

ME 274: Basic Mechanics II

Week 6 – Monday, February 16

Particle kinematics: 3D Rotating Reference Frames

Instructor: Manuel Salmerón

Notes for H.3.E

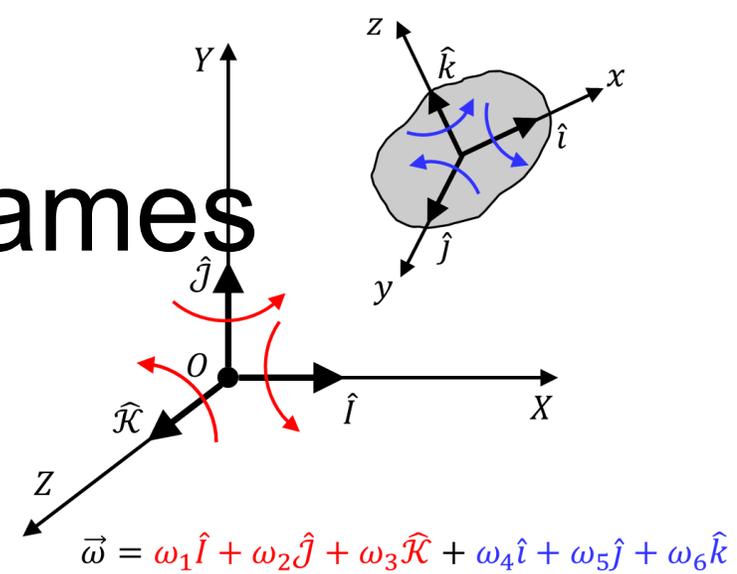
- You're asked to write the answer in the “lower-case” system
- In our Examples, the lower- and upper-case axes were aligned: we had, for example, $\hat{i} \equiv \hat{I}$, or $\hat{k} \equiv \hat{\mathcal{K}}$.
- In H.3.E, you need to use vector projections. Particularly, you will need to transform \hat{j} into “lower-case” coordinates using θ .

Today's Agenda

1. Recap: 3D Rotating Reference Frames
2. Angular Acceleration in 3D
3. Example
4. Note on the simplification formula

Solution Steps for Mov. Ref. Frames

1. Find the **moving** reference frame
2. Locate the axes of the **moving** and **fixed** reference frames
3. Describe the angular motion of the **moving** reference frame: $\vec{\omega}$ and $\vec{\alpha}$
4. Describe the relative motion of the **moving** reference frame: $(\vec{v}_{B/A})_{rel}$ and $(\vec{a}_{B/A})_{rel}$



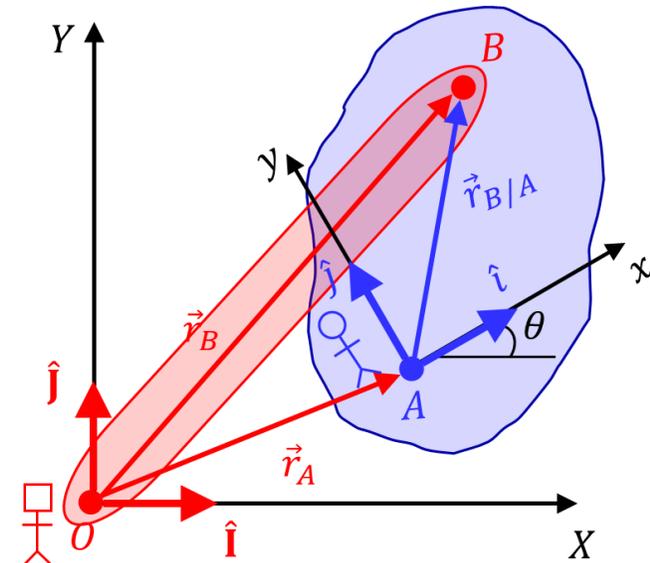
5. Solve the fundamental equations:

$$\vec{v}_B = \vec{v}_O + \vec{\omega} \times \vec{r}_{B/O}$$

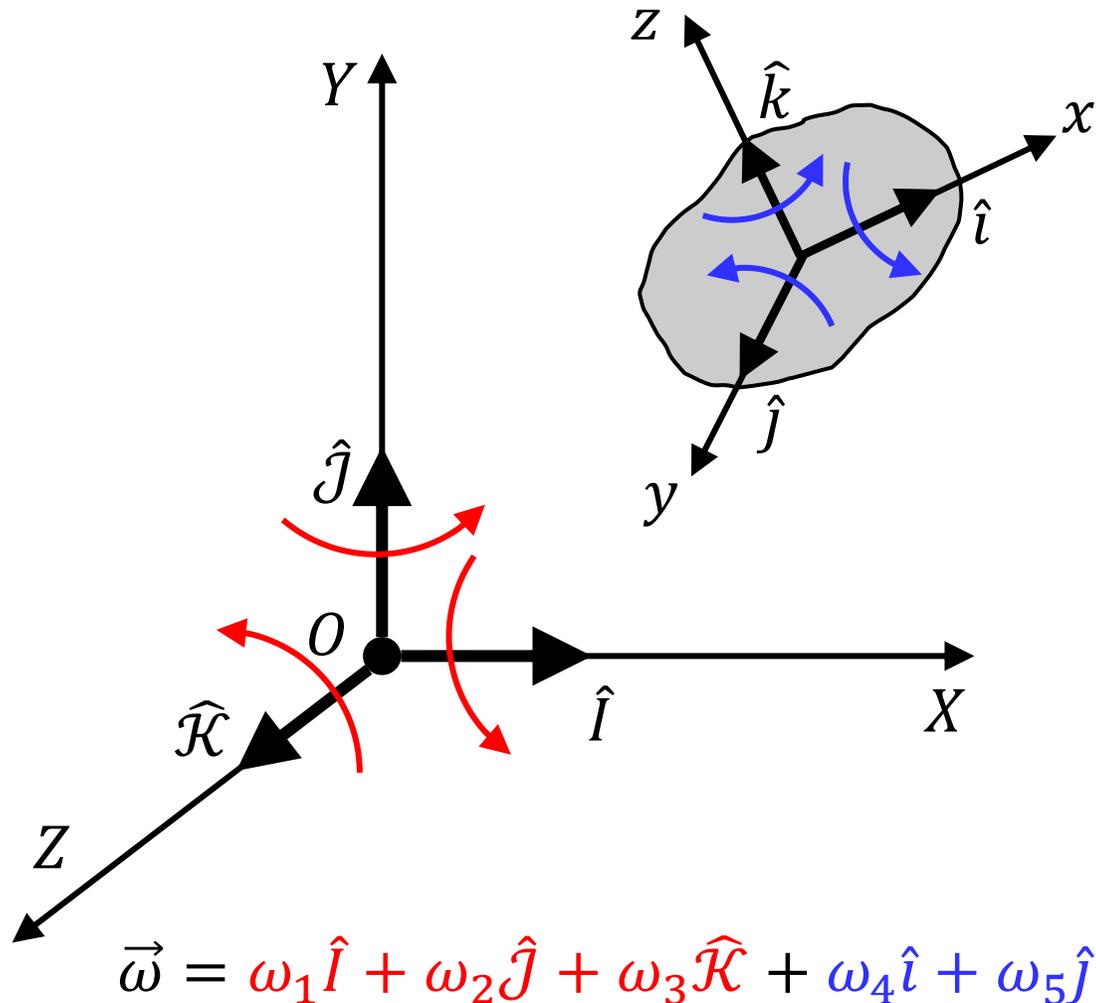
$$\vec{a}_B = \vec{a}_O + \vec{\alpha} \times \vec{r}_{B/O} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/O})$$

$$\vec{v}_B = \vec{v}_A + (\vec{v}_{B/A})_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$$



3D Rotating Reference Frames



We will use:

- cross-product properties,
- unit vector projections,
- the right-hand rule,
- the following derivatives:

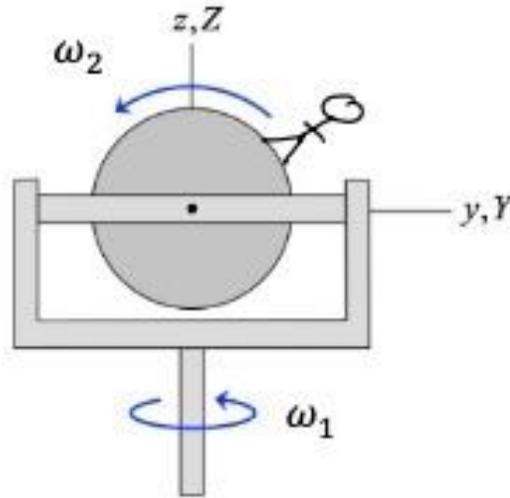
$$\frac{d\hat{i}}{dt} = \vec{\omega} \times \hat{i}$$

$$\frac{d\hat{j}}{dt} = \vec{\omega} \times \hat{j}$$

$$\frac{d\hat{k}}{dt} = \vec{\omega} \times \hat{k} \quad \text{(new!)}$$

Angular Acceleration in 3D

A gyro rotor has a constant rotation rate of ω_1 about a fixed vertical axis in addition to a constant rotation rate of ω_2 about the symmetry axis of the rotor. Let the xyz -axes be attached to the rotor, and the XYZ -axes be fixed to ground. Find the angular acceleration of the rotor.



Attach observer to rotor
 " xyz " "
 " $\Sigma I Z_1$ " ground

- $\vec{\omega} = \text{angular velocity of rotor}$
 $= \omega_1 \hat{K} + \omega_2 \hat{i}$; $\hat{K} = \text{fixed}$
 $\hat{i} = \text{moving}$
- $\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \dot{\omega}_1 \hat{K} + \omega_1 \dot{\hat{K}} + \dot{\omega}_2 \hat{i} + \omega_2 \dot{\hat{i}}$
 $= \omega_2 (\vec{\omega} \times \hat{i}) = \omega_2 (\omega_1 \hat{K} + \omega_2 \hat{i}) \times \hat{i}$
 $= \omega_1 \omega_2 \hat{K} \times \hat{i}$; $\hat{K} = \hat{k}$ @ this instant
 $= \omega_1 \omega_2 \hat{j}$ ← $\vec{\alpha}$

How is it possible to have constant angular velocity components but non-zero angular acceleration?

Because in 3D we are interested in changes of magnitude **AND** of direction.

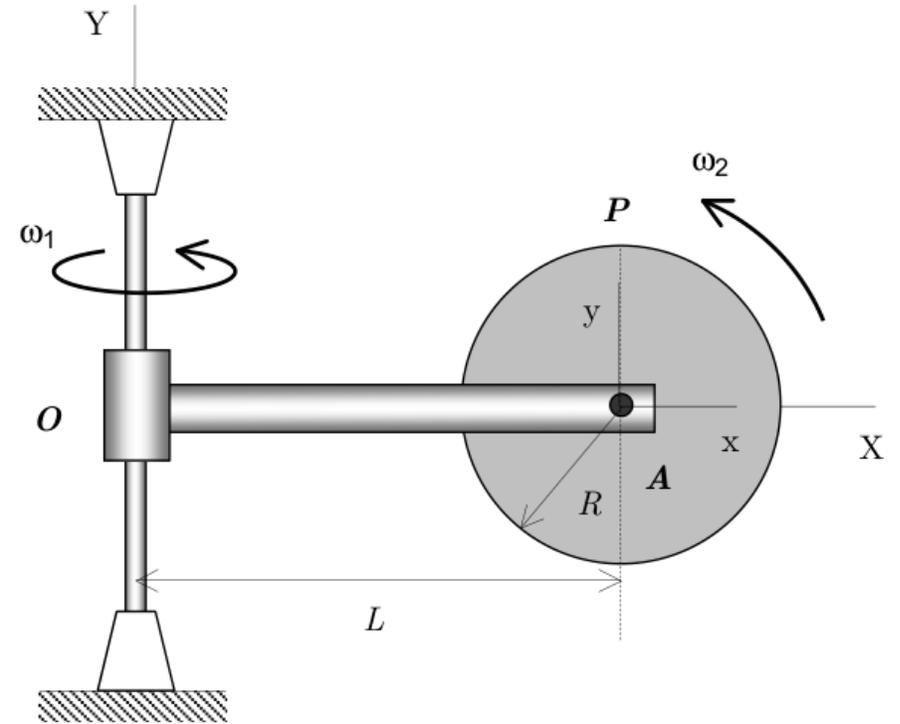
See the website animation [here](#).

Example 3.B.8

Given: The rotation rates ω_1 and ω_2 are constant.

Find: Determine:

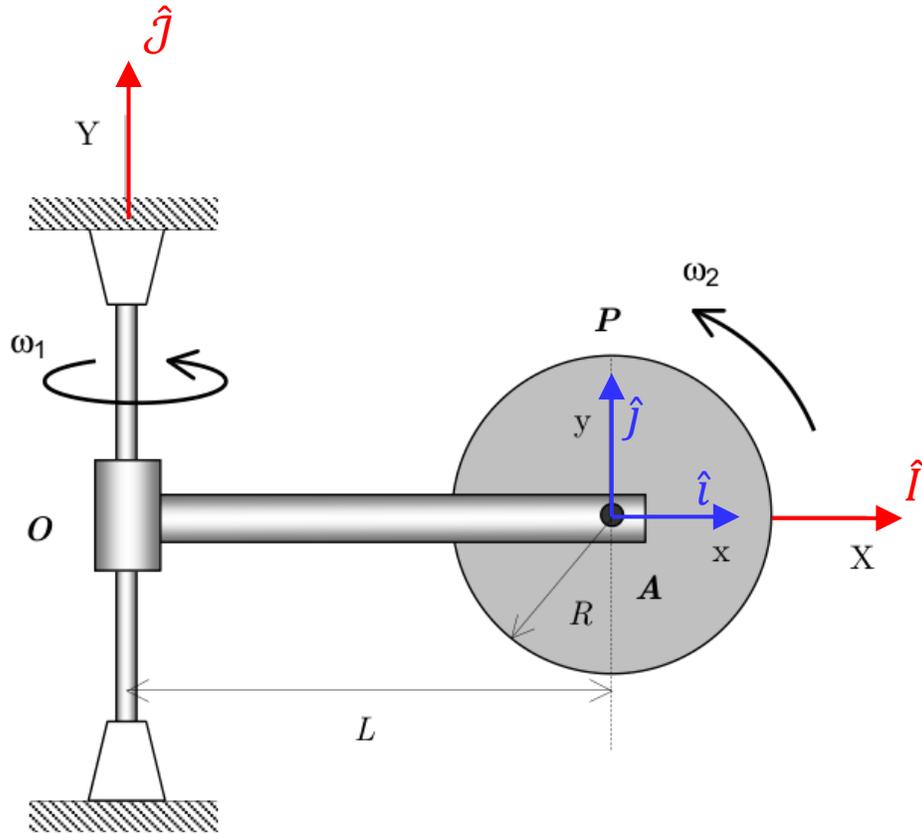
- The velocity of point P on the disk at the instant when P is directly above the center A of the disk; and
- The acceleration of point P at the same instant.



Example 3.B.8

Given: ω_1, ω_2 are constant

Find: (a) \vec{v}_P , and (b) \vec{a}_P



Solution:

$$\vec{v}_P = \vec{v}_A + \vec{\omega} \times \vec{r}_{P/A}$$

(No relative velocity term because we are on the same RB)

We need information on A:

$$\vec{v}_A = \vec{v}_O + \vec{\omega} \times \vec{r}_{A/O}$$

0, O is on the rotation axis

$$\vec{\omega}_{AO} \times \vec{r}_{A/O} = \begin{vmatrix} \hat{I} & \hat{J} & \hat{K} \\ 0 & \omega_1 & 0 \\ L & 0 & 0 \end{vmatrix} = -\omega_1 L \hat{K}$$

Now we need the angular velocity of the mov. ref. frame:

$$\vec{\omega} = \omega_1 \hat{J} + \omega_2 \hat{k}$$

Here: $\hat{k} \equiv \hat{K}$, so the cross product is:

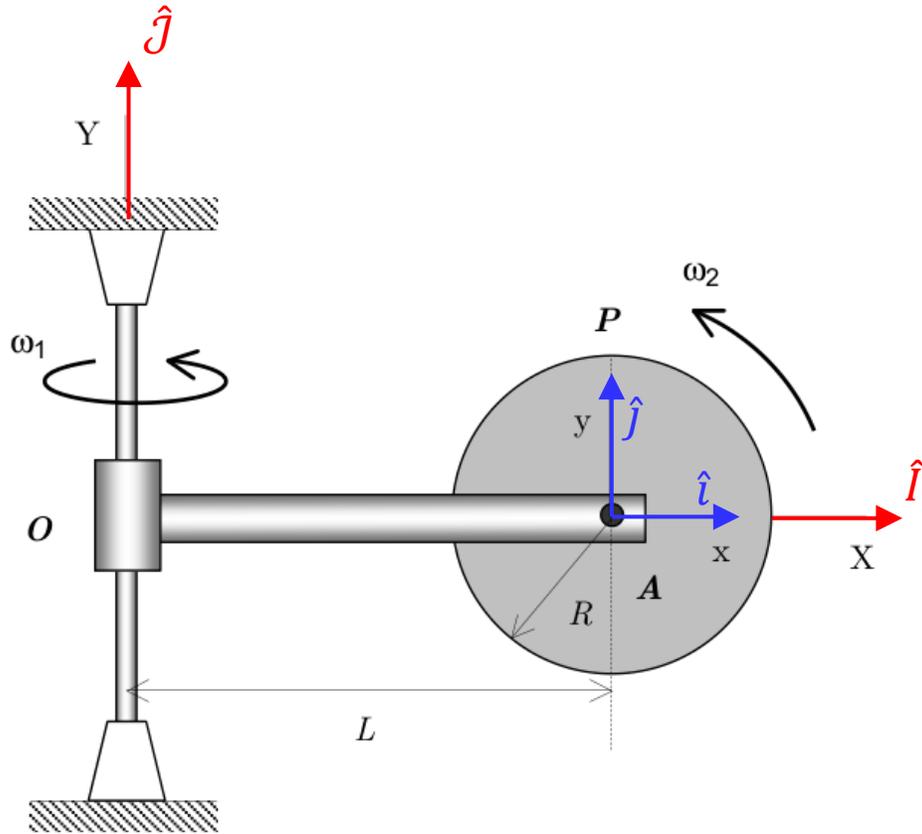
$$\vec{\omega} \times \vec{r}_{P/A} = \begin{vmatrix} \hat{I} & \hat{J} & \hat{K} \\ 0 & \omega_1 & \omega_2 \\ 0 & R & 0 \end{vmatrix} = -\omega_2 R \hat{I}$$

Finally: $\vec{v}_P = -\omega_2 R \hat{I} - \omega_1 L \hat{K}$

Example 3.B.8

Given: ω_1, ω_2 are constant

Find: (a) \vec{v}_P , and (b) \vec{a}_P



Solution:

$$\vec{a}_P = \vec{a}_A + \vec{\alpha} \times \vec{r}_{P/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A})$$

Again, we need information about A:

$$\vec{a}_A = \cancel{\vec{a}_O} + \vec{\alpha}_{AO} \times \vec{r}_{A/O} + \vec{\omega}_{AO} \times (\vec{\omega}_{AO} \times \vec{r}_{A/O})$$

$0, O$ is on the rotation axis

First cross product: $\vec{\alpha}_{AO} \times \vec{r}_{A/O} = -\dot{\omega}_1 L \hat{\mathcal{K}} = \vec{0}$ (constant ω_1)

Double cross product (step-by-step):

$$\vec{\omega}_{AO} \times (\vec{\omega}_{AO} \times \vec{r}_{A/O}) = \begin{vmatrix} \hat{I} & \hat{J} & \hat{\mathcal{K}} \\ 0 & \omega_1 & 0 \\ 0 & 0 & -\omega_1 L \end{vmatrix} = -\omega_1^2 L \hat{I} = -\omega_1^2 \vec{r}_{A/O}$$

Thus: $\vec{a}_A = -\omega_1^2 L \hat{I}$

Going back to \vec{a}_P : we need $\vec{\alpha}$:

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d}{dt} (\omega_1 \hat{J} + \omega_2 \hat{k}) = \omega_2 \frac{d\hat{k}}{dt} = \omega_2 (\vec{\omega} \times \hat{k})$$

Substituting $\vec{\omega}$: $\vec{\alpha} = \omega_2 [(\omega_1 \hat{J} + \omega_2 \hat{k}) \times \hat{k}]$

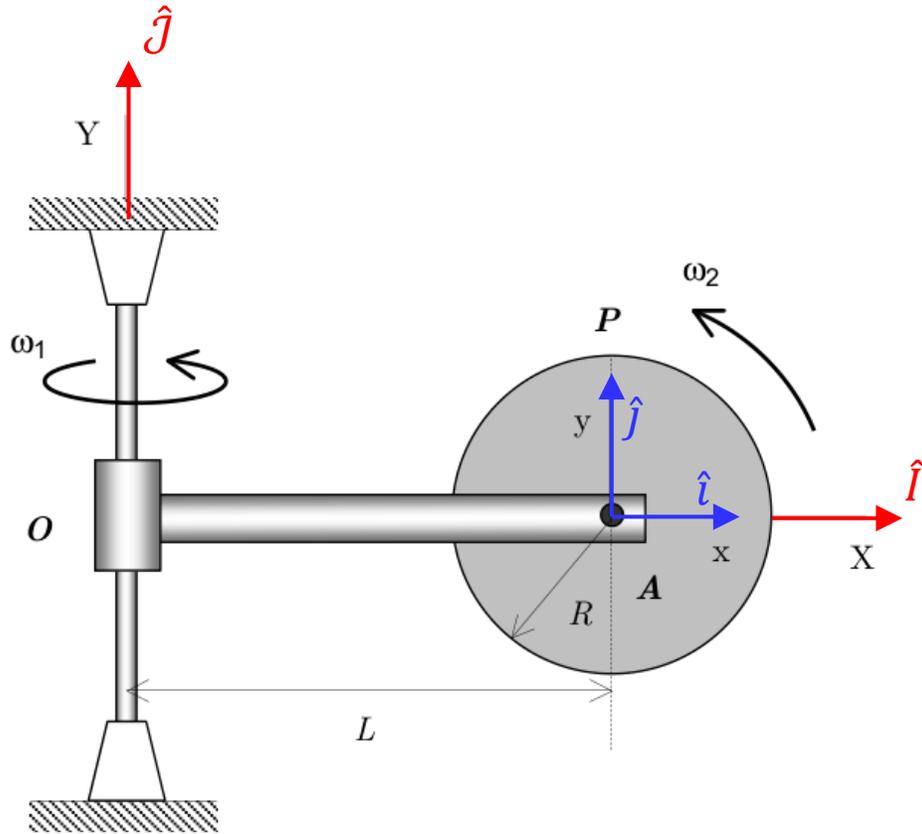
At this instant, $\hat{k} \equiv \hat{\mathcal{K}}$:

$$\vec{\alpha} = \omega_2 (\omega_1 \hat{I}) = \omega_1 \omega_2 \hat{I}$$

Example 3.B.8

Given: ω_1, ω_2 are constant

Find: (a) \vec{v}_P , and (b) \vec{a}_P



Solution:

$$\vec{a}_P = \vec{a}_A + \vec{\alpha} \times \vec{r}_{P/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A})$$

Things that we have computed so far:

$$\vec{\omega} = \omega_1 \hat{J} + \omega_2 \hat{K}$$

$$\vec{\alpha} = \omega_1 \omega_2 \hat{I}$$

$$\vec{\omega} \times \vec{r}_{P/A} = -\omega_2 R \hat{I}$$

First cross product:

$$\vec{\alpha} \times \vec{r}_{P/A} = \begin{vmatrix} \hat{I} & \hat{J} & \hat{K} \\ \omega_1 \omega_2 & 0 & 0 \\ 0 & R & 0 \end{vmatrix} = \omega_1 \omega_2 R \hat{K}$$

Second cross product:

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A}) = \begin{vmatrix} \hat{I} & \hat{J} & \hat{K} \\ 0 & \omega_1 & \omega_2 \\ -\omega_2 R & 0 & 0 \end{vmatrix} = -\omega_2^2 R \hat{J} + \omega_1 \omega_2 R \hat{K}$$

Thus:

$$\vec{a}_P = -\omega_1^2 L \hat{I} + \omega_1 \omega_2 R \hat{K} - \omega_2^2 R \hat{J} + \omega_1 \omega_2 R \hat{K}$$

What happened to our simplification formula $\vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A}) = -\omega^2 \vec{r}_{P/A}$?

Use cross product identity:

Definition of magnitude!

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = (\vec{\omega} \cdot \vec{r})\vec{\omega} - (\vec{\omega} \cdot \vec{\omega})\vec{r}$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \underbrace{(\vec{r} \cdot \vec{\omega})\vec{\omega}}_{\text{(Scaled) Projection of } \vec{r} \text{ onto } \vec{\omega}} - |\vec{\omega}|^2 \vec{r}$$

(Scaled) Projection
of \vec{r} onto $\vec{\omega}$

Thus, the simplification only holds if and only if \vec{r} and $\vec{\omega}$ are perpendicular (\vec{r} casts no “shadow” over $\vec{\omega}$):

$$\vec{r} \cdot \vec{\omega} = 0$$

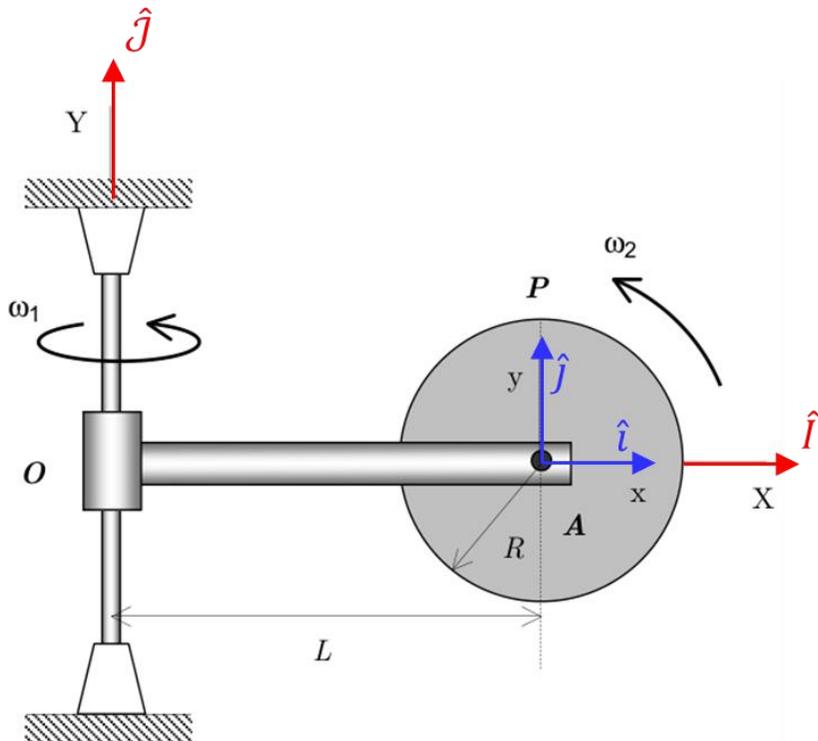
In 2D, this condition was trivially met because:

1. \vec{r} was always in the XY plane, and
2. $\vec{\omega}$ was always parallel to \hat{k}

In 3D, this condition does not always hold, as shown in our computation of $\vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A})$ in the previous example.

Using the simplification formula can save you time, but you must be certain that \vec{r} and $\vec{\omega}$ are perpendicular.

Advice: use the simplification only after verifying that \vec{r} and $\vec{\omega}$ are perpendicular. When in doubt, compute the full cross product.



ATTENDANCE

Choose all the pairs of \vec{r} and $\vec{\omega}$ for which the simplification formula $\vec{\omega} \times (\vec{\omega} \times \vec{r})$ is valid:

(a) $\vec{\omega} = \omega \hat{k}$, $\vec{r} = r_x \hat{i} + r_y \hat{j}$

(b) $\vec{\omega} = \omega \hat{k}$, $\vec{r} = r_z \hat{k}$

(c) $\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j}$, $\vec{r} = r_x \hat{i} + r_z \hat{k}$

(d) $\vec{\omega}$ and \vec{r} such that $\vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \omega & 0 \\ r_x & 0 & r_z \end{vmatrix}$

(e) $\vec{\omega}$ and \vec{r} such that $\vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ r_x & r_y & 0 \end{vmatrix}$

(f) $\vec{\omega}$ and \vec{r} such that $\vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & 0 \\ 0 & r_y & r_z \end{vmatrix}$

(g) $\vec{\omega}$ and \vec{r} such that $\vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & 0 \\ 0 & 0 & r_z \end{vmatrix}$

ME 274: Basic Mechanics II

Week 6 – Monday, February 16

Particle kinematics: 3D Rotating Reference Frames

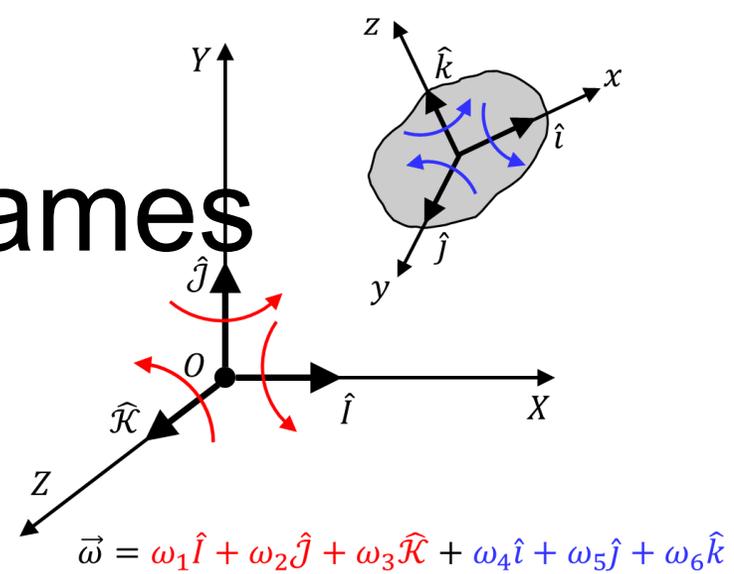
Instructor: Manuel Salmerón

Today's Agenda

1. Recap: Solution Steps
2. Example – different moving reference frames
3. Another example

Solution Steps for Mov. Ref. Frames

1. Find the **moving** reference frame
2. Locate the axes of the **moving** and **fixed** reference frames
3. Describe the angular motion of the **moving** reference frame: $\vec{\omega}$ and $\vec{\alpha}$
4. Describe the relative motion of the **moving** reference frame: $(\vec{v}_{B/A})_{rel}$ and $(\vec{a}_{B/A})_{rel}$



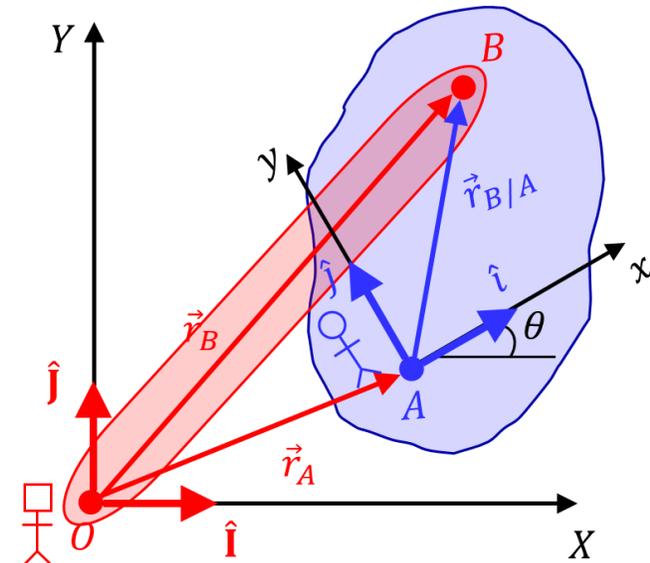
5. Solve the fundamental equations:

$$\vec{v}_B = \vec{v}_O + \vec{\omega} \times \vec{r}_{B/O}$$

$$\vec{a}_B = \vec{a}_O + \vec{\alpha} \times \vec{r}_{B/O} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/O})$$

$$\vec{v}_B = \vec{v}_A + (\vec{v}_{B/A})_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$$

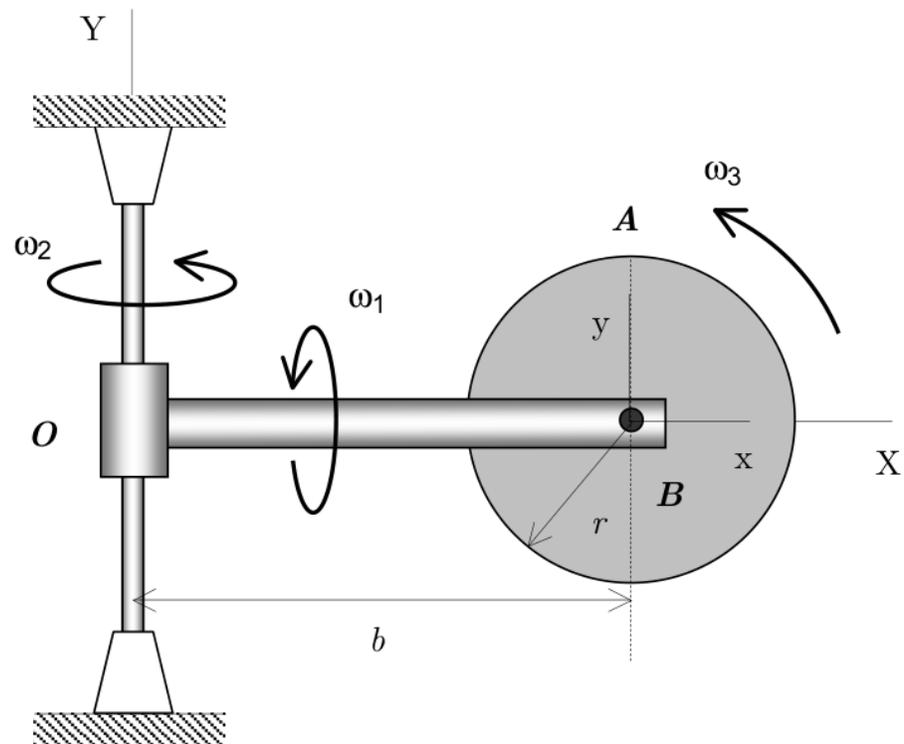


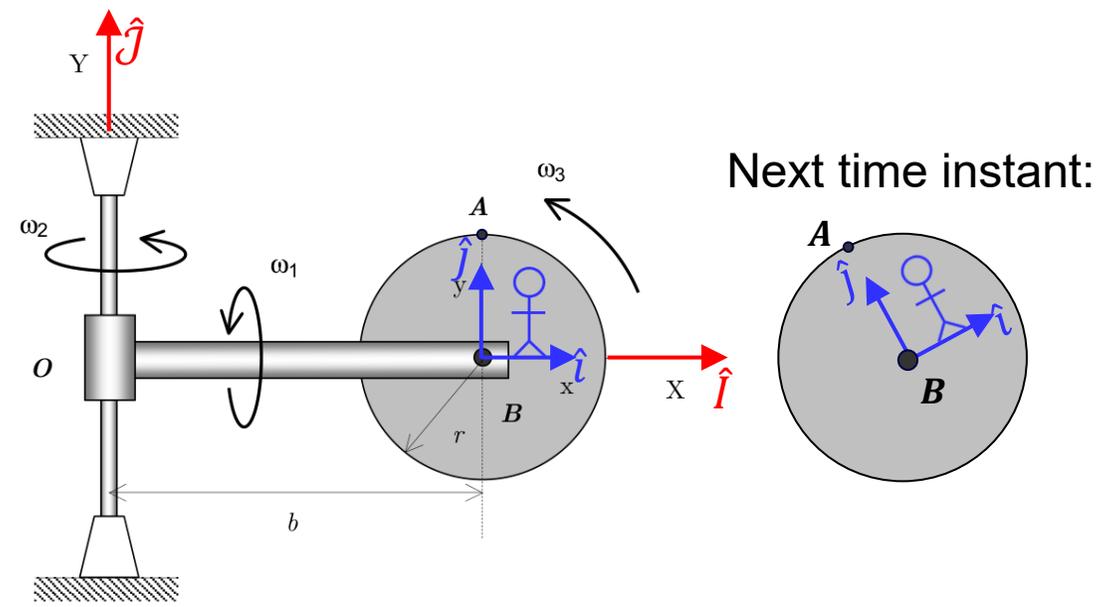
Example 3.B.11

Given: Rotation rates ω_1 , ω_2 and ω_3 are all constant. The XYZ axes are fixed, and the xyz axes are attached to the disk. At the instant shown, A is directly above the center B of the disk and the xyz and XYZ axes are aligned.

Find: Determine:

- (a) The velocity of point A ; and
- (b) The acceleration of point A .





Solution #1: m.r.f. attached to the **DISK**

$$\vec{v}_A = \vec{v}_B + (\vec{v}_{A/B})_{rel} + \vec{\omega} \times \vec{r}_{A/B}$$

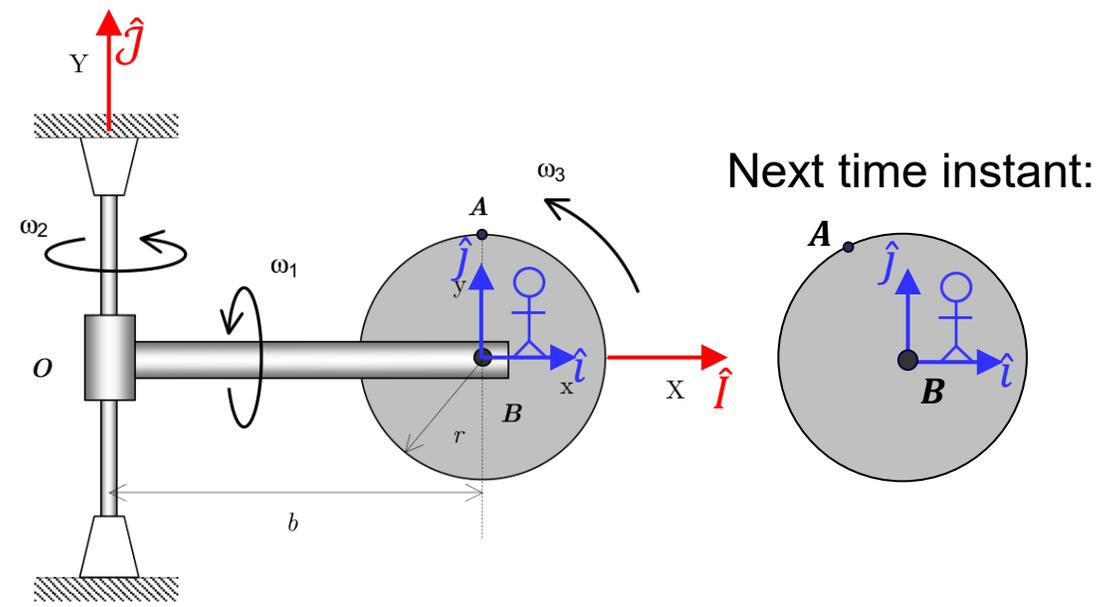
→ Information about B : $\vec{v}_B = \vec{v}_O + \vec{\omega}_{OB} \times \vec{r}_{B/O} = (\omega_2 \hat{J} \cancel{- \omega_1 \hat{I}}) \times (b \hat{I}) = -b\omega_2 \hat{K}$

→ Relative motion: $(\vec{v}_{A/B})_{rel} = \vec{0}$

→ Angular motion: $\vec{\omega} = -\omega_1 \hat{I} + \omega_2 \hat{J} + \omega_3 \hat{k}$

→ Compute & substitute: $\vec{\omega} \times \vec{r}_{A/B} = -r\omega_3 \hat{I} - r\omega_1 \hat{K}$

$$\vec{v}_A = -r\omega_3 \hat{I} - (r\omega_1 + b\omega_2) \hat{K}$$



Solution #2: m.r.f. attached to the **ARM**

$$\vec{v}_A = \vec{v}_B + (\vec{v}_{A/B})_{rel} + \vec{\omega} \times \vec{r}_{A/B}$$

→ Information about B : $\vec{v}_B = \vec{v}_O + \vec{\omega}_{OB} \times \vec{r}_{B/O} = (\omega_2 \hat{j} + \omega_1 \hat{i}) \times (b \hat{i}) = -b\omega_2 \hat{k}$

→ Relative motion: $(\vec{v}_{A/B})_{rel} = \vec{\omega}_{AB} \times \vec{r}_{A/B} = (\omega_3 \hat{k}) \times (r \hat{j}) = -r\omega_3 \hat{i} \neq \vec{0}$

→ Angular motion: $\vec{\omega} = -\omega_1 \hat{i} + \omega_2 \hat{j}$ **(no \hat{k} component!)**

→ Compute substitute: $\vec{\omega} \times \vec{r}_{A/B} = -r\omega_1 \hat{k}$ **(different)**

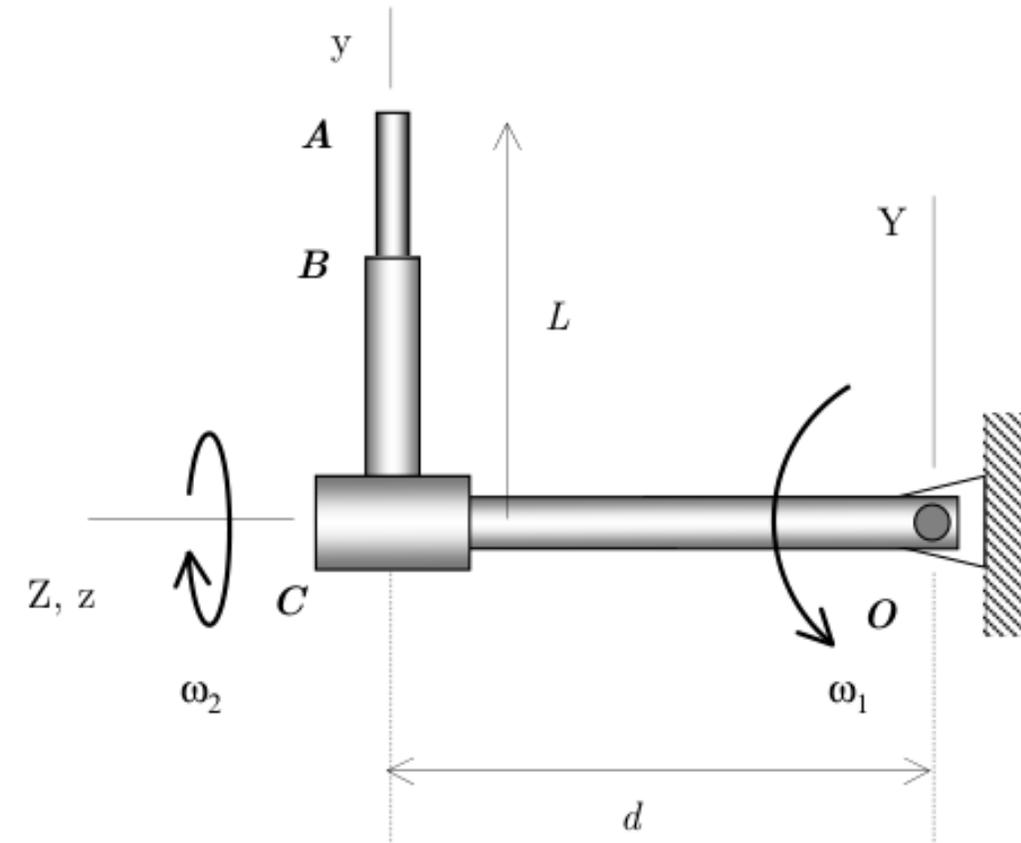
$\vec{v}_A = -r\omega_3 \hat{i} - (r\omega_1 + b\omega_2) \hat{k}$ **(same answer)**

Example 3.B.10

Given: $\dot{L} = 0.06 \text{ m/s} = \text{constant}$, $\omega_1 = 1.2 \text{ rad/s} = \text{constant}$ and $\omega_2 = 1.5 \text{ rad/s} = \text{constant}$. At the position shown, AC is aligned with the fixed Y-axis, $L = 0.12 \text{ m}$, and $d = 0.02 \text{ m}$.

Find: Determine:

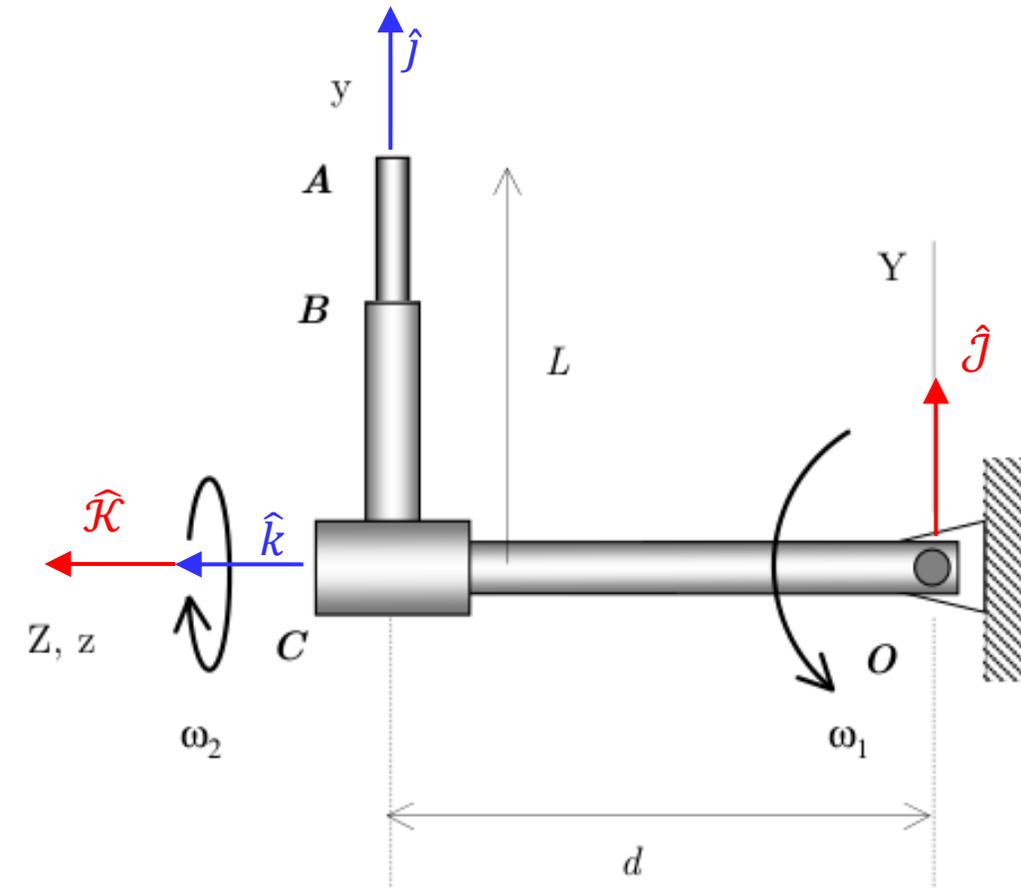
- The velocity of end A of the telescoping rod AC; and
- The acceleration of the same point.



Example 3.B.10

Given: \dot{L} , ω_1 , ω_2 constant

Find: (a) \vec{v}_A , (b) \vec{a}_A



Solution:

$$\vec{v}_A = \vec{v}_C + (\vec{v}_{A/C})_{rel} + \vec{\omega} \times \vec{r}_{A/C}$$

→ Information about C :

$$\vec{v}_C = \vec{v}_O + \vec{\omega}_{OC} \times \vec{r}_{C/O} = -\omega_1 d \hat{J}$$

→ Angular motion of the mov. ref.:

$$\vec{\omega} = \omega_1 \hat{I} - \omega_2 \hat{k}$$

$$\vec{\omega} \times \vec{r}_{A/C} = \omega_2 L \hat{i} + \omega_1 L \hat{k}$$

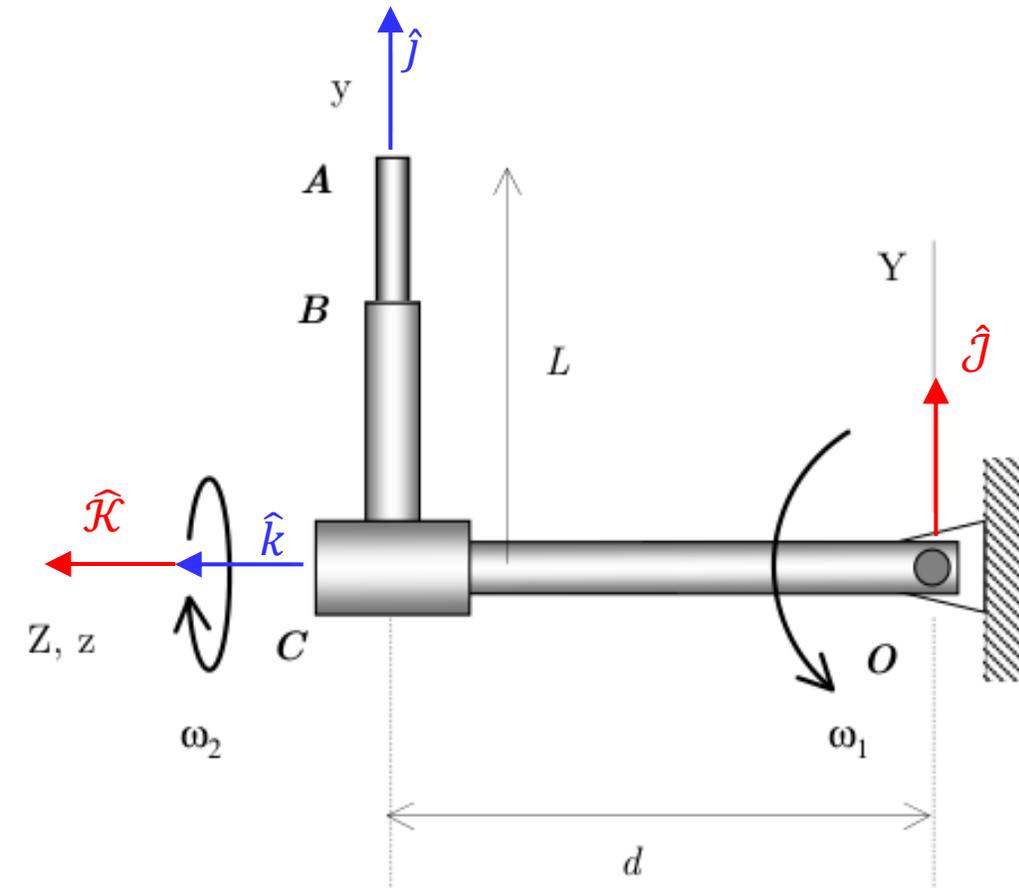
→ Relative motion of the mov. ref.:

$$(\vec{v}_{A/C})_{rel} = \dot{L} \hat{j}$$

Example 3.B.10

Given: \dot{L} , ω_1 , ω_2 constant

Find: (a) \vec{v}_A , (b) \vec{a}_A



Solution:

$$\vec{a}_A = \vec{a}_C + (\vec{a}_{A/C})_{rel} + \vec{\alpha} \times \vec{r}_{A/C} + 2\vec{\omega} \times (\vec{v}_{A/C})_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/C})$$

→ Information about C:

$$\vec{a}_C = \vec{a}_O + \vec{\alpha}_{OC} \times \vec{r}_{C/O} - \omega_1^2 \vec{r}_{C/O} = \vec{0} + \vec{0} - \omega_1^2 d \hat{\mathcal{K}}$$

→ Relative motion of the mov. ref.: $(\vec{a}_{A/C})_{rel} = \vec{0}$

→ Angular motion of the mov. ref.:

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d}{dt} (\omega_1 \hat{I} - \omega_2 \hat{k}) = -\omega_2 (\vec{\omega} \times \hat{k})$$

$$\vec{\alpha} = -\omega_2 [(\omega_1 \hat{I} - \omega_2 \hat{k}) \times \hat{k}] = -\omega_2 [(\omega_1 \hat{i} - \omega_2 \hat{k}) \times \hat{k}] = \omega_1 \omega_2 \hat{j}$$

→ Compute remaining terms

$$\vec{\alpha} \times \vec{r}_{A/C} = (\omega_1 \omega_2 \hat{j}) \times (L \hat{j}) = \vec{0}$$

$$2\vec{\omega} \times (\vec{v}_{A/C})_{rel} = 2(\omega_1 \hat{I} - \omega_2 \hat{k}) \times (\dot{L} \hat{j}) = 2\omega_1 \dot{L} \hat{k} + 2\omega_2 \dot{L} \hat{i}$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/C}) = -(\omega_1^2 + \omega_2^2) L \hat{j}$$

ME 274: Basic Mechanics II

Week 6 – Friday, February 20

Particle kinematics: Particle Kinetics (Intro)

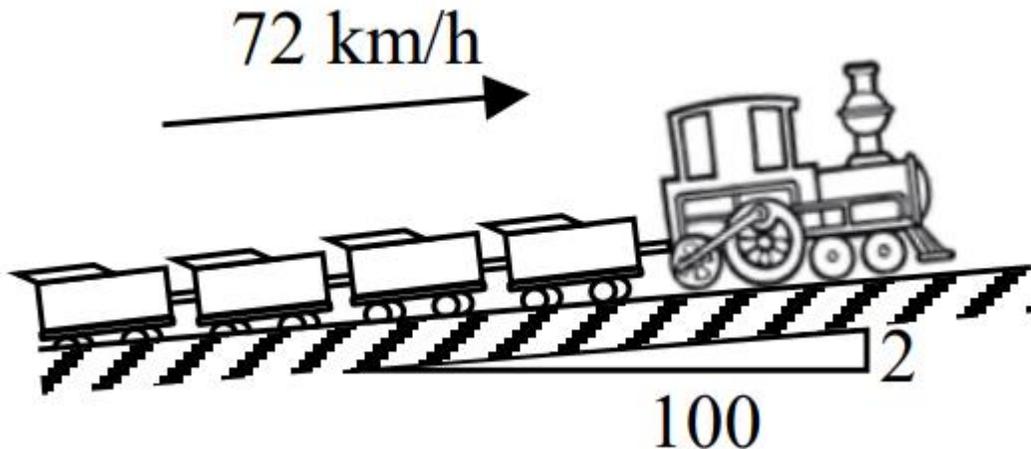
Instructor: Manuel Salmerón

Today's Agenda

1. Survey reminder
2. Statics Recap
3. From Statics to Kinetics
4. Summary of Steps

Statics Recap

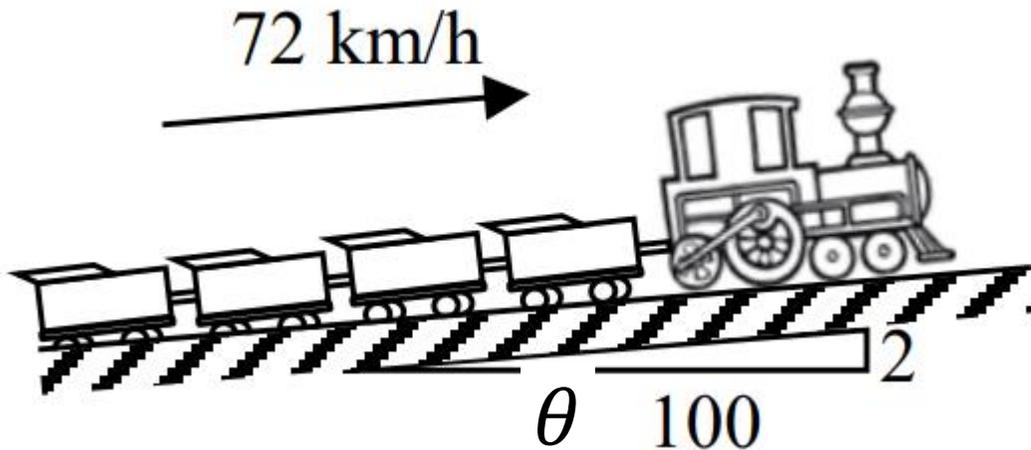
A train travels along a straight track with a 2% slope, moving at a constant speed of 72 km/h. The locomotive weighs 80 tons, and the total weight of the cars is 1000 tons. Determine all the external forces acting on the locomotive.



Taken from: Juan Ocáriz Castelazo, *Mecánica Clásica*, UNAM, <https://profesores.dcb.unam.mx/juanoc/>

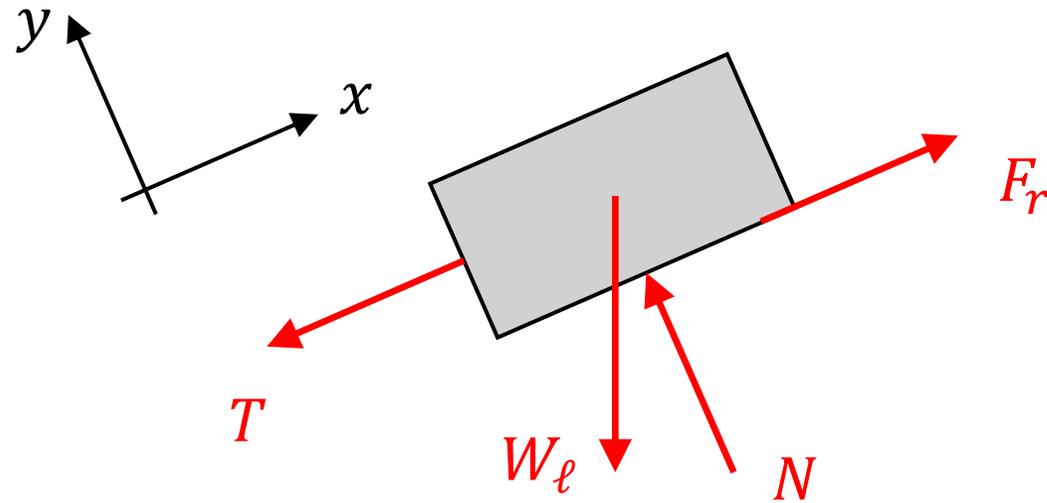
Given: slope, $v_0 \equiv \text{constant}$,
 $W_\ell = 80 \text{ t}$, $W_c = 1000 \text{ t}$, θ

Find: all the external forces
acting on the locomotive



Solution:

Free body diagram (FBD) of locomotive:



Fundamental Equations:

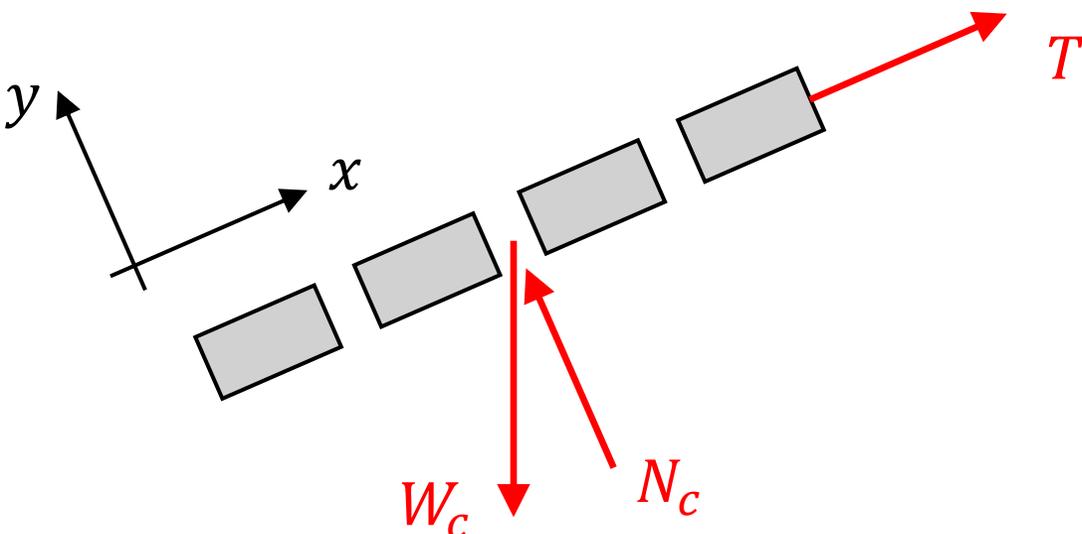
$$\sum F_x = 0 \quad \sum F_y = 0$$

$$\text{In } x: -T + E_r - W_\ell \sin \theta = 0$$

$$\text{In } y: N - W_\ell \cos \theta = 0$$

3 unknowns, 2 equations

FBD of the carts:



Fundamental Equations:

$$\sum F_x = 0 \quad \sum F_y = 0$$

$$\text{In } x: T - W_c \sin \theta = 0$$

Answer:

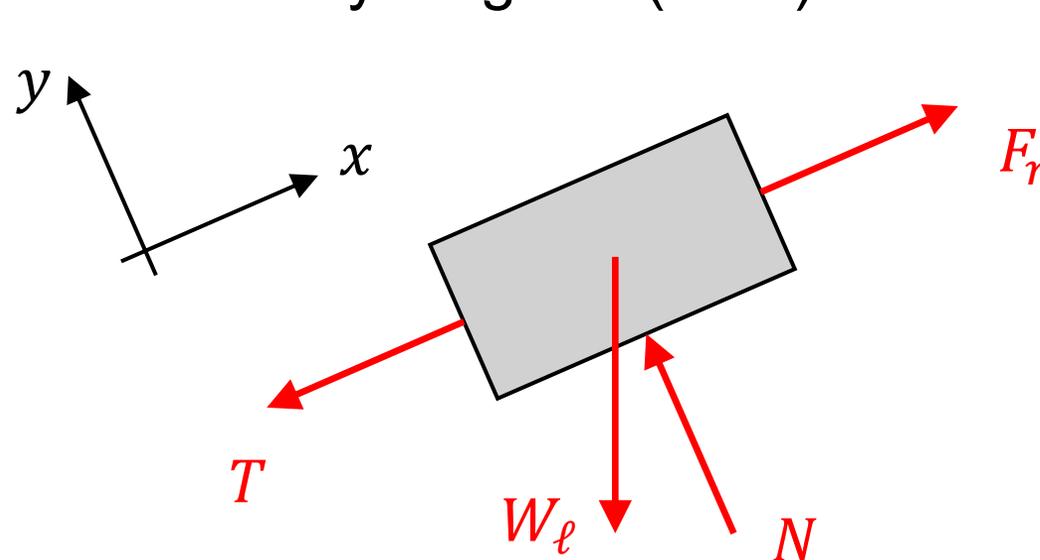
$$T = W_c \sin \theta$$

$$F_r = T + W_t \sin \theta$$

$$N = W_\ell \cos \theta$$

Solution:

Free body diagram (FBD) of locomotive:



Fundamental Equations:

$$\sum F_x = 0 \quad \sum F_y = 0$$

$$\text{In } x: -T + F_r - W_\ell \sin \theta = 0$$

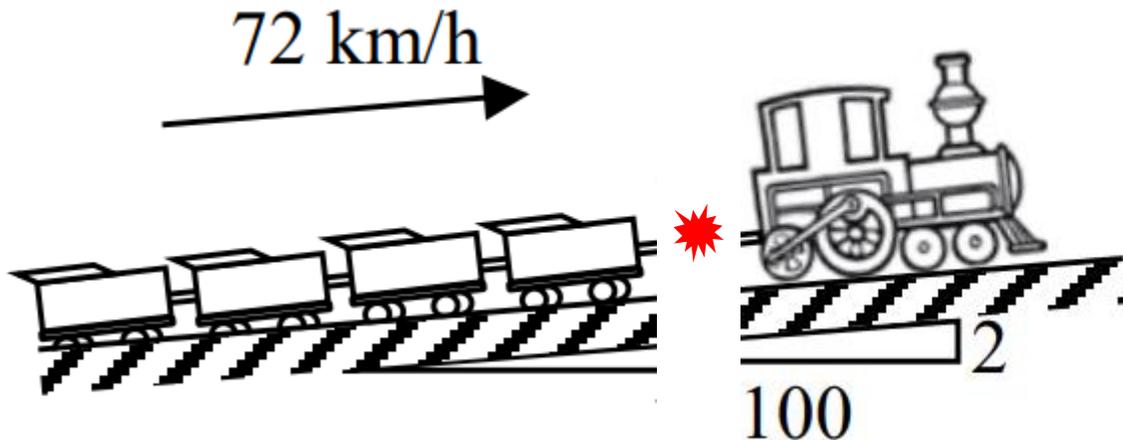
$$\text{In } y: N - W_\ell \cos \theta = 0$$

3 unknowns, 2 equations

From Statics to Kinetics

At a given instant, the link between the locomotive and the carts breaks.
Find:

- (a) the acceleration of the locomotive,
- (b) the acceleration of the carts,
- (c) the time it will take for the carts to return to the rupture position



Taken from: Juan Ocáriz Castelazo, *Mecánica Clásica*, UNAM, <https://profesores.dcb.unam.mx/juanoc/>

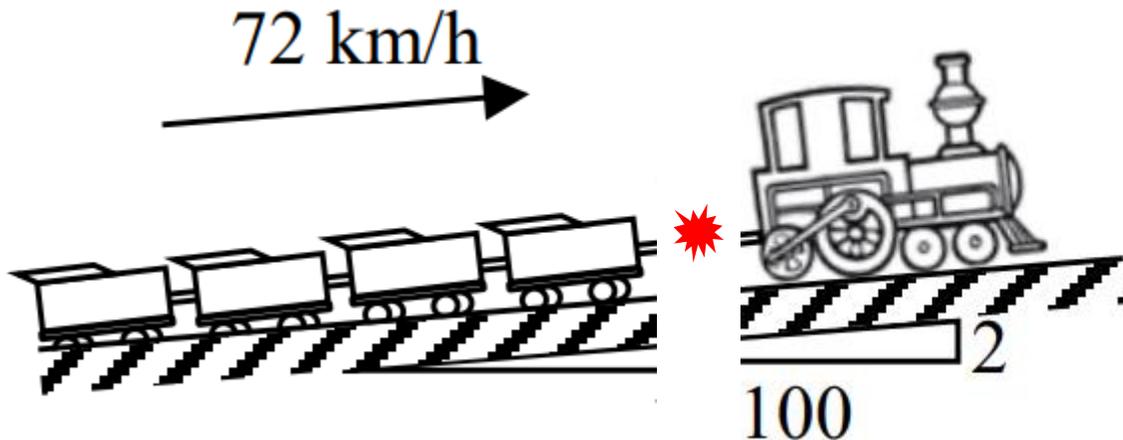
Given: same, but the link broke!

Find:

(a) a_{locom}

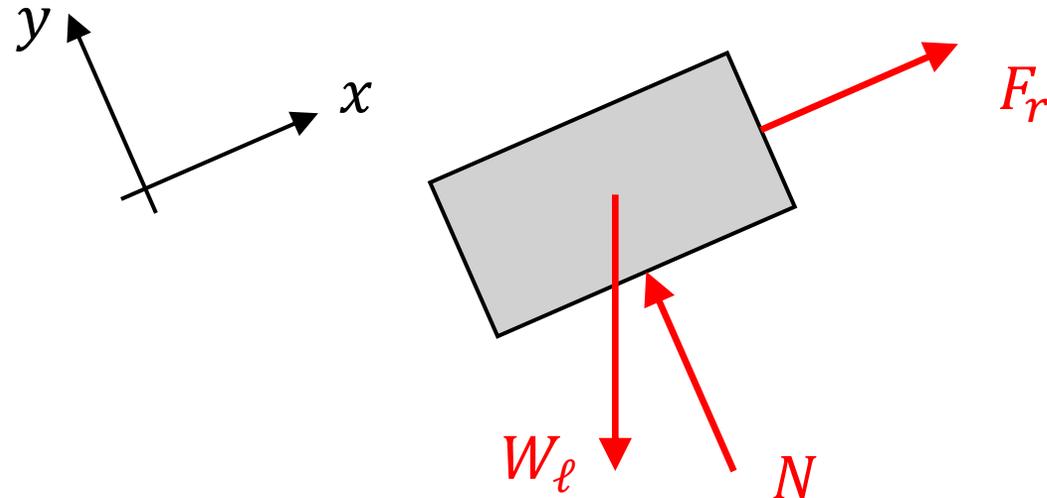
(b) a_{carts}

(c) t for carts to return to break position



Solution:

Free body diagram (FBD) of locomotive:



Fundamental Equations:

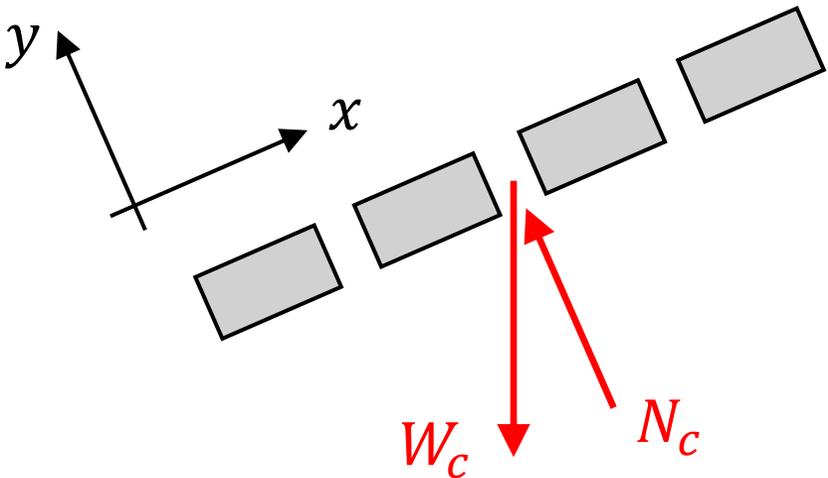
$$\sum F_x = ma_x \quad \sum F_y = ma_y$$

$$\text{In } x: F_r - W_\ell \sin \theta = ma_x = \left(\frac{W_\ell}{g}\right) a_x$$

Thus:

$$a_{locom} = \frac{F_r g}{W_\ell} - g \sin \theta$$

FBD of the carts:



Fundamental Equations:

$$\sum F_x = ma_x \quad \sum F_y = ma_y$$

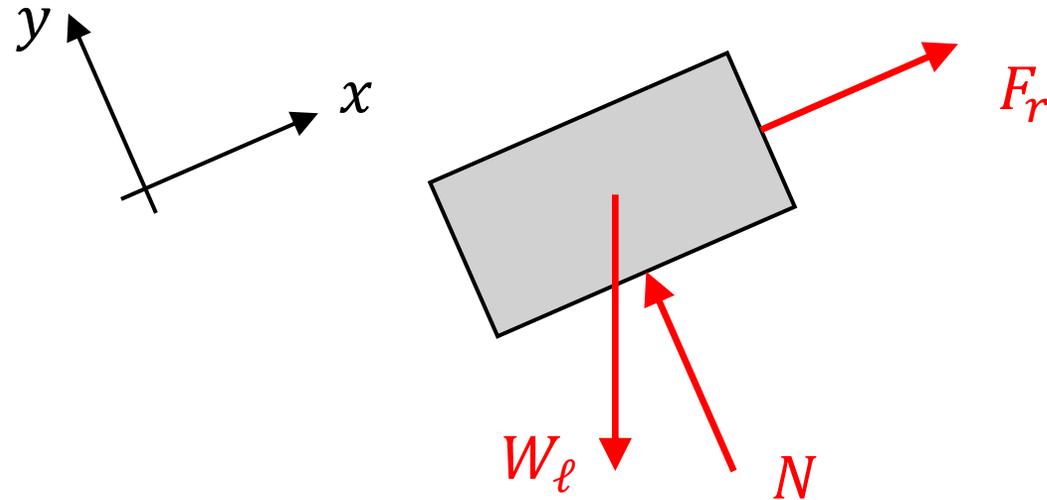
$$\text{In } x: -W_c \sin \theta = ma_x = \left(\frac{W_c}{g}\right) a_x$$

Thus:

$$(b) a_{carts} = -g \sin \theta$$

Solution:

Free body diagram (FBD) of locomotive:



Fundamental Equations:

$$\sum F_x = ma_x \quad \sum F_y = ma_y$$

$$\text{In } x: F_r - W_\ell \sin \theta = ma_x = \left(\frac{W_\ell}{g}\right) a_x$$

Thus:

$$(a) a_{locom} = \frac{F_r g}{W_\ell} - g \sin \theta$$

(c) Time it will take for the carts to return to the rupture position

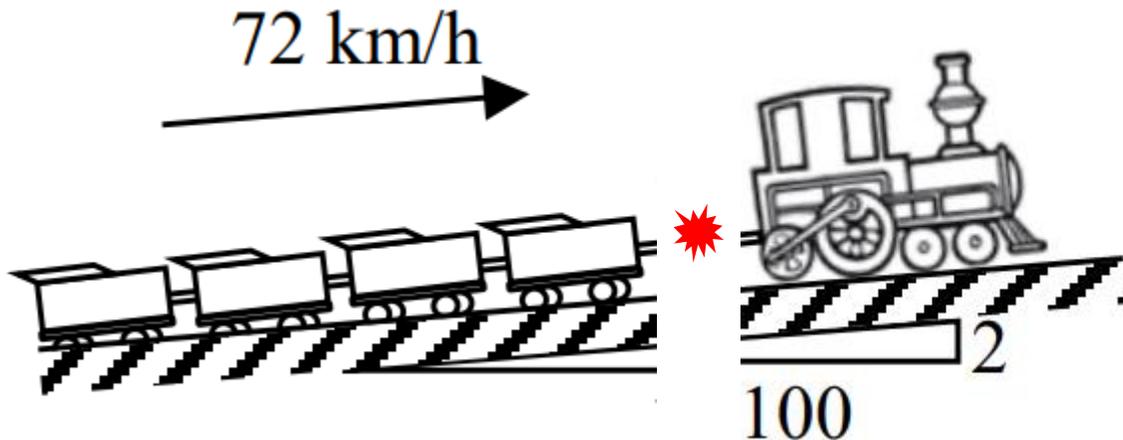
What is that?

At the instant of rupture:

$$a_{carts} = -g \sin \theta$$

Negative acceleration does not mean “negative” displacement!

Let's do the math.



$$a(t) = -g \sin \theta \quad (\text{constant})$$

We know:

$$a(t) = \frac{dv}{dt}$$

Thus:

$$a(t)dt = dv \quad \text{and} \quad \int_{t_0}^t a(t)dt = \int_{v_0}^v dv$$

Let $t_0 = 0$ be the rupture time. At this instant:

$$v_0 = v(t_0) \equiv \text{given in statics problem}$$

Integrating:

$$\int_0^t (-g \sin \theta) dt = -gt \sin \theta = v - v_0$$

Expression for the velocity:

$$v(t) = v_0 - gt \sin \theta$$

(c) Time it will take for the carts to return to the rupture position

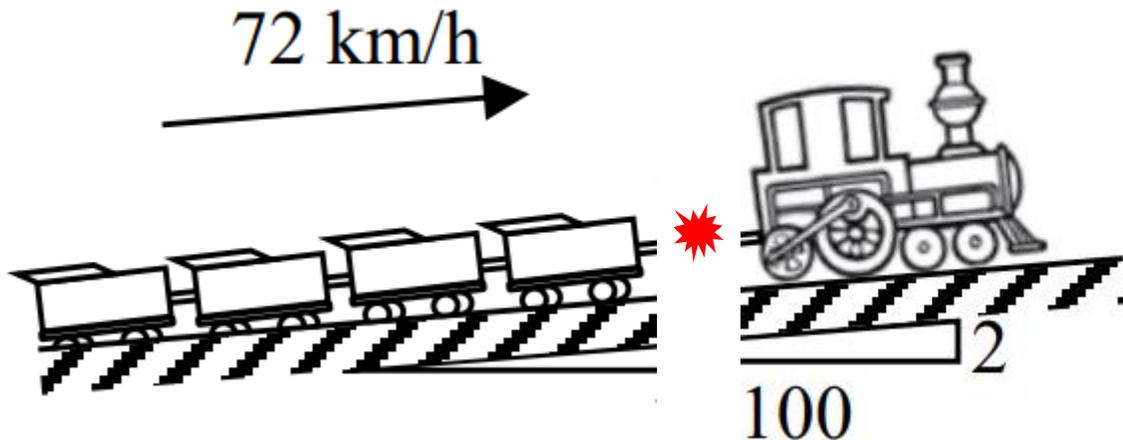
What is that?

At the instant of rupture:

$$a_{carts} = -g \sin \theta$$

Negative acceleration does not mean “negative” displacement!

Let's do the math.



Expression for the acceleration:

$$a(t) = -g \sin \theta$$

Expression for the velocity:

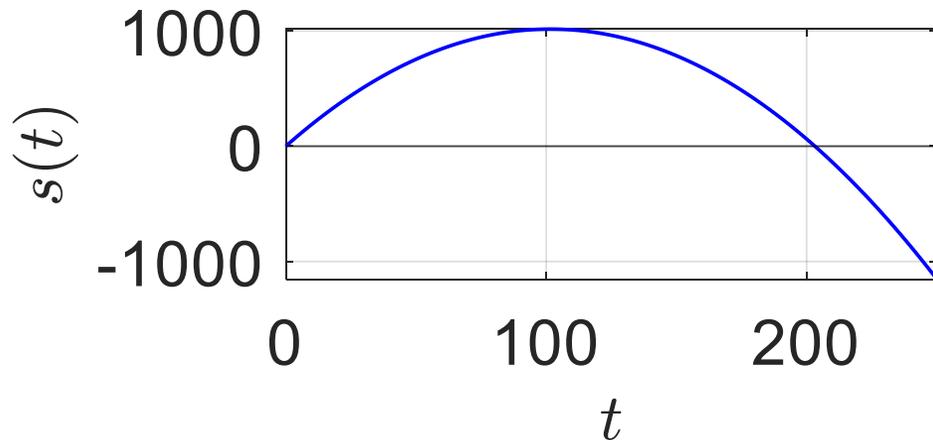
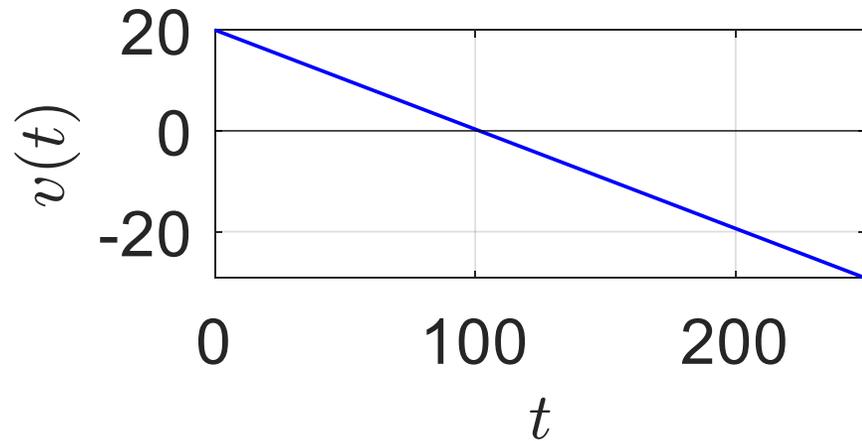
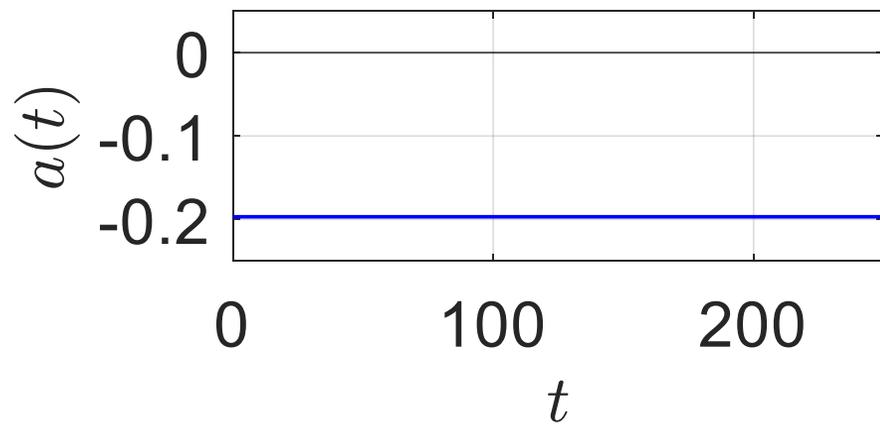
$$v(t) = v_0 - gt \sin \theta$$

For the position:

$$v(t) = \frac{ds}{dt} \rightarrow \int_0^t v(t) dt = \int_{s_0}^s ds$$

Assuming $s_0 = s(t_0) = 0$ at the rupture time:

$$s(t) = v_0 t - \frac{1}{2} g t^2 \sin \theta$$



Expression for the acceleration:

$$a(t) = -g \sin \theta$$

Expression for the velocity:

$$v(t) = v_0 - gt \sin \theta$$

For the position:

$$v(t) = \frac{ds}{dt} \rightarrow \int_0^t v(t) dt = \int_{s_0}^s ds$$

Assuming $s_0 = s(t_0) = 0$ at the rupture time:

$$s(t) = v_0 t - \frac{1}{2} g t^2 \sin \theta$$

(c) Time it will take for the carts to return to the rupture position:

$$s(t) = v_0 t - \frac{1}{2} g t^2 \sin \theta = 0$$

Answer: 204 seconds

Format of examples

Step 1: find all the forces

Step 2: balance all the forces

Step 3: what kinematic relations are useful?

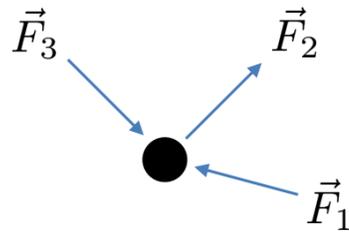
Example	Given	Find	Kinematic Relation(s)
Train (a),(b)	v_0	a	None!
Train (c)	v_0	t	$a = \frac{dv}{dt}, v = \frac{ds}{dt}$

Step 4: solve (usually integrate and clear)

Summary of Steps

1. Free Body Diagram (FBD)

Newton's Second Law



2. Kinetics

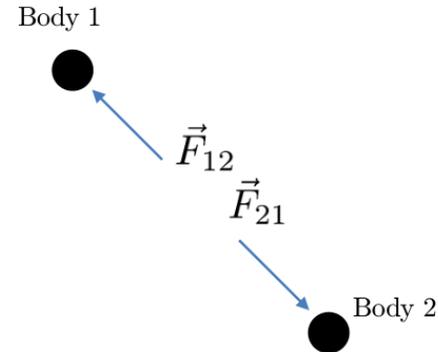
$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = m\vec{a} \quad \vec{F}_{12} = -\vec{F}_{21}$$

3. Kinematics

$$a = \frac{dv}{dt} = \frac{dv}{ds} \frac{ds}{dt} = v \frac{dv}{ds} = \frac{d^2s}{dt^2}$$

4. Solve – for whatever is asked!

Newton's Third Law



Manuel's Findings

Example	Given	Find	Kinematic Relation(s)
Train (a),(b)	v_0	a	None!
Train (c)	v_0	t	$a = \frac{dv}{dt}, v = \frac{ds}{dt}$
4.A.1	v_1, v_2, F	s, t	$a = \frac{dv}{dt} = \frac{dv}{ds} \frac{ds}{dt} = v \frac{dv}{ds}$
4.A.2	$v_1 = 0$	v	$\ddot{s} = \frac{d\dot{s}}{dt} = \frac{d\dot{s}}{ds} \frac{ds}{dt} = v \frac{d\dot{s}}{ds}$
4.A.3	v_0	t	$a = \frac{dv}{dt}$
4.A.4	a	a	None!
4.A.5	F	a, N	None!