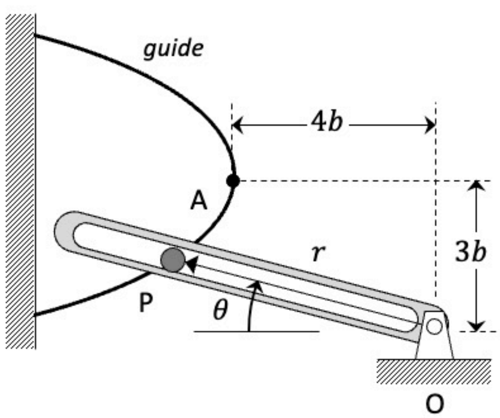
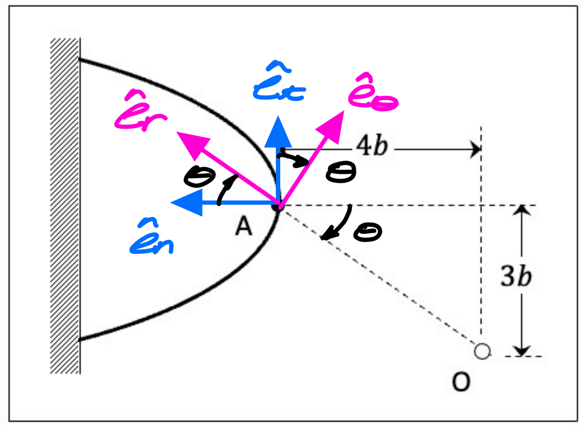
ME 274 Exam No. 1 Name $\underline{\text{SOLUTION}}$

February 12, 2026

PROBLEM NO. 1 *(20 points)*

Given: Particle P is constrained to move along a curved guide. In addition, P is constrained to move in a slotted arm with the arm being pinned to ground at end O. The arm is being rotated in the clockwise direction with a *constant* rate of $\dot{\theta} = \omega_0$. The position of P is described in terms of the radial distance r and the angle θ of the arm. The radius of curvature of the guide at point A is known to be $\rho = 2b$.





$r = 5b$; $\cos\theta = \frac{4}{5}$; $\sin\theta = \frac{3}{5}$

Find: For this problem, you are asked to determine $\dot{r}$, $\ddot{r}$, the speed of P (v), and the rate of change of speed of P ($\dot{v}$) when P is *at position A* on the guide, where A is at the extreme right position of the guide. You are asked to follow the steps outlined below. Failure to follow these steps can result in a loss of points.

Solution:

(a) On the above right figure, show the polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$ and the path unit vectors $\hat{e}_t$ and $\hat{e}_n$ *at position A* on the guide.

(b) Write the polar unit vectors in terms of the path unit vectors for when P is at position A.

$$\hat{e}_r = \sin\theta\,\hat{e}_t + \cos\theta\,\hat{e}_n$$
$$e_\theta = \cos\theta\,\hat{e}_t - \sin\theta\,\hat{e}_t$$

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PROBLEM NO. 1 (*continued)*

(c) Determine the values for $\dot{r}$ and v. Express these in terms of b and ω_0.

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$v\hat{e}_t = \dot{r}(\sin\theta\,\hat{e}_t + \cos\theta\,\hat{e}_n) + r\dot{\theta}(\cos\theta\,\hat{e}_t - \sin\theta\,\hat{e}_n)$$

$$\begin{cases} \hat{e}_n: & 0 = \dot{r}\cos\theta - r\dot{\theta}\sin\theta \Rightarrow \dot{r} = \frac{15}{4}b\omega_0 \longleftarrow \dot{r} \\ \hat{e}_t: & v = \dot{r}\sin\theta + r\dot{\theta}\cos\theta = \frac{25}{4}b\omega_0 \longleftarrow v \end{cases}$$

(d) Develop a set of two algebraic equations that can be used to solve for the values for $\ddot{r}$ and $\dot{v}$ in terms of ρ_0, b and ω_0. *You need not solve these equations.*

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\cancel{\ddot{\theta}}^{\,0} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$$\dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n = (\ddot{r} - r\dot{\theta}^2)(\sin\theta\,\hat{e}_t + \cos\theta\,\hat{e}_n) + 2\dot{r}\dot{\theta}(\cos\theta\,\hat{e}_t - \sin\theta\,\hat{e}_n)$$

$$\begin{cases} \hat{e}_n: & \frac{v^2}{\rho} = (\ddot{r} - r\omega_0^2)\cos\theta - 2\dot{r}\omega_0\sin\theta \longleftarrow \\ \hat{e}_t & \dot{v} = (\ddot{r} - r\omega_0^2)\sin\theta + 2\dot{r}\omega_0\cos\theta \longleftarrow \end{cases}$$

PROBLEM NO. 1 (*continued*)

(c) Determine the values for $\dot{r}$ and v. Express these in terms of b and ω_0.

$\bar{v} = v\hat{e}_t$

- $r\dot{\theta} = \bar{v}\cdot\hat{e}_\theta = (v\hat{e}_t)\cdot(\cos\theta\,\hat{e}_t - \sin\theta\,\hat{e}_n) = v\cos\theta$

  $v = \dfrac{5b\omega_0}{\cos\theta} = \dfrac{25}{4}b\omega_0 \longleftarrow v$

- $\dot{r} = \bar{v}\cdot\hat{e}_t = (v\hat{e}_t)\cdot(\sin\theta\,\hat{e}_t + \cos\theta\,\hat{e}_n) = \dfrac{15}{4}b\omega_0 \longleftarrow \dot{r}$

(d) Develop a set of two algebraic equations that can be used to solve for the values for $\ddot{r}$ and $\dot{v}$ in terms of ρ_0, b and ω_0. *You need not solve these equations.*

$\bar{a} = \dot{v}\hat{e}_t + \dfrac{v^2}{\rho}\hat{e}_n$

(1) $\ddot{r} - r\dot{\theta}^2 = \bar{a}\cdot\hat{e}_r = \left(\dot{v}\hat{e}_t + \dfrac{v^2}{\rho}\hat{e}_n\right)\cdot(\sin\theta\,\hat{e}_t + \cos\theta\,\hat{e}_n)$

$= \dot{v}\sin\theta + \dfrac{v^2}{\rho}\cos\theta$

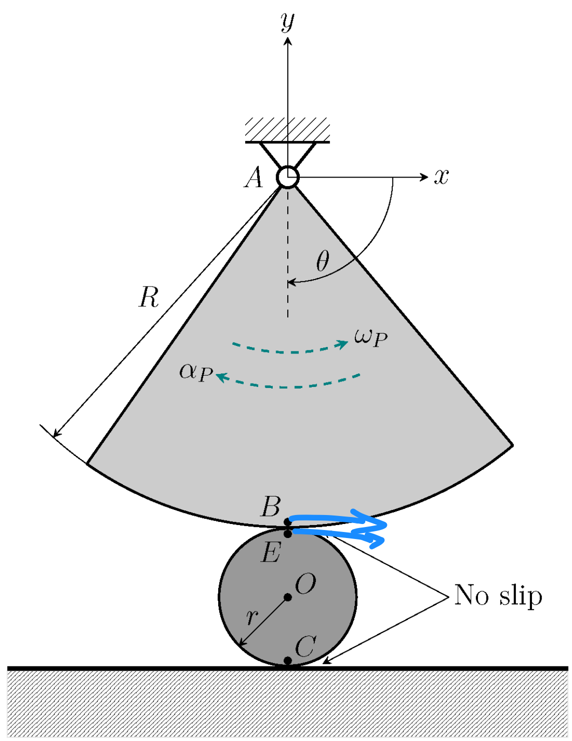
(2) $2\dot{r}\dot{\theta} = \bar{a}\cdot\hat{e}_\theta = \left(\dot{v}\hat{e}_t + \dfrac{v^2}{\rho}\hat{e}_n\right)\cdot(\cos\theta\,\hat{e}_t - \sin\theta\,\hat{e}_n)$

$= \dot{v}\cos\theta - \dfrac{v^2}{\rho}\sin\theta$

Solve (2) for $\dot{v}$, followed by solving (1) for $\ddot{r}$

Problem 1.2

The circular arc pendulum AB rotates about a fixed axis through A with angular velocity ω_P and angular acceleration α_P with directions as indicated in the figure. The bottom surface of the pendulum is in contact with a disk centered at O, which rolls without slipping on its top and bottom contact points (E and C, respectively). Determine the angular velocity $\vec{\omega}_D$ and angular acceleration $\vec{\alpha}_D$ of the disk at the instant shown, where points A, B, E, O and C are aligned vertically exactly at $\theta = 90°$. Express your answer in vector form.



Use the following numerical values in your analysis: $R = 12$ cm, $r = 3$ cm, $\omega_P = 3$ rad / s, $\alpha_P = 5$ rad / s^2

1) determine velocity @ pt. B using rigid body eqns:

$$\vec{V}_B = \cancel{\vec{V}_A}^{\,0} + \vec{\omega}_P \times \vec{r}_{B/A}$$

$$\vec{V}_B = \omega_P \hat{k} \times (-R\hat{j}) = \omega_P R \hat{i}$$

alternatively, using ICs:

A is the IC of the pendulum

$$V_B = |\vec{r}_{B/A}|\omega_p = \omega_p R$$

CW rotation indicates V_B is in the $+\hat{\imath}$ direction

$$\rightarrow \vec{V}_B = \omega_p R\hat{\imath}$$

2) solving for $\vec{\omega}_D$

using rigid body eqns:

$$\vec{V}_E = \vec{V}_B = \omega_p R\hat{\imath}$$

$$\vec{V}_E = \cancel{\vec{V}_c} + \vec{\omega}_D \times \vec{r}_{E/c}$$

$$= \omega_D\hat{k} \times (2r\hat{\jmath})$$

$$\omega_p R\hat{\imath} = 2\omega_D r\hat{\imath}$$

$$\omega_D = \frac{\omega_p R}{2r} = \frac{3(12)}{-2(3)} = -6$$

$$\vec{\omega}_D = -6\hat{k} \text{ rad/s}$$

alternatively, using ICs:

C is the IC of the disk

$$\vec{V}_E = \vec{V}_B = \omega_p R\hat{\imath}$$

$$\omega_D = \frac{|\vec{V}_E|}{|\vec{r}_{E/c}|} = \frac{\omega_p R}{2r}$$

direction of $\vec{V}_E$ indicates CW rotation $\rightarrow -k$

$$\vec{\omega}_D = -6\hat{k} \text{ rad/s}$$

3) Solving for $\vec{a}_B$

$$\vec{a}_B = \cancel{\vec{a}_A}^{\,0} + \vec{\alpha}_p \times r_{B/A} - \omega_p^2 r_{B/A}$$

$$= -\alpha_p \hat{k} \times -R\hat{j} - \omega_p^2 R\hat{j}$$

$$= -\alpha_p R\hat{i} + \omega_p^2 R\hat{j}$$

4) solving for $\vec{\alpha}_D$

$$\vec{a}_E = \vec{a}_c + \vec{\alpha}_D \times \vec{r}_{E/c} - \omega_D^2 \vec{r}_{E/c}$$

$$a_{Ex}\hat{i} + a_{Ey}\hat{j} = a_c\hat{j} + \alpha_D \hat{k} \times 2r\hat{j} - \omega_D^2 2r\hat{j}$$

$$= a_c\hat{j} - 2r\alpha_D\hat{i} - \omega_D^2 2r$$

$$a_{Ex}\hat{i} = a_{Bx}\hat{i} = -\alpha_p R\hat{i}$$

$$\hat{i}: \; -\alpha_p R = -2r\alpha_D$$

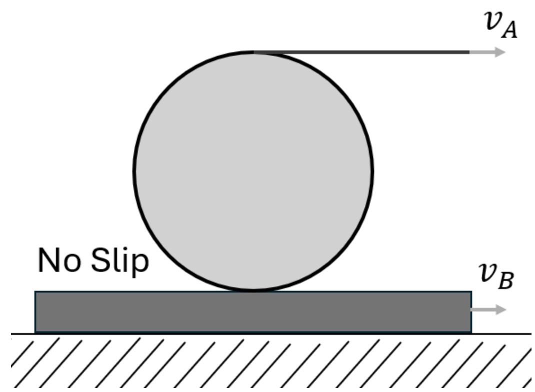
$$\alpha_D = \frac{\alpha_p R}{2r} = \frac{5(12)}{2(3)} = 10$$

$$\vec{\alpha}_D = 10\hat{k} \text{ rad/s}^2$$

Problem #3 (20 Points)

Part 3A

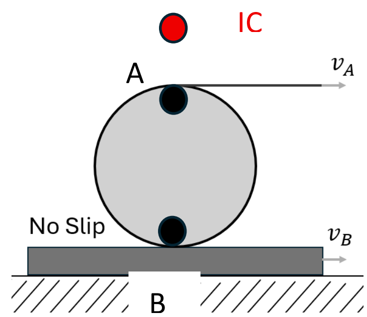
A disk has a cable wrapped around its outside that is pulled at a speed v_A in the +x-direction. Additionally, this disk sits on a block which has a velocity in the +x-direction of v_B. The disk and block obey a no slip-condition. Assume that the cable is not slipping on the disk.



Part 3A.1 If $v_B > v_A$, in what direction is the disk rotating? *(1 Point)*

a) CW
b) CCW
c) The disk is not rotating
d) Not enough information

IC is somewhere above A and both A and B have velocity to the right. Disk is then rotating CCW around IC

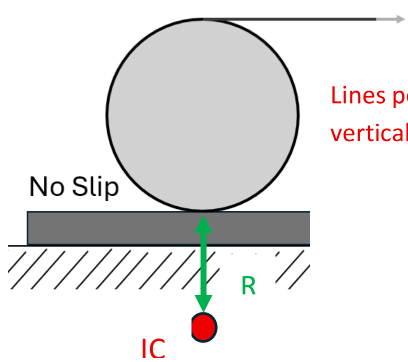


Part 3A.2 If $v_A = v_B$, in what direction is the disk rotating? *(1 Point)*

a) CW
b) CCW
c) The disk is not rotating
d) Not enough information

In order for these points to have the same velocity, $\omega_d = 0$, or the disk is not rotating

Part 3A.3 If $v_A = 3v_B$, mark the location of the instant center of the disk in the diagram below. *(2 Points)*



Lines perpendicular to $\vec{v}_A$ and $\vec{v}_B$ are vertical.

$$\omega_d = v_a/d_A = v_B/d_B$$

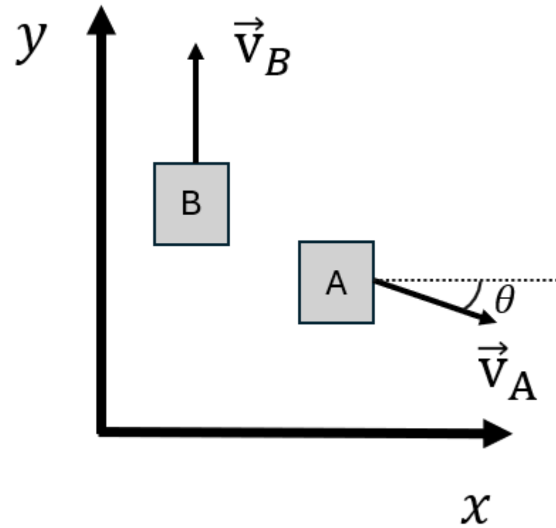
$$3v_B/(d_B + 2R) = v_B/d_B$$

$$3/(d_B + 2R) = 1/d_B$$

$d_B = R$ and $d_A = 3R$

Part 3B *(3 Points)*

Given: The velocity of block B is defined with the equation $\dot{y}\,\hat{j}$. The velocity of block A is defined with the equation $v_A\hat{e}_t$ with the direction of $\hat{e}_t$ defined in the accompanying figure.



$$\vec{v}_B = \dot{y}\,\hat{j}$$

$$\vec{v}_A = v_A(\cos(\theta)\hat{\imath} - \sin(\theta)\hat{\jmath})$$

Find:

The velocity of block B relative to an observer on block A ($\vec{v}_{B/A}$) in **Cartesian coordinates**. Your answer should be in terms of $v_A, \dot{y}$, and θ. *(3 Points)*

$$\vec{v}_{B/A} =$$

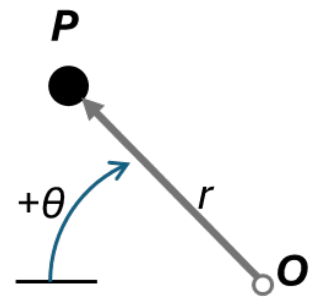
$$Ans: \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$

$$\vec{v}_{B/A} = \dot{y}\hat{\jmath} - [v_A cos\theta\ \hat{\imath} - v_A sin\theta\,\hat{\jmath}]$$

$$\vec{v}_{B/A} = cos\theta\ \hat{\imath} + (\dot{y} + v_A sin\theta)\hat{\jmath}$$

Part C *(6 points)*

The following polar description applies to point P at the position shown in the figure.



$$\vec{v}_p = 4\hat{e}_r + 4\hat{e}_\theta \qquad \vec{a}_p = -10\hat{e}_\theta$$

Part 3C.1 Which option most accurately describes the speed of P? *(1 Point)*

(a) The speed of P is increasing
(b) The speed of P is not changing
(c) The speed of P is decreasing

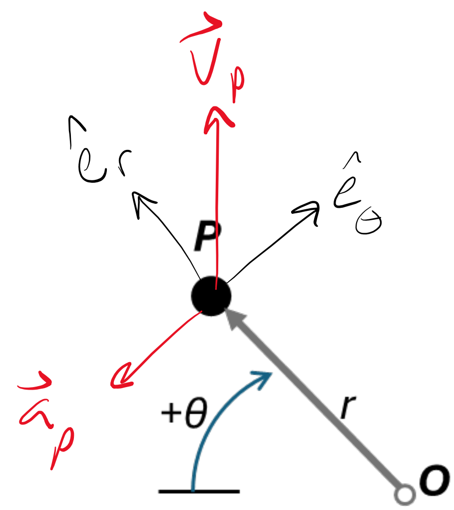
Acceleration is >90 degrees away from velocity

Part 3C.2 From the perspective of the particle, which direction is the particle turning? *(1 Point)*

(a) Left
(b) Right
(c) Not turning

Component of acceleration which is 90 degrees to velocity is in left direction

$\vec{v}_p$ is in between two unit vectors
$\vec{a}_p$ is in negative theta direction



Part 3C.3 What is true about $\dot{\theta}$? *(1 Point)*

(a) $\dot{\theta} > 0$
(b) $\dot{\theta} = 0$
(c) $\dot{\theta} < 0$

Component of velocity in theta direction is positive

Part 3C.4 What is true about $\dot{r}$? *(1 Point)*

(a) $\dot{r} > 0$
(b) $\dot{r} = 0$
(c) $\dot{r} < 0$

Component of velocity in r direction is positive

Part 3C.5 What is true about $\ddot{\theta}$? *(1 Point)*

(a) $\ddot{\theta} > 0$
(b) $\ddot{\theta} = 0$
(c) $\ddot{\theta} < 0$

Component of acceleration in theta direction is negative

Part 3C.6 What is true about $\ddot{r}$? *(1 Point)*

(a) $\ddot{r} > 0$
(b) $\ddot{r} = 0$
(c) $\ddot{r} < 0$

Equation method: $\vec{a}_p = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta = -10\hat{e}_\theta$

$\hat{e}_r: \ddot{r} - r\dot{\theta}^2 = 0$

$\underline{\ddot{r} = r\dot{\theta}^2, r > 0, \therefore \ddot{r} > 0}$

Intuitive method: Even though given acceleration of $-10\hat{e}_\theta$ doesn't contain a radial component, it is turning the velocity leftwards to point more directly in the radial direction, which means positive $\ddot{r}$

Part 3D

Blocks A and B are connected through a series of pulleys and inextensible cables. Block B remains flat and at one instant in time moves upward at 2 m/s with an acceleration upward of 3 m/s^2.

Part 3D.1 Which direction is block A moving? *(1 Points)*

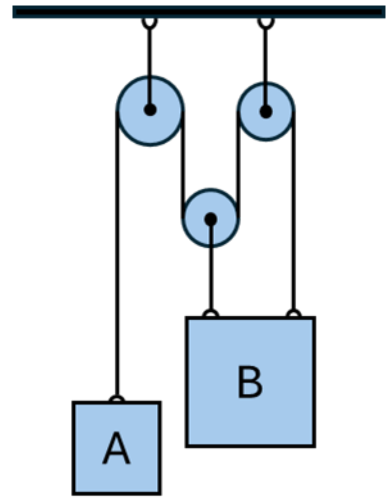
(a) Up
(b) Down
(c) Not moving

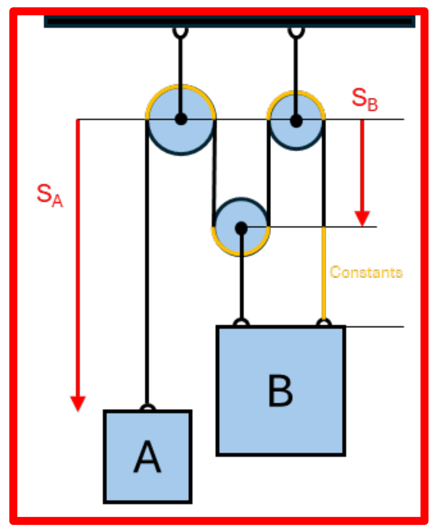
Part 3D.2 What is the speed of block A? *(1 Point)*

(a) 0 m/s
(b) 2 m/s
(c) 3 m/s
(d) 6 m/s
(e) 9 m/s

Part 3D.3 What is the acceleration of block A? *(1 Point)*

(a) 0 m/s^2
(b) 3 m/s^2
(c) 6 m/s^2
(d) 9 m/s^2
(e) 12 m/s^2





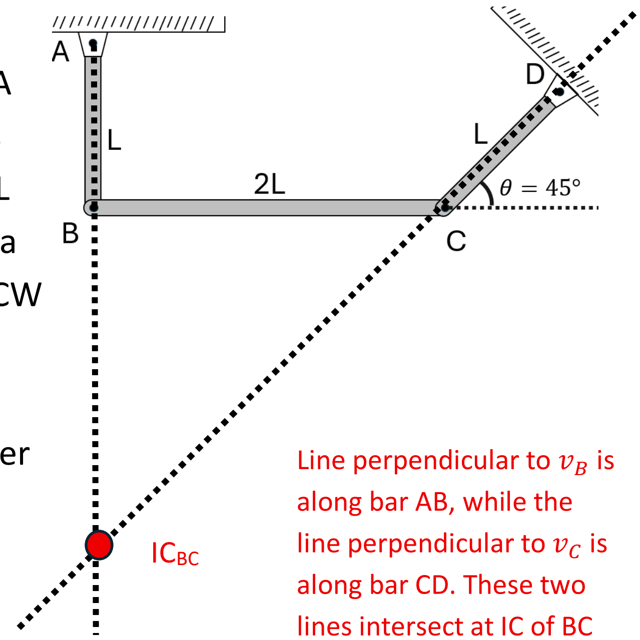
$L = s_A + 3s_B + \text{constants}$

$dL/dt = v_A + 3v_B = 0 \rightarrow v_A = 6$ m/s

$d^2L/dt^2 = a_A + 3a_B = 0 \rightarrow v_B = 9$ m/s^2

Part 3E

Link AB with length L hangs vertically with end A attached to the ceiling. Link BC with length 2L is in a horizontal orientation. Link CD with length L has end D attached to a wall and is currently at a $45°$ angle from horizontal. Link AB is rotating CCW at an angular ***speed*** of ω_{AB}.



Part 3E.1 Mark the location of the instant center of link BC in the figure above. *(2 points)*

Line perpendicular to v_B is along bar AB, while the line perpendicular to v_C is along bar CD. These two lines intersect at IC of BC

Part 3E.2 How is the **speed** of points B and C related? *(1 Point)*

a) $v_C > v_B$
b) $v_C < v_B$
c) $v_C = v_B$
d) $v_C > v_B$
e) Not enough information

$$v_B = \omega_{BC}\, d_B$$

$$v_C = \omega_{BC}\, d_C$$

$$v_B / v_C = d_B / d_C$$

Because distance from C to IC (d_C) is larger than distance from B to IC (d_B) or $\frac{d_B}{d_C} < 1$ this means $\frac{v_B}{v_C} < 1$ which is the same as $v_B < v_C$

Part 3E.3 In what direction is link BC rotating? *(1 Point)*

a) CW
b) CCW
c) Link BC is not rotating
d) Not enough information

With link AB rotating CCW, $\vec{v}_B$ is to the right. Which then means that $\vec{v}_C$ is going down and to the right b/c both points are rotating about the IC. With the direction of these two velocities know, link BC is rotating CW around the IC