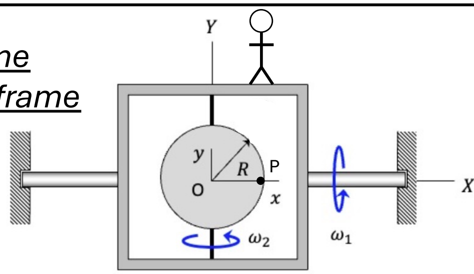
Problem 3.B SOLUTION

A square frame rotates about a fixed horizontal axis with a constant rate of ω_1. A disk rotates about a diametral axis with a constant rate of ω_2 with respect to the frame. Determine the acceleration of point P on the perimeter of the disk using:

$$\vec{a}_P = \vec{a}_O + (\vec{a}_{P/O})_{rel} + \vec{\alpha}\times\vec{r}_{P/O} + 2\vec{\omega}\times(\vec{v}_{P/O})_{rel} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{P/O})$$

Attach observer to the <u>square frame</u>

Attach the *xyz*-axes to the <u>square frame</u>



$\vec{\omega}$ = angular velocity of observer

$= \omega_1 \hat{I}$

$\vec{\alpha}$ = angular acceleration of observer

$= \cancel{\dot{\omega}_1}^{0}\hat{I} + \omega_1 \cancel{\dot{\hat{I}}}^{0} = \vec{0}$

$(\vec{v}_{P/O})_{rel}$ = velocity of P as seen by the observer

$= -R\omega_2\hat{k}$

$(\vec{a}_{P/O})_{rel}$ = acceleration of P as seen by the observer

$= -R\omega_2^2\hat{i}$

$\vec{a}_P = \cancel{\vec{a}_O}^{0} + (\vec{a}_{P/O})_{rel} + \cancel{\vec{\alpha}}^{0}\times\vec{r}_{P/O} + 2\vec{\omega}\times(\vec{v}_{P/O})_{rel} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{P/O})$

$= \vec{0} - R\omega_2^2\hat{i} + \vec{0} + 2(\omega_1\hat{i})\times(-R\omega_2\hat{k})$

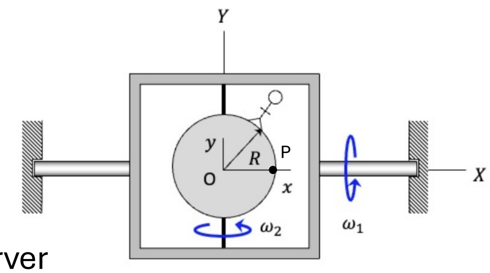
$+ (\omega_1\hat{i})\times(\omega_1\hat{i}\cancel{\times}^{0} R\hat{i})$

$= -R\omega_2^2\hat{i} + 2R\omega_1\omega_2\hat{j}$

SAME! ☺

Attach observer to the <u>disk</u>

Attach the *xyz*-axes to the <u>disk</u>



$\vec{\omega}$ = angular velocity of observer

$= \omega_1\hat{I} + \omega_2\hat{j}$

$\vec{\alpha}$ = angular acceleration of observer

$= \cancel{\dot{\omega}_1}^{0}\hat{I} + \omega_1\cancel{\dot{\hat{I}}}^{0} + \cancel{\dot{\omega}_2}^{0}\hat{j} + \omega_2\dot{\hat{j}} = \omega_2(\vec{\omega}\times\hat{j})$

$= \omega_2(\omega_1\hat{i} + \omega_2\hat{j})\times\hat{j} = \omega_1\omega_2\hat{k}$

$(\vec{v}_{P/O})_{rel}$ = velocity of P as seen by the observer

$= \vec{0}$

$(\vec{a}_{P/O})_{rel}$ = acceleration of P as seen by the observer

$= \vec{0}$

$\vec{a}_P = \cancel{\vec{a}_O}^{0} + \underbrace{\cancel{(\vec{a}_{P/O})_{rel}}^{0} + \vec{\alpha}\times\vec{r}_{P/O}}_{R\omega_1\omega_2\hat{j}} + 2\vec{\omega}\times\cancel{(\vec{v}_{P/O})_{rel}}^{0}$

$+ \vec{\omega}\times(\vec{\omega}\times\vec{r}_{P/O})$

$= (\omega_1\omega_2\hat{k})\times(R\hat{i})$

$+ \underbrace{(\omega_1\hat{i} + \omega_2\hat{j})\times[\underbrace{(\omega_1\hat{i} + \omega_2\hat{j})\times(R\hat{i})}_{-R\omega_2\hat{k}}]}_{-R\omega_1\omega_2\hat{i} + R\omega_1\omega_2\hat{j}}$

$= -R\omega_2^2\hat{i} + 2R\omega_1\omega_2\hat{j}$