ME 274 Lecture 20

Particle Kinetics – Work/energy Part 2

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03/02/26

Housekeeping/Announcements

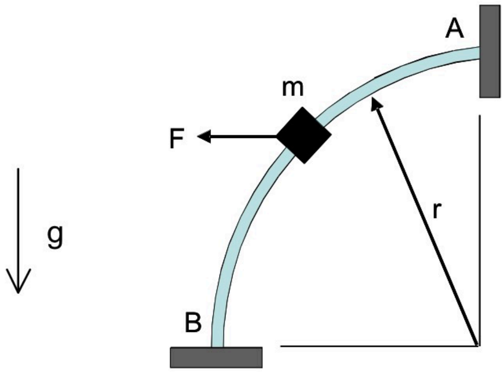
***Reminder for Henny to wear a mic during the lecture.

1. **HW 19 (4.G and 4.H) due today!!**
2. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME.
    - Feel free to come to OHs if you need help with HW or discussing exam problems.
3. Bonus quiz grade at end of the semester if we get a good response rate to QR code surveys at the end of lecture.
    - If you are unable to attend lecture on that day/forget to fill it out:
        - feel free to give feedback based on the content of that lecture's slides.
        - Way of you reviewing previous content and giving feedback to me.

Homework H.4.G

Given: The collar, shown below, of mass m, starts from rest at point A. A constant force F is applied to the collar in the direction shown. Note that the mechanism lies in the vertical plane. Assume all surfaces to be smooth.

Find: Determine the speed of the collar when it reaches point B.



Please leave your answer in terms of, at most, m, g, F and r.

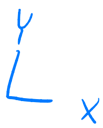
o p. 211.

① Use cartesian. despite curved path.

② go page 211

↳ you can break up "x" & "y" components of work

③ In problem H4.6, $\vec{F}$ has only x components:



$F_y = 0$

$$U^{(F)}_{1\to 2} = \int_1^2 F_x \, dx \qquad F_x = -F$$

$$= -F \Delta x$$

Kinetics: Four-step problem solving method

1. **FBDs:**
   - Draw appropriate FBD**(s).**
   - Choose your coordinate system.
2. **Kinetics:**
   - Choose what solution method for the particular problem at hand **(we will go over these in the coming days…)**:
     - Newton/Euler (lectures 15-18)
     - Work/Energy (lectures 19-20)
     - Linear impulse/momentum
     - Angular impulse/momentum
3. **Kinematics:**
   - Perform needed kinematic analysis (position/velocity/acceleration)
   - Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem
4. **Solve:**
   - Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
     a) Draw more FBDs
     b) You will need to do more kinematic analysis

[pg. 184 content]

Particle Kinetics: Work-Energy Equation

1. Work-energy equation for analyzing the *change in speed of a particle as a results of forces on the particle* (or systems of particles).

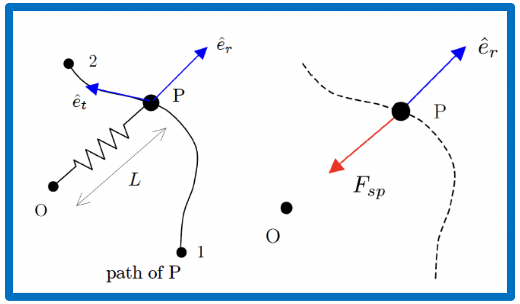
$$T_2 + V_2 = T_1 + V_1 + U_{1\to 2}^{(nc)}$$

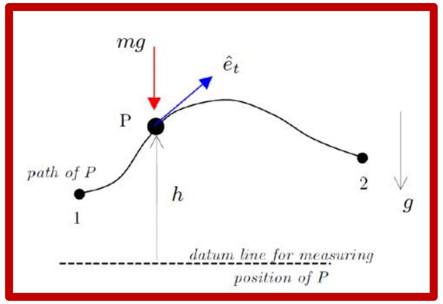
- T = Kinetic Energy.
- V = Potential Energy.
- U = Total Work done on the system

2. When to use W/E equation? Given a ***change in position*** we are asked to find a ***change in speed***

3. Conservative Forces - Class of forces for which *their work between positions 1 and 2* ***is independent of the path*** *over which the forces act as the particle moves from position 1 to position 2.*

- Examples of **conservative forces:**
  1. Force due to the action of a ***spring onto a particle***
  2. Force due to the ***weight of a particle***





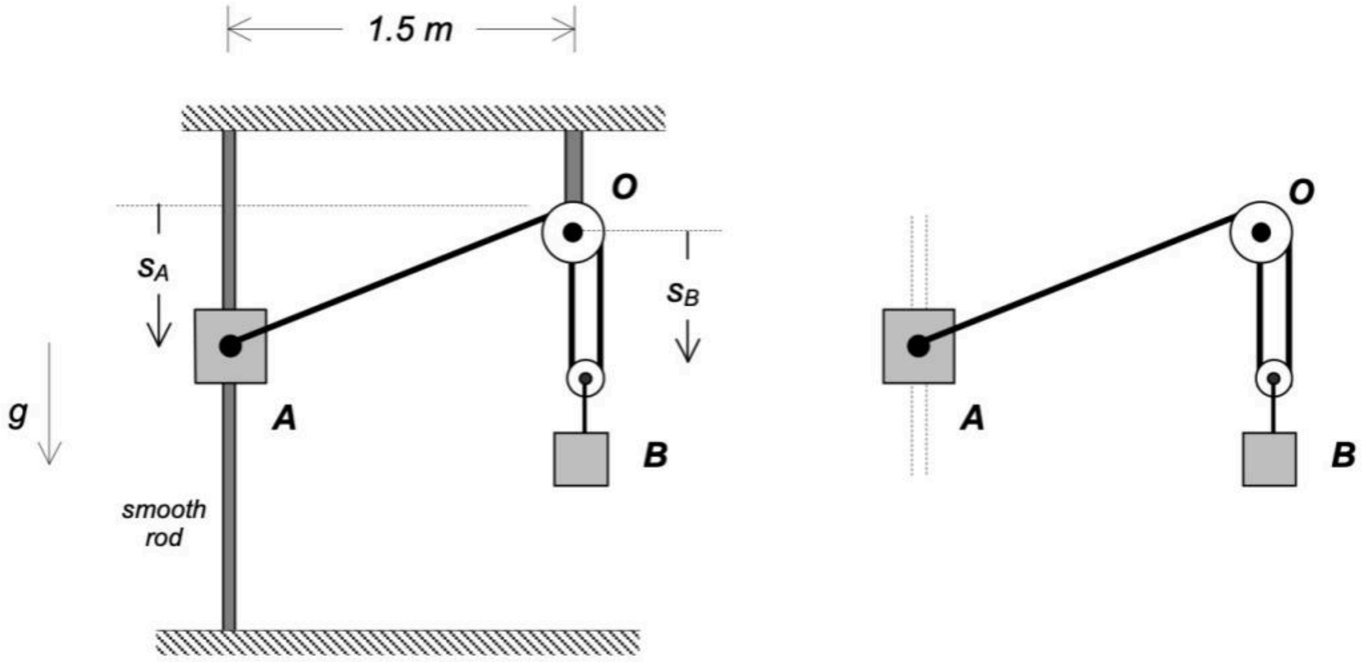
4. Friction always opposes direction of motion [p. 216]
5. Normal force always has zero work. Since it is perpendicular to the path [p. 216]

[pg. 209 content]

In-class Example 4.2

Given: Particles A and B (having masses of m_A and m_B) are interconnected by the cable-pulley system shown in the figure. Both particles are constrained to vertical motion with particle A able to slide on a smooth vertical rod. The system is released at $s_A = 0$ m with A traveling downward with a speed of v_{A1}. Assume the pulleys to be small, massless and frictionless.

Find: Find the speed of particle A when A has reached the position of s_A.



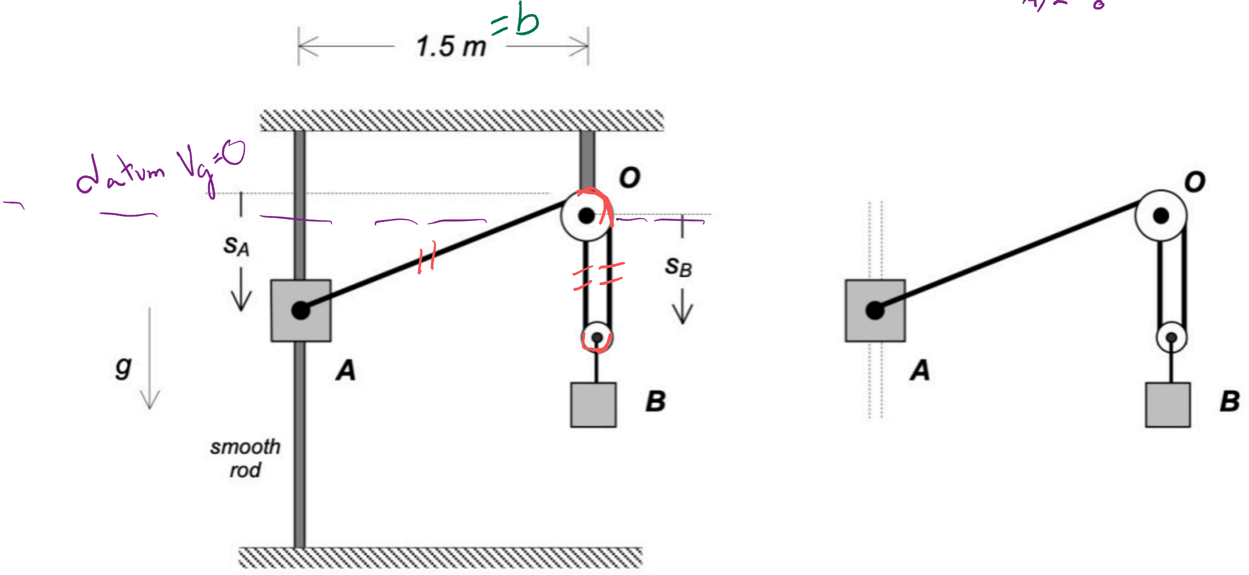
Use the following parameters in your analysis: $m_A = 10$ kg, $m_B = 10$ kg, $v_{A1} = 5$ m/s and $s_A = 2$ m.

In-class Example 4.2 (Similar to hw due Thu)

Tip: Put everything in FBD w/ W/E probs

Given: Particles A and B (having masses of m_A and m_B) are interconnected by the cable-pulley system shown in the figure. Both particles are constrained to vertical motion with particle A able to slide on a smooth vertical rod. The system is released at $s_A = 0$ m with A traveling downward with a speed of v_{A1}. Assume the pulleys to be small, massless and frictionless.

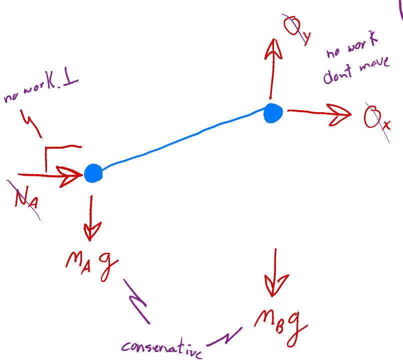
Find: Find the speed of particle A when A has reached the position of s_A. $v_{A,2}$?



Use the following parameters in your analysis: $m_A = 10$ kg, $m_B = 10$ kg, $v_{A1} = 5$ m/s and $s_A = 2$ m.

$m_A = 10$ kg $v_{A1} = 5$ m/s
$m_B = 10$ kg $S_A = 2$ m

① FBD include A, B, cable, pulleys



Why no tension forces?
↳ they're internal forces that cancel each other out

② Kinetics W/E

$$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$$

$T_1 = \frac{1}{2} m_A v_{A1}^2 + \frac{1}{2} m_B v_{B1}^2$

$V_1 = 0 - m_B g (s_B)$; datum

$U_{1\to2}^{(NC)} = 0$; (mech. energy is conserved)

$T_2 = \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2$

$V_2 = -m_A g s_{A,2} - m_B g s_{B,2}$; datum

③ Plug-in W/E

$$\frac{1}{2} m_A v_{A,1}^2 + \frac{1}{2} m_B v_{B,1}^2 - m_B g s_{B,1} + 0 = \frac{1}{2} m_A v_{A,2}^2 + \frac{1}{2} m_B v_{B,2}^2 - m_A g s_{A,2} - m_B g s_{B,2} \quad (1)$$

④ Kinematics   Pulley Lengths

L = Length of cable

$= \sqrt{s_A^2 + b^2} + s_B + s_B + \text{Constants} = \text{constants}$

⑤ $\frac{dL}{dt}$

$$\frac{dL}{dt} = \frac{1}{2} \frac{2 s_A \dot{s}_A}{\sqrt{s_A^2 + b^2}} + 2\dot{s}_B = 0$$

$\Rightarrow \dot{s}_B = -\frac{s_A}{2\sqrt{s_A^2 + b^2}} \dot{s}_A$; solve for $\dot{s}_B$

⑥ Get eqns for v_{B1} & v_{B2}. (2) & (3) respectively

$\Rightarrow |\dot{s}_{B1}| = v_{B1} = \frac{s_{A1}}{\sqrt{s_{A1}^2 + b^2}} v_{A1} \quad (2)$ $|\dot{s}_{B2}| = v_{B2} = \frac{s_{A2}}{\sqrt{s_{A2}^2 + b^2}} v_{A2} \quad (3)$

⑦ Get an eqn relating $s_{B,1}$ & $s_{B,2}$

$\Rightarrow s_{B1} - s_{B2} = \frac{1}{2}\sqrt{s_{A,1}^2 + b^2} - \frac{1}{2}\sqrt{s_{A,2}^2 + b^2} \quad (4)$

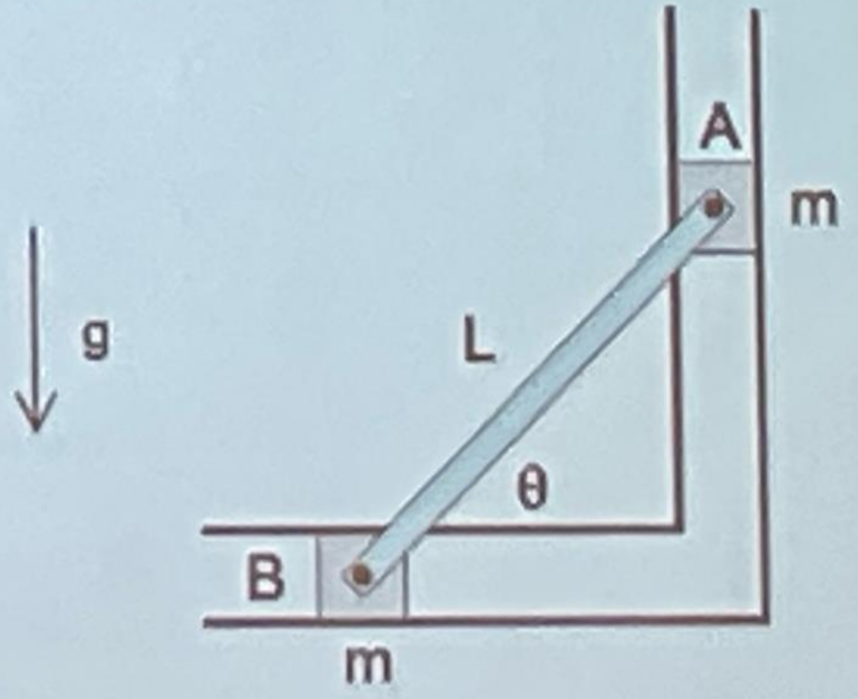
⑧ Solve

$(1-4) \Rightarrow v_{A,2} = \underline{?}$

Homework H.4.H

Given: The mechanism shown below, consisting of two blocks A and B and a massless bar, is released from rest in the configuration shown.

Find: Determine the speed of block A when it reaches the bottom of the slot.



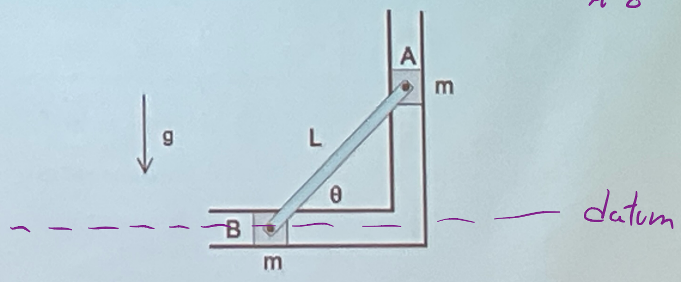
Use the following parameters in your analysis: $m = 6$ kg, $\theta = 53.13°$ and $L = 2$ m.

H4.H - similar problem to hwk. Wed night

Homework H.4.H

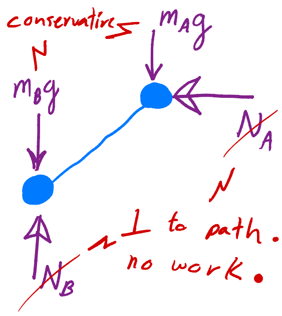
Given: The mechanism shown below, consisting of two blocks A and B and a massless bar, is released from rest in the configuration shown.

Find: Determine the speed of block A when it reaches the bottom of the slot. v_A ?



Use the following parameters in your analysis: $m = 6$ kg, $\theta = 53.13°$ and $L = 2$ m.

① FBD entire system A, B, bar



② Kinetics W/E

$$T_1 + V_1 + U^{(nc)}_{1\to2} = T_2 + V_2$$

③ Terms & datum

$T_1 = 0$ (RFR)

$V_1 = + mg\,L\sin\theta$

$U^{(NC)}_{1\to2} = 0$

$T_2 = \frac{1}{2} m v_{A,2}^2 + \frac{1}{2} m v_{B,2}^2$

$V_2 = 0$; @ datum

③ Plug-in

$$mg\,L\sin\theta = \frac{1}{2} m v_{A,2}^2 + \frac{1}{2} m v_{B,2}^2$$

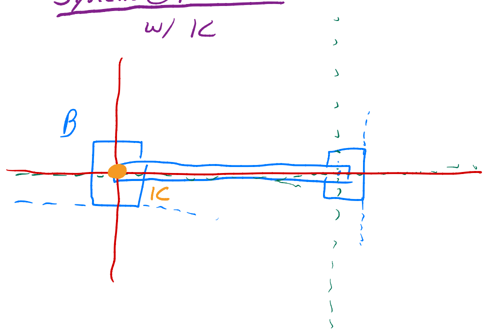
1 eqn 2 unkns...

④ Kinematics

Where's IC ?

$B = IC_{AB} \Rightarrow v_B = 0$

System @ position 2: w/ IC



⑤ Solve 1 eqn 1 unkn

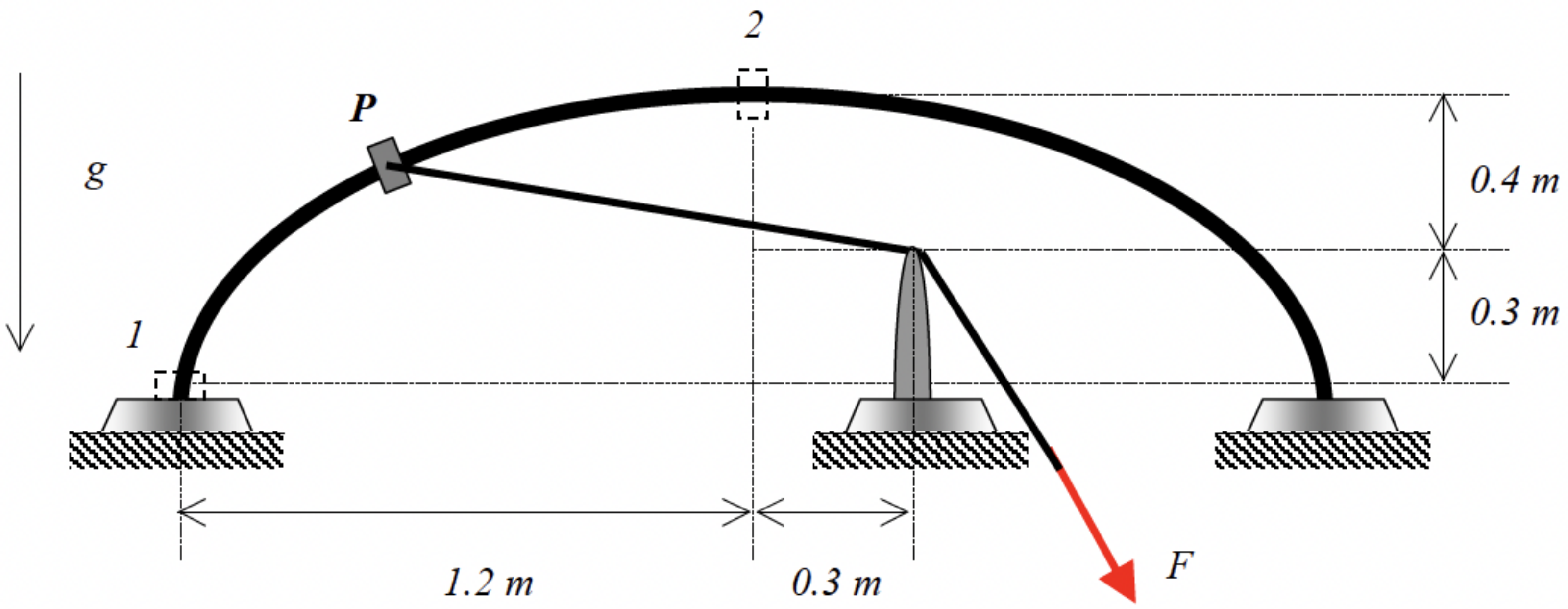
$$mg\,L\sin\theta = \frac{1}{2} m v_{A2}^2$$

$$\Rightarrow v_{A2} = \sqrt{2gL\sin\theta}$$

Example 4.B.5

Given: Slider P, having a mass of $m = 0.6$ kg, moves freely along the fixed, smooth, curved rod from position A to position B in the vertical plane under the action of the constant $F = 20$ N tension in the cord.

Find: Determine the speed of the slider at position 2, if the slider is released from rest at position 1.

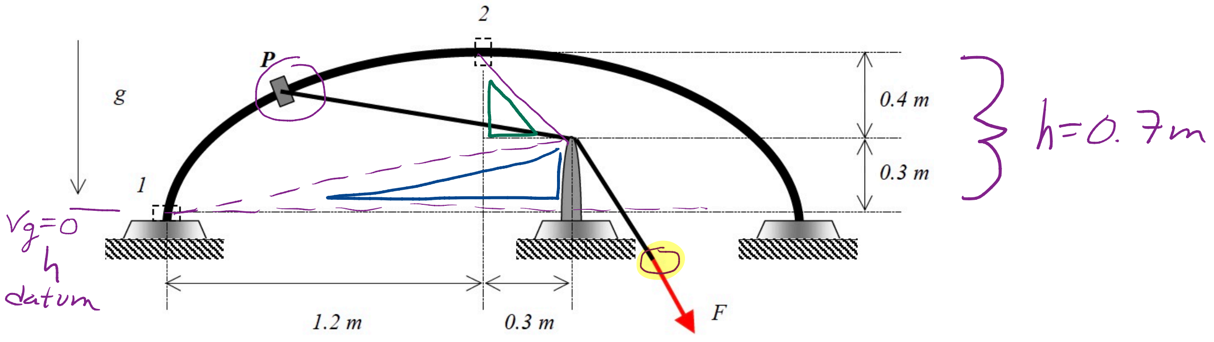


[pg. 223]

Example 4.B.5 p. 223 — one of Rhoad's fav examples

Given: Slider P, having a mass of $m = 0.6$ kg, moves freely along the fixed, smooth, curved rod from position A to position B in the vertical plane under the action of the constant $F = 20$ N tension in the cord.

Find: Determine the speed of the slider at position 2, if the slider is released from rest at position 1. v_2?



$m = 0.6$ kg

$F = 20$ N

Kinematics

d_1, 0.3, 1.5

0.4, $d_2 = 0.5$, 0.3

$d_1 = \sqrt{1.5^2 + 0.3^2}$

① FBD

conservative: mg

non-conservative: F

N — no work

Kinetics

② W/E Eqn

$$T_1 + V_1 + U^{NC}_{1\to 2} = T_2 + V_2$$

③ Determine terms, datum.

$T_1 = 0$; (RFR) released from rest

$V_1 = 0$

$$U^{NC}_{1\to 2} = \int \vec{F} \cdot d\vec{r}$$

$$= Fd$$; look at place where F is applied instead of P

$$= F(d_1 - d_2)$$

$$T_2 = \frac{1}{2} m v_2^2$$

$$V_2 = mg(0.7)$$

④ Solve

$$\Rightarrow F(d_1 - d_2) = \frac{1}{2} m v_2^2 + 0.7\,mg$$

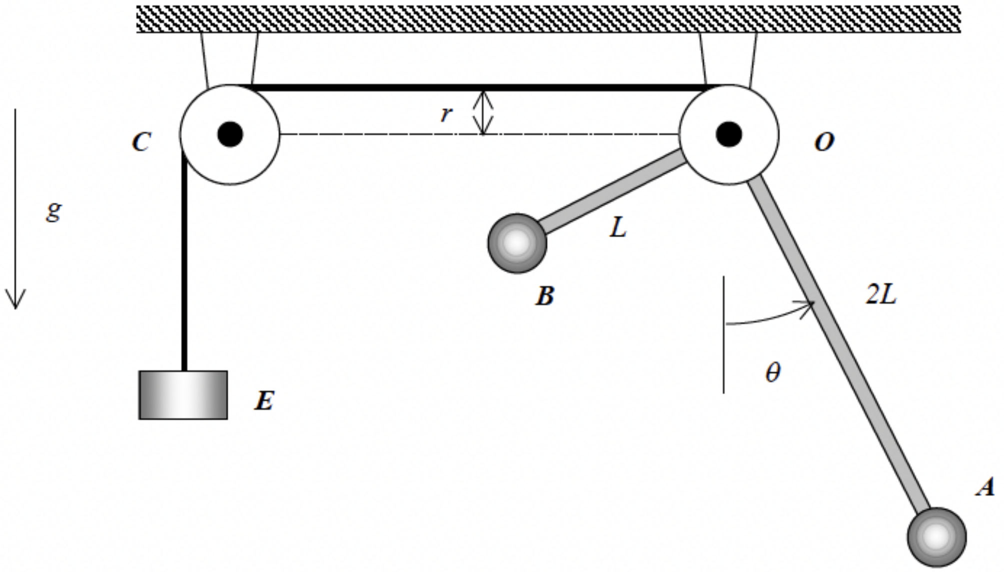
$$\Rightarrow v_2$$

Example 4.B.6

Given: The system shown below is made up of particles A, B and E (having masses of 5 kg, 20 kg and 80 kg, respectively). Lightweight bars OB and OA are welded together such that there is a right angle between the two bars, and these bars are welded to the pulley at O. Consider the mass of the pulleys to be negligible. The system is released from rest when $\theta = 0$.

Find: Determine the speed of particle E when $\theta = 90°$.

Use the following parameters in your analysis: $r = 0.1$ m and $L = 0.5$ m.



[pg. 224]

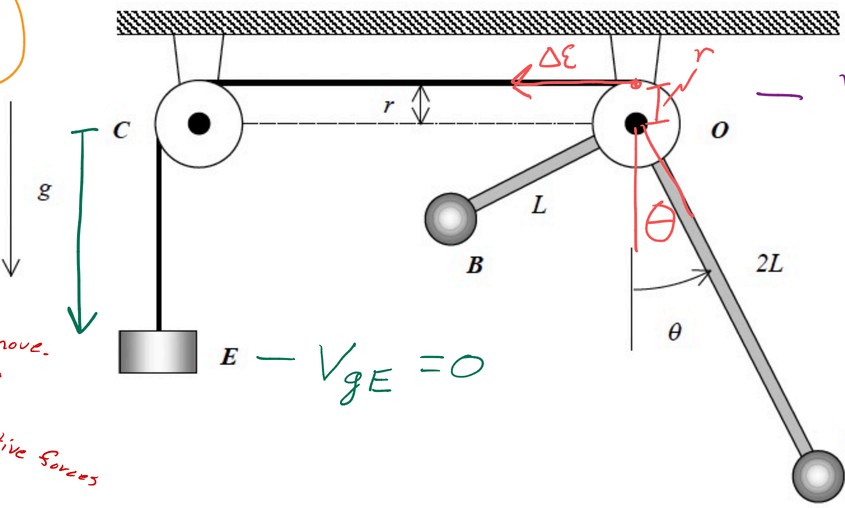
Example 4.B.6 p. 224

Given: The system shown below is made up of particles A, B and E (having masses of 5 kg, 20 kg and 80 kg, respectively). Lightweight bars OB and OA are welded together such that there is a right angle between the two bars, and these bars are welded to the pulley at O. Consider the mass of the pulleys to be negligible. The system is released from rest when $\theta = 0$.

Find: Determine the speed of particle E when $\theta = 90°$. v_E ?

Use the following parameters in your analysis: $r = 0.1$ m and $L = 0.5$ m.

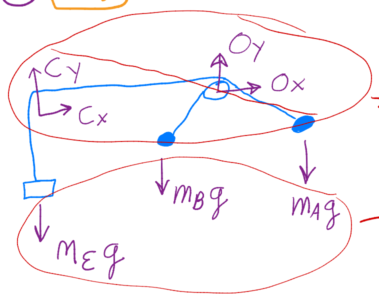
$m_A = 5\ kg$, $r = 0.1\ m$
$m_B = 20\ kg$, $L = 0.5\ m$
$m_E = 80\ kg$



− $V_g = 0$

E − $V_{gE} = 0$

① (FBD)



— don't move. no work.

— conservative forces

② W/E. Bc looking for speed after change in position

$T_1 + V_1 + U^{NC}_{1\to2} = T_2 + V_2$

③ Terms, Define datum. 2 datums

$T_1 = 0$ (RFR)

$V_1 = -m_A g(2L)$

$U^{NC}_{1\to2} = 0$

$T_2 = \frac{1}{2} m_E v_E^2 + \frac{1}{2} m_B v_B^2 + \frac{1}{2} m_A v_A^2$

$V_2 = -m_B g L - m_E g \Delta E$ (ΔE: tbd)

④ Solve

$\Rightarrow -2 m_A g L = \frac{1}{2} m_E v_E^2 + \frac{1}{2} m_B v_B^2 + \frac{1}{2} m_A v_A^2 - m_B g L - m_E g \Delta E \leftarrow$ (1)

?

⑤ Aside: Use arc length, find ΔE & eqns for v_B, v_A, $\dot{\theta}$

Arc Length = s = rθ

$\Delta E = r\theta$
$= \frac{\pi r}{2}$ (2)

$v_B = L\dot{\theta}$
$v_A = 2L\dot{\theta}$
$v_E = r\dot{\theta}$
$\Longrightarrow$

$\dot{\theta} = \frac{v_E}{r}$ (3)

$v_B = \frac{v_E L}{r}$ (4)

$v_A = \frac{2 v_E L}{r}$ (5)

⑥ 5 eqns 5 unkns.

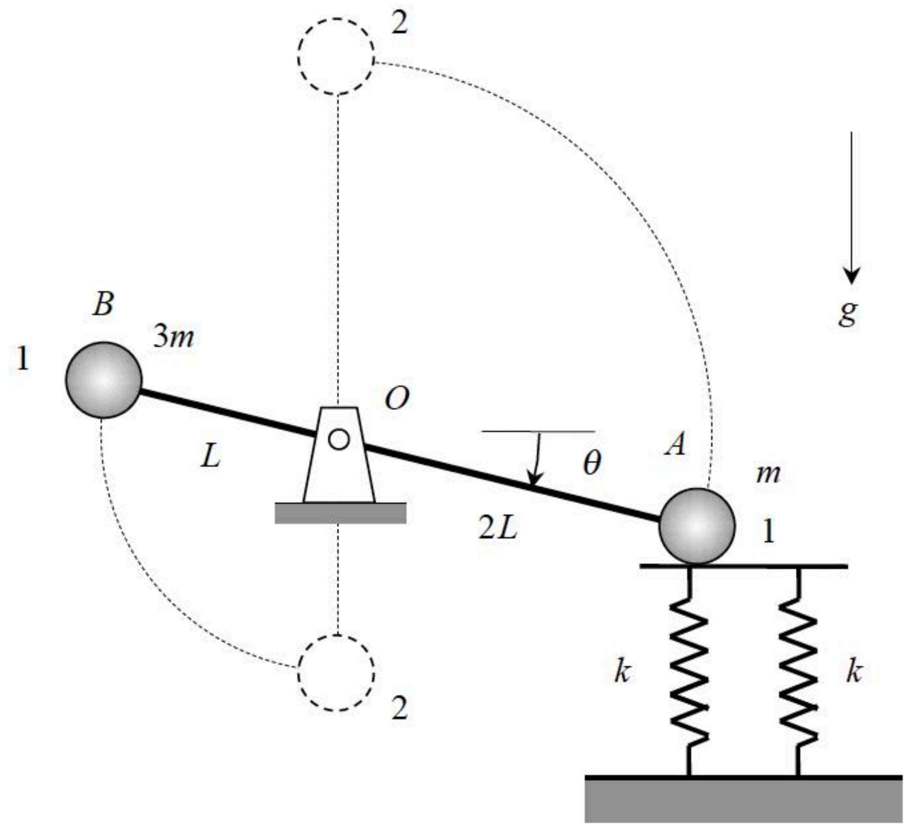
Solve for v_E

Example 4.B.7

Given: Particles A and B, having masses of m and $2m$, respectively, are connected by rigid bar AB, with AB having negligible mass. Bar AB is pinned to ground with a pin joint at O. This system is released from rest at position 1 with $\theta = \theta_1$, with A in contact with a pair of identical springs, as shown in the figure. Each spring has a stiffness of k, and the springs are unstretched when $\theta = 0$. Assume the dimensions of the particles to be negligible.

Find: Determine the speeds of particles A and B at position 2, where in position 2 particle A is directly above O.

Use the following parameters in your analysis: $\theta_1 = 36.87°$, $L = 0.1$ m, $m = 10$ kg and $k = 100$ N/m.



[pg. 225]

Example 4.B.7 p. 225

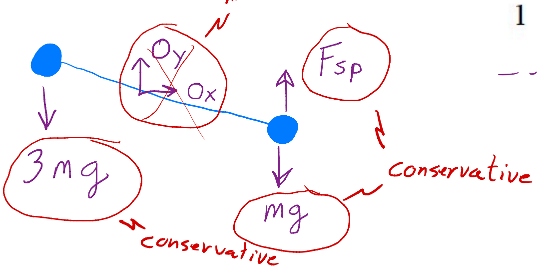
Given: Particles A and B, having masses of m and $2m$, respectively, are connected by rigid bar AB, with AB having negligible mass. Bar AB is pinned to ground with a pin joint at O. This system is released from rest at position 1 with $\theta = \theta_1$, with A in contact with a pair of identical springs, as shown in the figure. Each spring has a stiffness of k, and the springs are unstretched when $\theta = 0$. Assume the dimensions of the particles to be negligible.

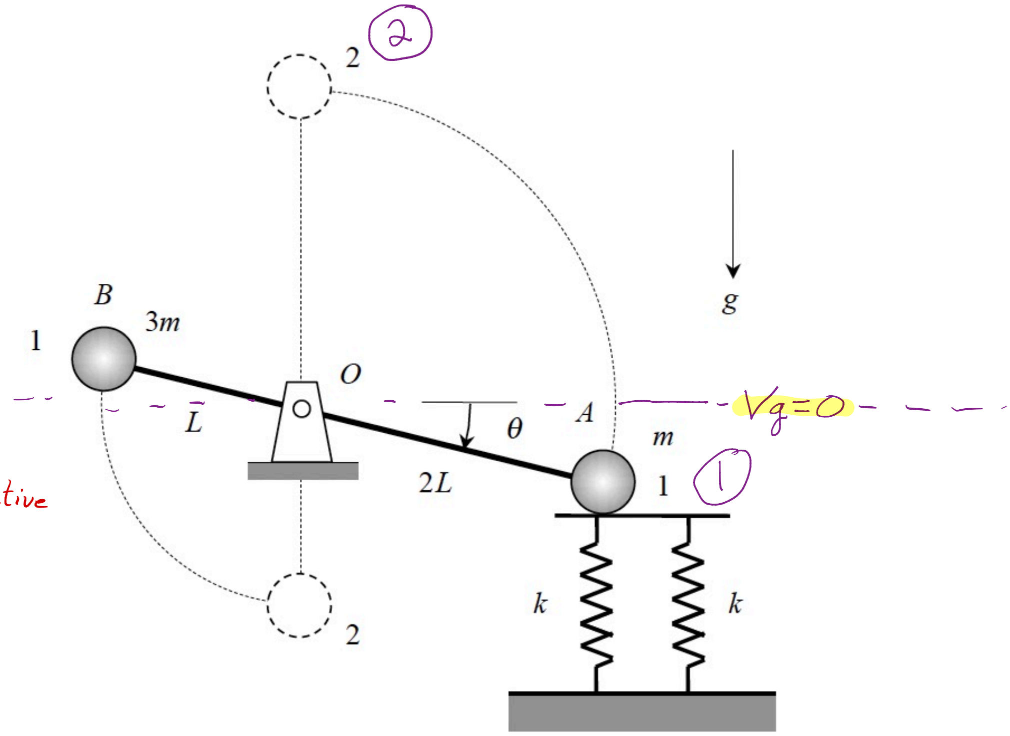
Find: Determine the speeds of particles A and B at position 2, where in position 2 particle A is directly above O. $v_{A,2}$? $v_{B,2}$?

Use the following parameters in your analysis: $\theta_1 = 36.87°$, $L = 0.1$ m, $m = 10$ kg and $k = 100$ N/m.

$\theta_1 = 36.87°$
$L = 0.1\,m$
$m = 10\,Kg$
$K = 100\,N/m$

① FBD





Kinetics

② W/E.

$$T_1 + V_1 + U_{1\to2}^{NC} = T_2 + V_2$$

③ Terms. datum

$T_1 = 0$ (RFR)

$$V_1 = \underbrace{-mg(2L\sin\theta)}_{\text{part A}} + \underbrace{3mgL\sin\theta}_{\text{part B}} + \underbrace{\tfrac{1}{2}(2k)(2L\sin\theta)^2}_{\text{spring}}$$

(2k: stiffness; 2L sin θ: displacement/compression)

$U_{1\to2}^{NC} = 0$; no non-cons forces

$T_2 = \frac{1}{2} m v_A^2 + \frac{1}{2}(3m) v_B^2$; particles A & B

$V_2 = mg2L - 3mgL$; particles A & B. no spring

④ Solve

$$\Rightarrow -2mgL\sin\theta + 3mgL\sin\theta + 4kL^2\sin^2\theta = \frac{1}{2} m v_A^2 + \frac{3}{2} m v_B^2 - mgL \quad (1)$$

⑤ Kinematics Use arc length formula. $S = r\theta$. $\frac{ds}{dt}$. r = const.

$V_A = 2L\dot{\theta}$

$v_B = 2\dot{\theta}$

$\Rightarrow v_A = 2v_B \quad (2)$

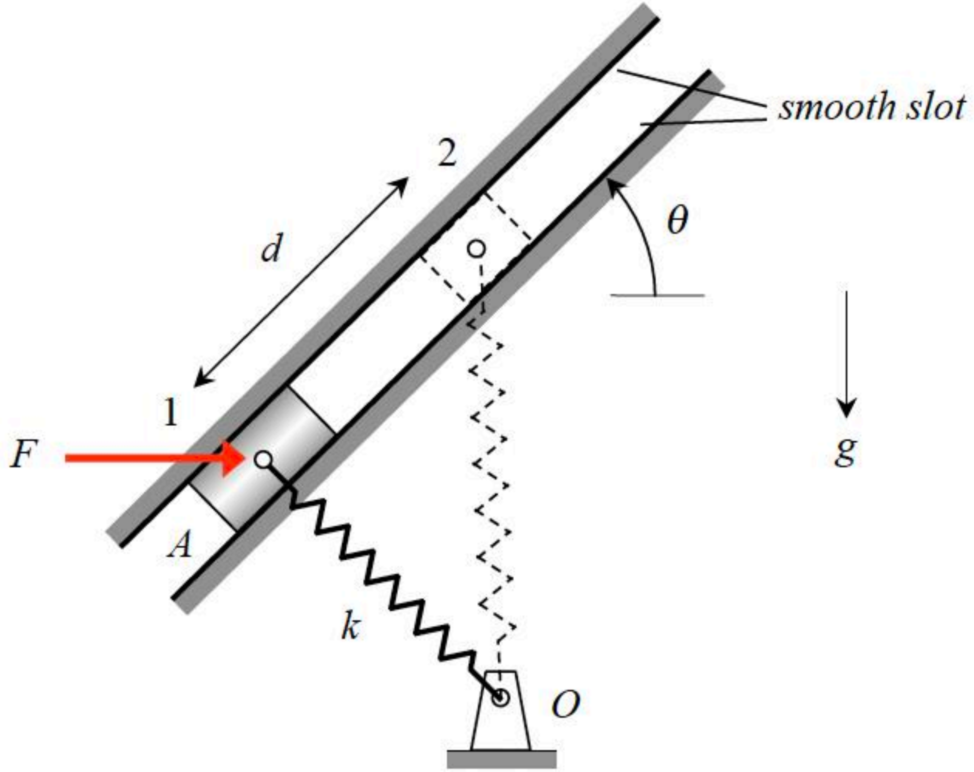
⑥ 2 eqns. 2 unkns. Solve for v_A & v_B

Example 4.B.8

Given: Block A, having a mass of m, is able to slide within a smooth, inclined slot. A spring of stiffness k is attached between A and a fixed pin at O, as shown in the figure. A constant horizontal force F acts on the block. The system is released from rest at position 1, where at position 1 the spring is perpendicular to the slot and is unstretched. At position 2, block A is directly above O.

Find: Assuming that block A reaches position 2, determine the speed of A at position 2. If A does not reach position 2, explain why not.

Use the following parameters in your anlaysis: $m = 5$ kg, $k = 50$ N/m, $\theta = 53.13°$, $d = 0.2$ m and $F = 300$ N.



[pg. 226]

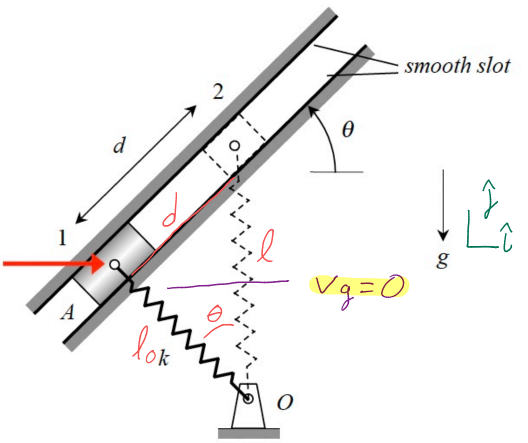
Example 4.B.8 p.226

Given: Block A, having a mass of m, is able to slide within a smooth, inclined slot. A spring of stiffness k is attached between A and a fixed pin at O, as shown in the figure. A constant horizontal force F acts on the block. The system is released from rest at position 1, where at position 1 the spring is perpendicular to the slot and is unstretched. At position 2, block A is directly above O.

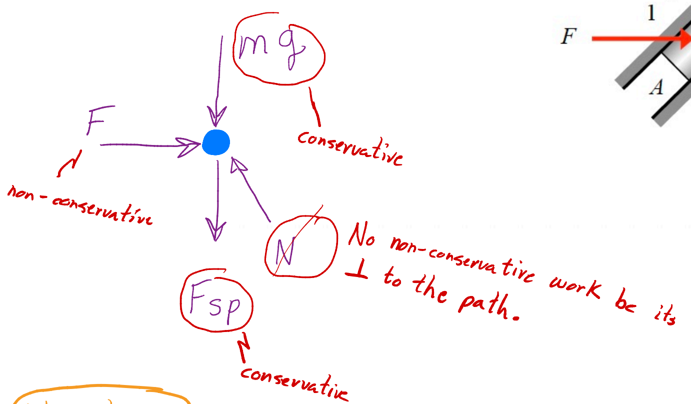
Find: Assuming that block A reaches position 2, determine the speed of A at position 2. If A does not reach position 2, explain why not. v_2? explain

Use the following parameters in your anlaysis: $m = 5$ kg, $k = 50$ N/m, $\theta = 53.13°$, $d = 0.2$ m and $F = 300$ N.

$m = 5$ kg, $d = 0.2$ m
$K = 50$ N/m, $F = 300$ N
$\theta = 53.13°$



① FBD



Kinetics

② W/E.

$$T_1 + V_1 + U_{1\to2}^{NC} = T_2 + V_2$$

③ Terms. datum.

$T_1 = 0$ (RFR)

$V_1 = 0$; datum. spring @ rest

$U_{1\to2}^{NC} = \int \vec{F} \cdot d\vec{r}$; Applied force, $\vec{F}$. differential displacement, $d\vec{r}$

$= \int F\hat{i} \cdot (dx\,\hat{i} + dy\,\hat{j})$; define cartesian unit vectors. $\hat{i}\cdot\hat{i} = 1$. $\hat{i}\cdot\hat{j} = 0$.

$= F\Delta x$; F. amount of change in position in x

$= F d\cos\theta$

$T_2 = \frac{1}{2} m v_2^2$

Aside to find Δ / displacement of spring

$V_2 = mg(d\sin\theta) + \frac{1}{2}k\Delta^2$; mgh. spring: $\int F\cdot dr$. integral of hooks law (F)

Aside:

$\tan\theta = \frac{d}{l_0}$

$\Rightarrow l_0 = \frac{d}{\tan\theta}$

$l = \sqrt{l_0^2 + d^2}$

$\Delta = l - l_0$

④ Solve

$\Rightarrow F d\cos\theta = \frac{1}{2} m v_2^2 + mgd\sin\theta + \frac{1}{2}k(l - l_0)^2$

$\Rightarrow v_2$; is it possible it could not reach this state?

Do we take sqrt() of a negative #?

if yes, then never reached

Summary: Work-Energy Equation 2

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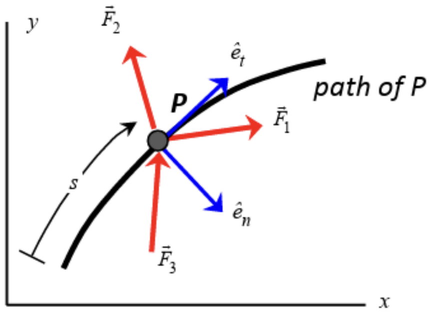
FUNDAMENTAL equation: the work-energy equation

$$T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2$$



Lec 20 Short Feedback Form:

SOLUTION PROCESS:

1. Draw free body diagram (FBD) for system of your choice (see comment below on system choice).
2. Write down the work-energy equation.
3. Write down the appropriate kinematics (velocity) equations for the problem.
4. If you have enough equations, solve for the desired unknowns. If you do not have enough equations, then you have probably missed some information from kinematics.

SYSTEM CHOICE: Make your choice of system as "large" as reasonable – you want to make workless forces INTERNAL to the system.

CONSERVATION: If no work is done on the system, $U_{1\to 2}^{(nc)} = 0$, *then energy is conserved.*