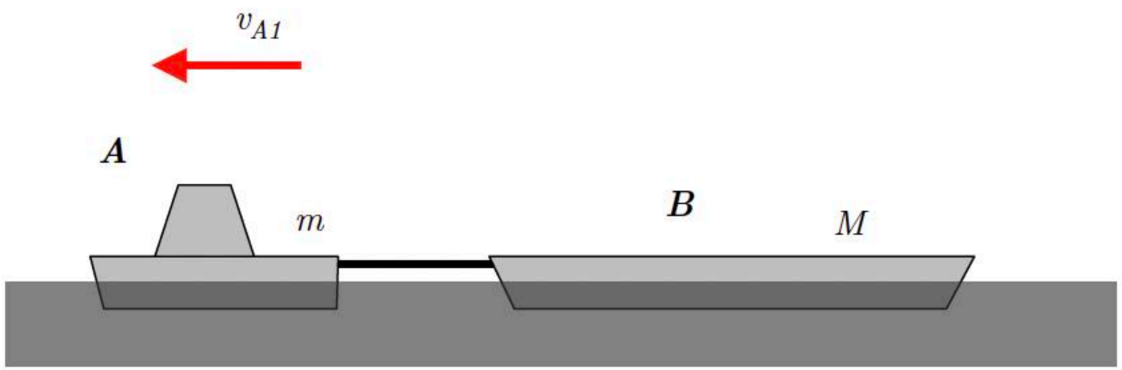
Given: A tug having a mass of m is towing a coal barge (mass M) with a speed of v_{A1}. During a short period of time Δt when the engines of the tug are turned off, the stern winch takes in the towing cable at a rate of $v_{B/A}$.

Find: Determine:

(a) The speed of the tug during this period of time; and

(b) The average value of the tension in the towing cable during this time.

Ignore the water resistance on the tug and barge during this time period. Also, assume that the towing cable remains taut at all times.



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Example 4.C.3 p.238

Given: A tug having a mass of m is towing a coal barge (mass M) with a speed of v_{A1}. During a short period of time Δt when the engines of the tug are turned off, the stern winch takes in the towing cable at a rate of $v_{B/A}$.

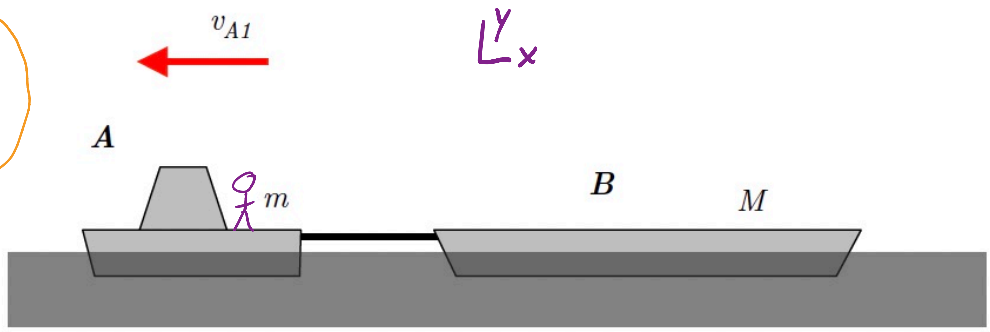
Find: Determine:

(a) The speed of the tug during this period of time; and $v_{A,2}$?

(b) The average value of the tension in the towing cable during this time. T_{Avg} ?

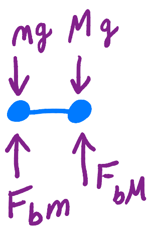
Ignore the water resistance on the tug and barge during this time period. Also, assume that the towing cable remains taut at all times.

$m;\ M;\ v_{A1};\ \Delta t;\ v_{B/A}$



(a)

① FBD tug + barge



② Kinetics LIM x-dir

No external forces. No Impulse

$\Sigma m v_1 + \cancel{\int (T) dt} = \Sigma m v_2$

$m v_{A1} + M v_{B1} = m v_{A2} + M v_{B2}$ (1)

③ Kinematics Use $v_{B/A}$ to find $v_{B,2}$

$v_{B/A} = v_B - v_A$

$\Rightarrow v_{B2} = v_{B/A} + v_{A2}$ (2)

④ Solve Use (1) & (2). Solve $v_{A,2}$

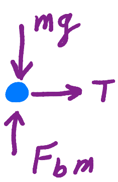
$\Rightarrow (m+M) v_{A1} = m v_{A2} + M (v_{B/A} + v_{A2})$

$\Rightarrow (m+M) v_{A1} = (m+M) v_{A2} + M v_{B/A}$

$\Rightarrow v_{A2}$

(b)

① FBD tug alone



② Kinetics LIM x-dir

$m v_{A1} + \int T dt = m v_{A2}$

③ Kinematics

Replace $\int T dt$ for: $T_{Avg} \Delta t$

④ Solve

$\Rightarrow m v_{A1} + T_{Avg} \Delta t = m v_{A2}$

$\Rightarrow T_{Avg} = \dfrac{m (v_{A2} - v_{A1})}{\Delta t}$

Summary: Linear Impulse-Momentum Equation 1

FUNDAMENTAL equation: the linear impulse-momentum equation:

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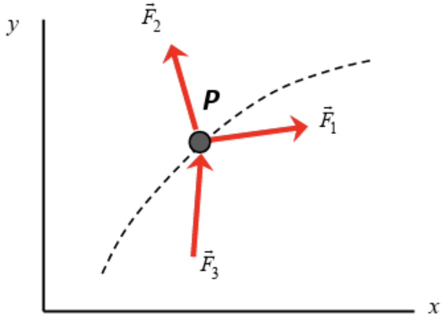
$$m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \left(\sum \vec{F}\right) dt$$



Lec 21 Short Feedback Form:

CONSERVATION: If there is no net force acting of the system in a given direction (say x), $\int_1^2 \left(\sum \vec{F}\right)_x dt = 0$ *, then linear momentum in that direction is conserved.*

SYSTEM CHOICE: Make your choice of system as "large" as reasonable – you want to make as many forces as possible INTERNAL to the system.