Announcements

- MIDTERM: Thursday, 02/12, 8 – 9:30 PM
- If you have accommodations, you must have received an email. Let me know if you didn't!
- Review sessions:
  - Thursday, 02/05, 6:30 – 7:30 PM, hosted by Pi Tau Sigma at WTHR 172
  - Tuesday, 02/10, 7:00 PM, by Prof. Krousgrill, Zoom and in-person (meeting room TBD)

ME 274: Basic Mechanics II

Week 4 *– Monday, February 2*

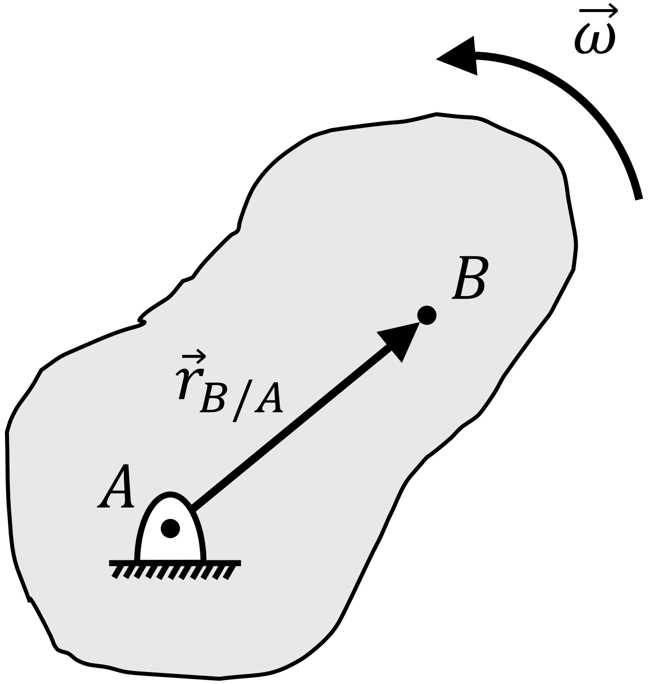
Particle kinematics: Instantaneous Centers of Rotation

Instructor: Manuel Salmerón

Today’s Agenda

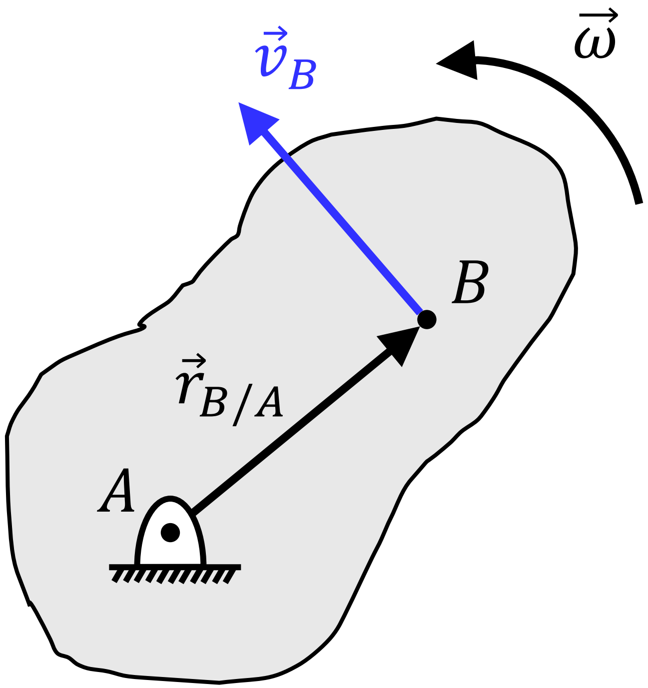
1. Rigid Body Background
2. Instantaneous Centers of Rotation
3. Example 2.B.1
4. Example 2.B.2

Rigid Body Background



What do you know about the **rigid body** on the left?

Rigid Body Background



$\vec{r}_{B/A} = r_x\hat{\imath} + r_y\hat{\jmath}$

What do you know about the **rigid body** on the left?

(1) $|\vec{r}_{B/A}| = \text{constant}$

(2) $\vec{v}_A = \vec{0}$

Fundamental equation for $\vec{v}_B$:

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

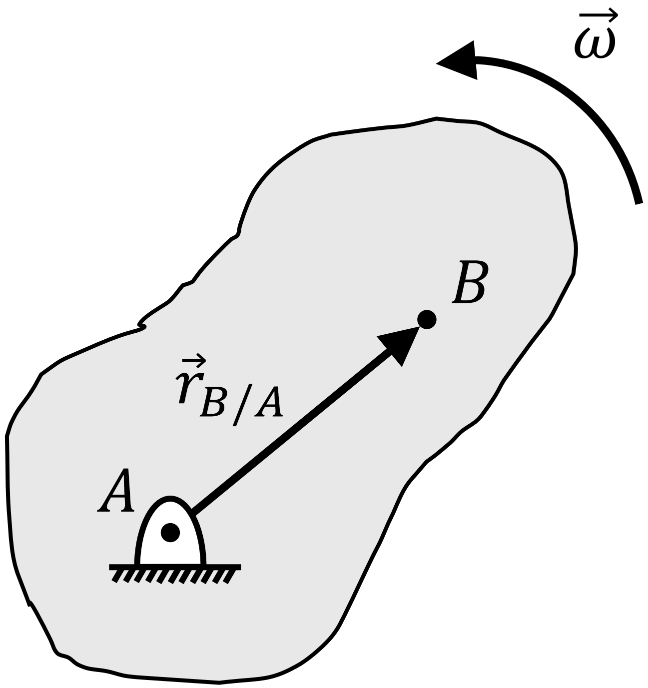
$\vec{0}$

$$\vec{v}_B = \vec{\omega} \times \vec{r}_{B/A} \text{ (i.e., } \vec{v} \perp \vec{r}_{B/A}\text{)}$$

$$\vec{v}_B = -\omega r_y\hat{\imath} + \omega r_x\hat{\jmath}$$

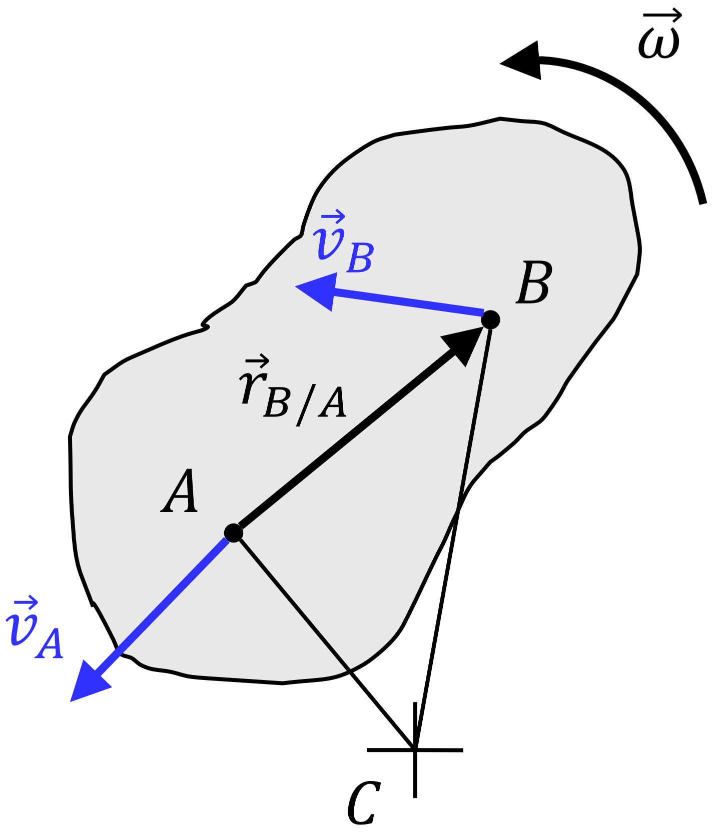
$$|\vec{v}_B| = \sqrt{\omega^2 r_y^2 + \omega^2 r_x^2} = \omega\sqrt{r_x^2 + r_y^2} = \omega|\vec{r}_{B/A}|$$

Instantaneous Centers of Rotation



- Assume that A is no longer fixed

Instantaneous Centers of Rotation

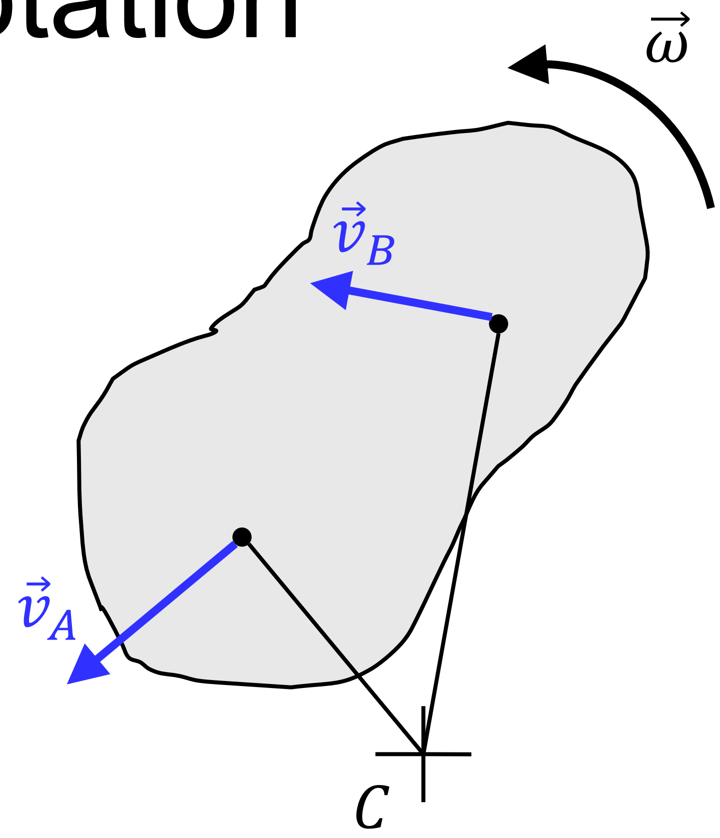


- Assume that A is no longer fixed
- Assume A and B have velocities in the directions shown
- At a given instant of time, we can "imagine" that the body is in pure rotation.
- At this instant, the body rotates around a point C.

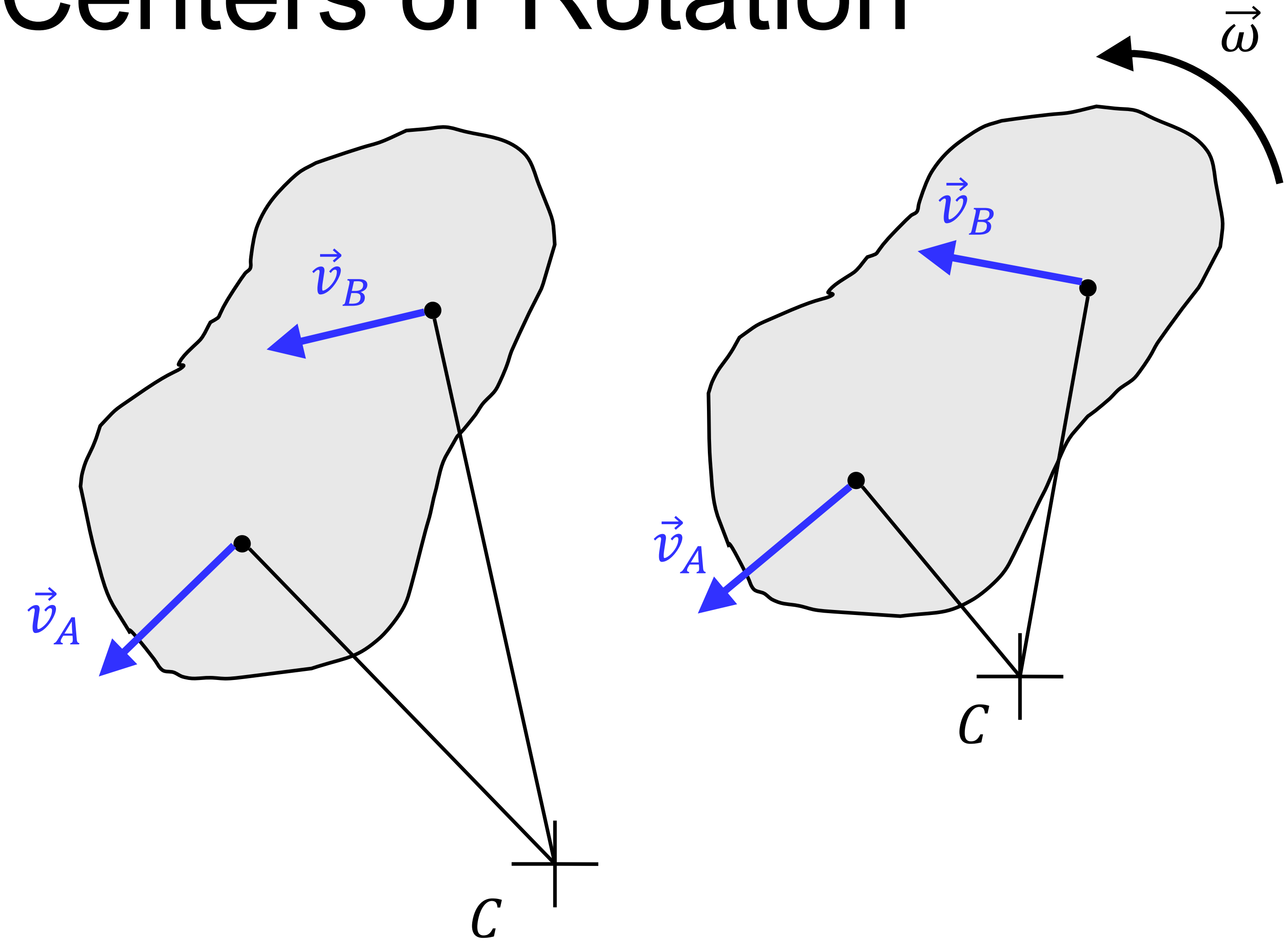
FACTS:

- C is fixed: $\vec{v}_C = \vec{0}$
- C lies on a line perpendicular to $\vec{v}_A$ **and** on a line perpendicular to $\vec{v}_B$
- The angular speed is $\omega = \frac{|\vec{v}_A|}{|\vec{r}_{A/C}|} = \frac{|\vec{v}_B|}{|\vec{r}_{B/C}|}$

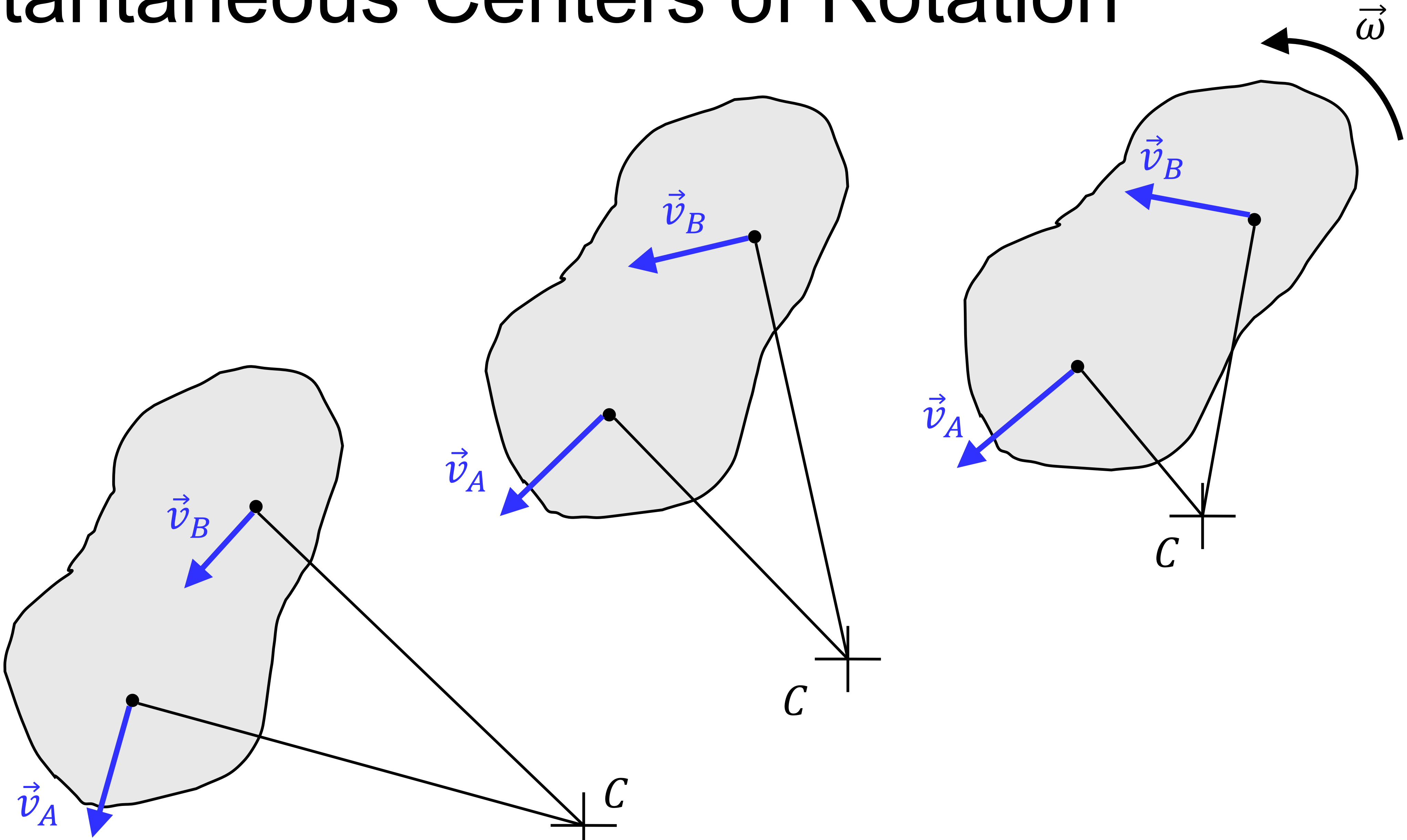
Instantaneous Centers of Rotation



Instantaneous Centers of Rotation



Instantaneous Centers of Rotation

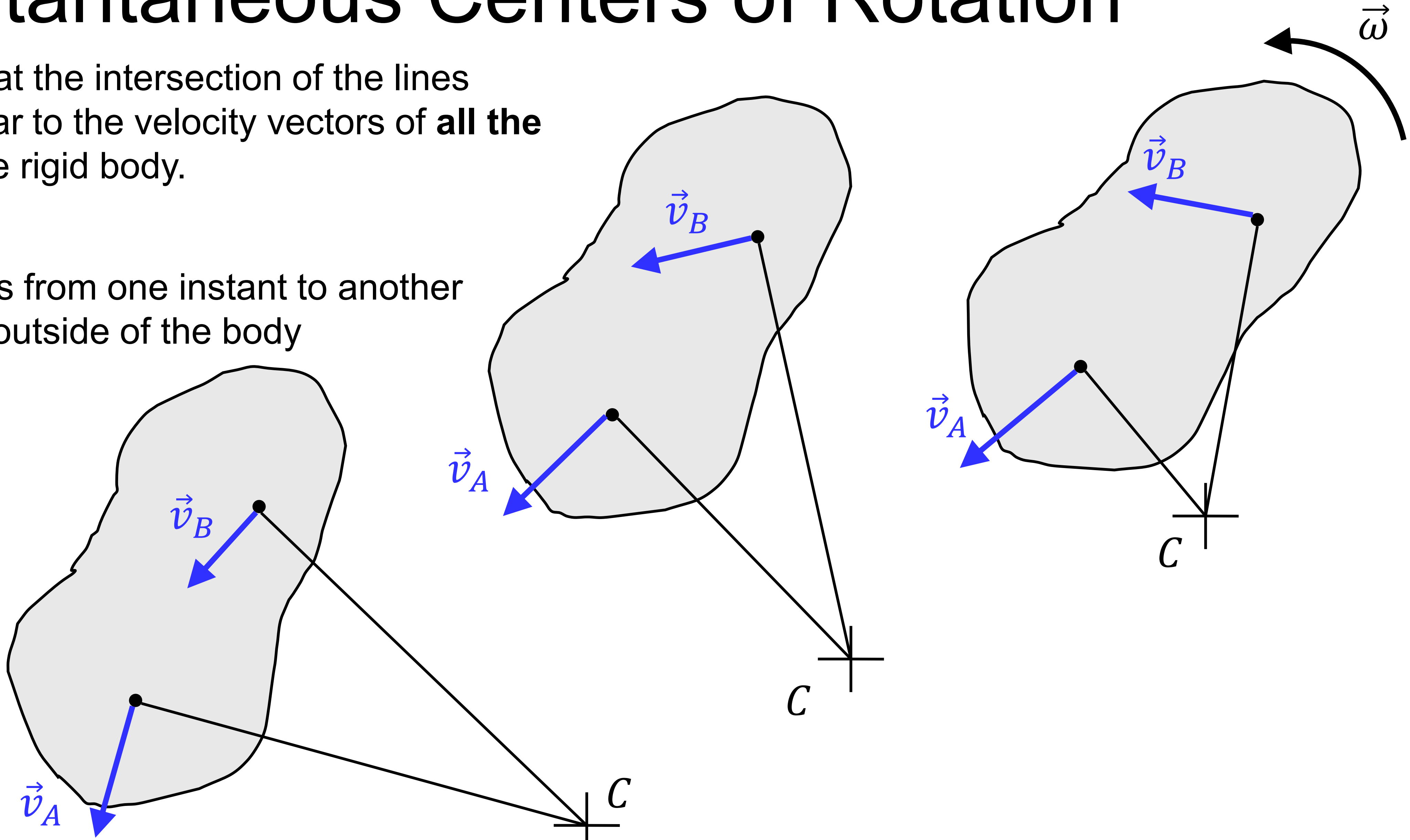


Instantaneous Centers of Rotation

C is always at the intersection of the lines perpendicular to the velocity vectors of **all the points** in the rigid body.

Thus:

- C changes from one instant to another
- C can lie outside of the body

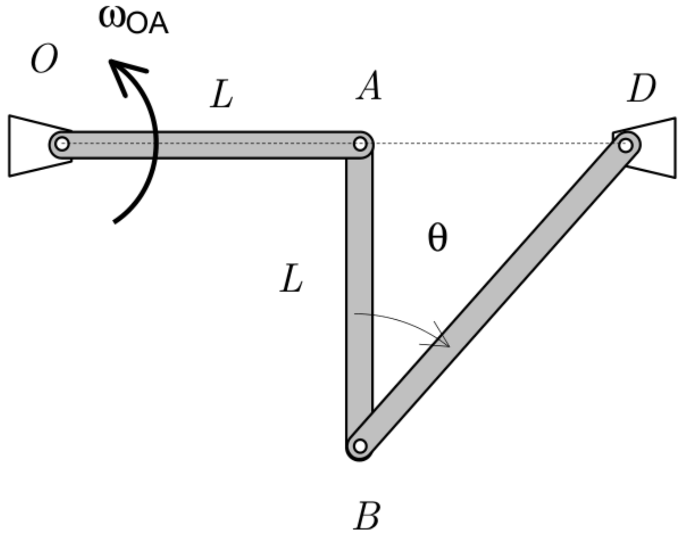


Example 2.B.1.

Given: Link OA rotates with an angular speed of $\omega_{OA} = 3$ rad/s with a counterclockwise sense about pin O. At the instant shown, link OA is horizontal, AB is vertical and $\theta = 36.87°$.

Find:

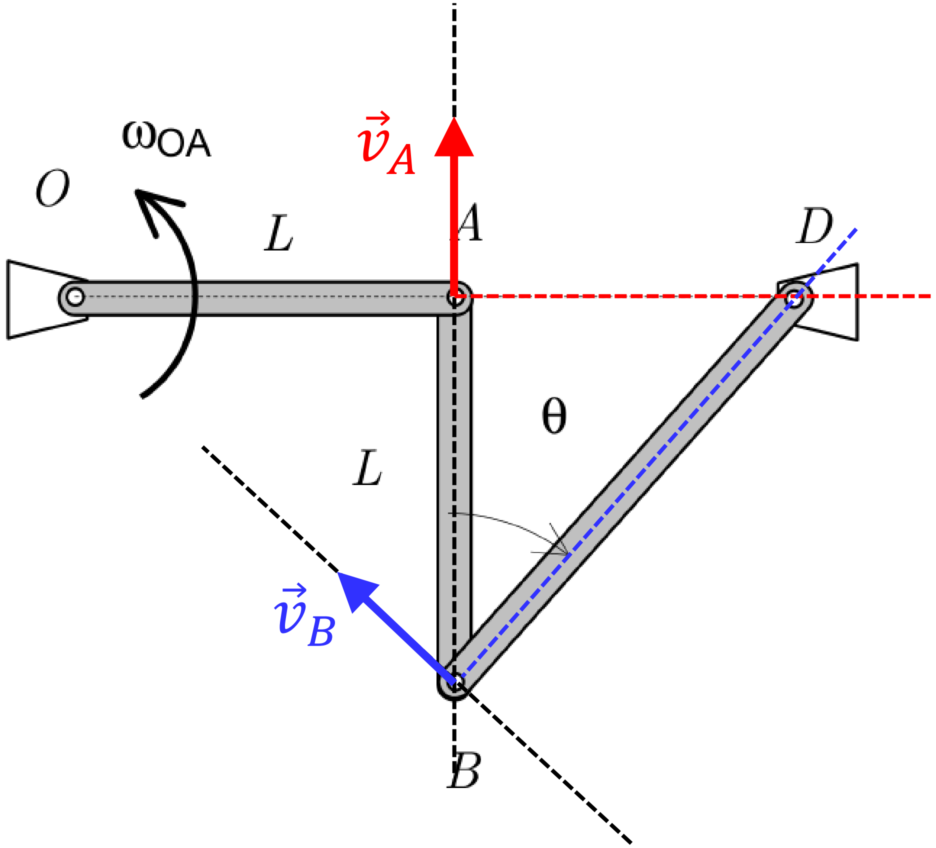
(a) Locate the instant center IC_{AB} for link AB.
(b) Using the location of IC_{AB}, determine the angular velocities of links AB and DB.



Example 2.B.1.

Given: $\omega_{OA} = 3$ rad/s, $\theta = 36.87°$

Find: (a) IC_{AB}, (b) $\vec{\omega}_{AB}$, $\vec{\omega}_{DB}$



Solution:

(a) Find the IC_{AB}:

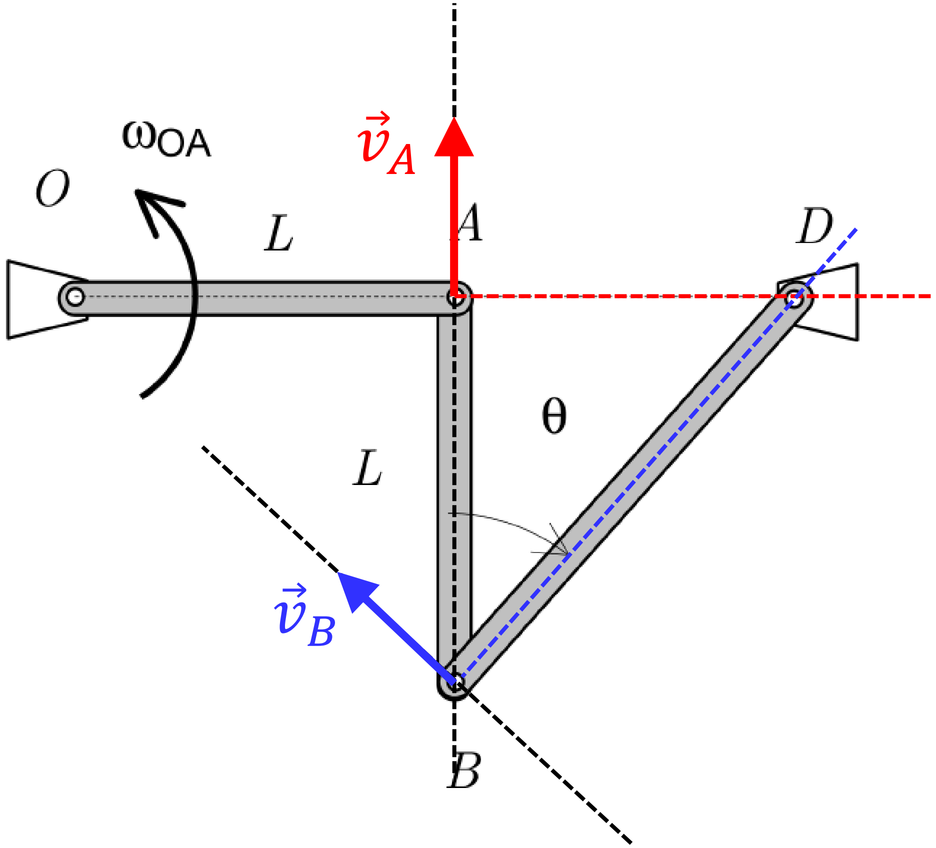
1. Identify direction of $\vec{v}_A$
2. Identify direction of $\vec{v}_B$
3. Draw perpendiculars to each velocity
4. The intersection is the IC

In this case, $IC_{AB} = D$

Example 2.B.1.

Given: $\omega_{OA} = 3$ rad/s, $\theta = 36.87°$

Find: (a) IC_{AB}, (b) $\vec{\omega}_{AB}$, $\vec{\omega}_{DB}$



Solution:

(b) For $\vec{\omega}_{AB}$:

$$\omega_{AB} = \frac{|\vec{v}_A|}{|\vec{r}_{A/D}|} = \frac{|\vec{v}_B|}{|\vec{r}_{B/D}|}$$

We have information about $\vec{v}_A$:

$$\vec{v}_A = \vec{v}_O + \vec{\omega}_{OA} \times \vec{r}_{A/O}$$

$$\vec{v}_A = \vec{\omega}_{OA} \times \vec{r}_{A/O}$$

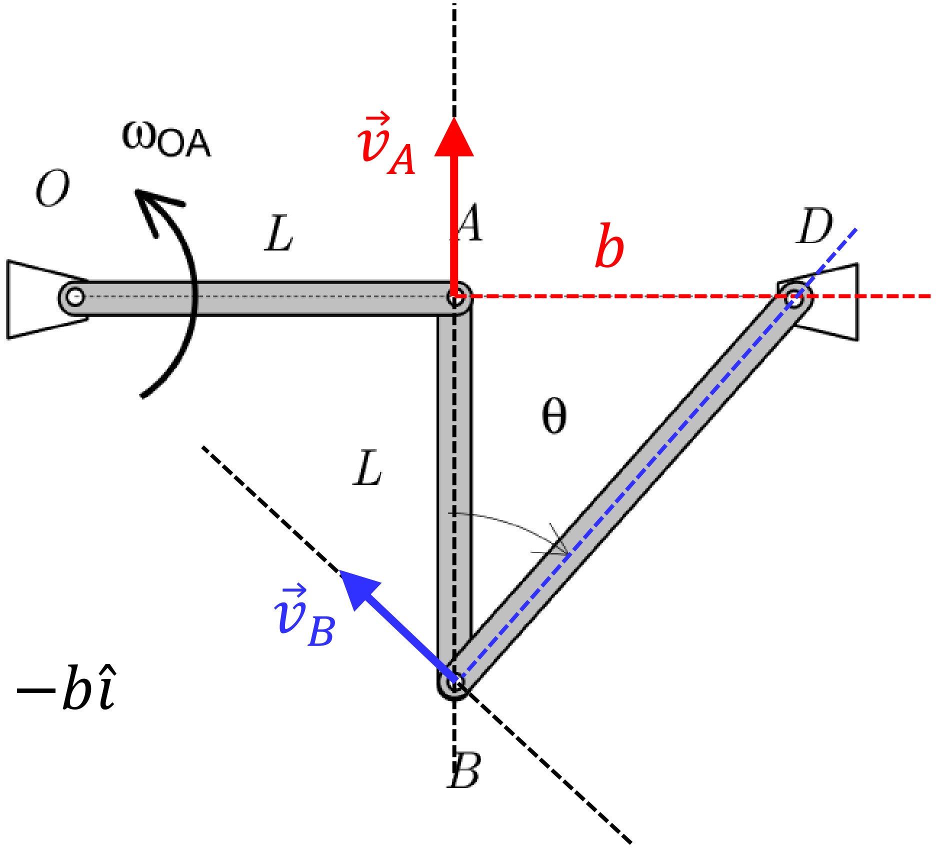
$$\vec{v}_A = (\omega_{OA}\hat{k}) \times (L\hat{\imath})$$

$$\vec{v}_A = L\omega_{OA}\hat{\jmath}$$

Example 2.B.1.

Given: $\omega_{OA} = 3$ rad/s, $\theta = 36.87°$

Find: (a) IC_{AB}, (b) $\vec{\omega}_{AB}$, $\vec{\omega}_{DB}$



$\vec{v}_A$

$\vec{v}_B$

$\vec{r}_{A/D} = -b\hat{\imath}$

Solution:

(b) For $\vec{\omega}_{AB}$:

$$\omega_{AB} = \frac{|\vec{v}_A|}{|\vec{r}_{A/D}|} = \frac{|\vec{v}_B|}{|\vec{r}_{B/D}|}$$

Thus:

$$\omega_{AB} = \frac{L\omega_{OA}}{b}$$

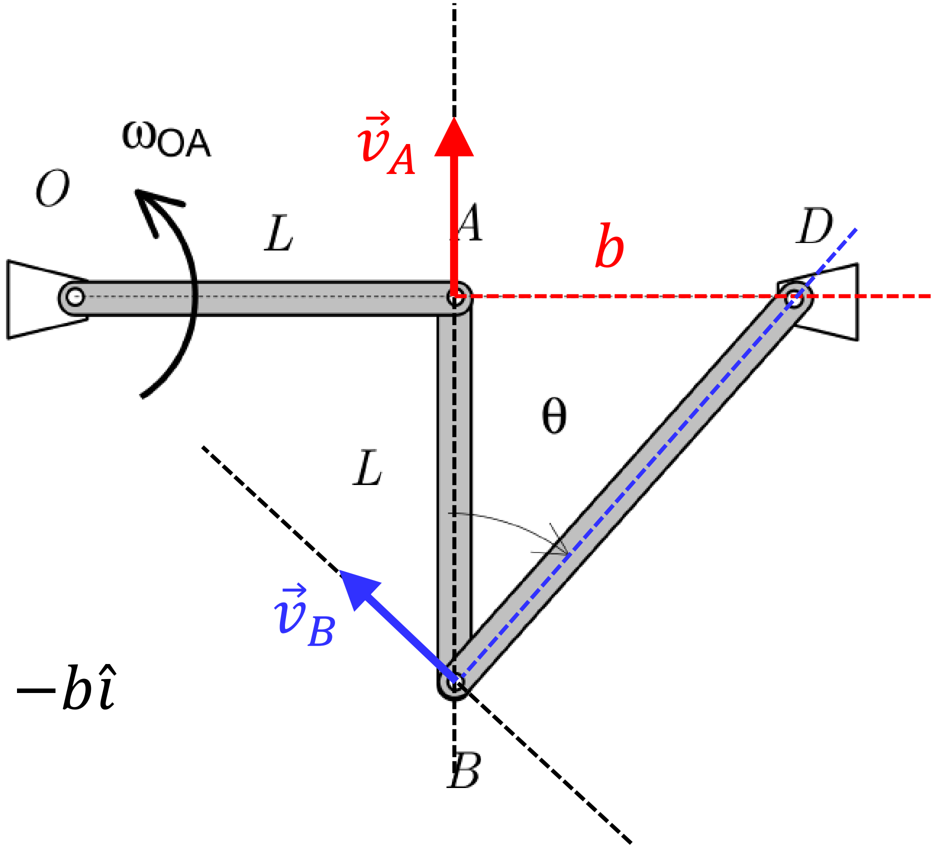
The IC_{AB} is D, thus the direction is CW:

$$\vec{\omega}_{AB} = -\frac{L\omega_{OA}}{b}\hat{k}$$

Example 2.B.1.

Given: $\omega_{OA} = 3$ rad/s, $\theta = 36.87°$

Find: (a) IC_{AB}, (b) $\vec{\omega}_{AB}$, $\vec{\omega}_{DB}$



$\vec{r}_{A/D} = -b\hat{\imath}$

Solution:

(b) For $\vec{\omega}_{BD}$:

It is easily seen that the IC_{BD} of the body BD is D. Thus,

$$\omega_{DB} = \frac{|\vec{v}_B|}{|\vec{r}_{B/D}|}$$

But from the previous body we know that:

$$\omega_{AB} = \frac{|\vec{v}_B|}{|\vec{r}_{B/D}|}$$

Thus: $\omega_{DB} = \omega_{AB}$, and $\vec{\omega}_{DB} = -\frac{L\omega_{OA}}{b}\hat{k}$

Example 2.B.2

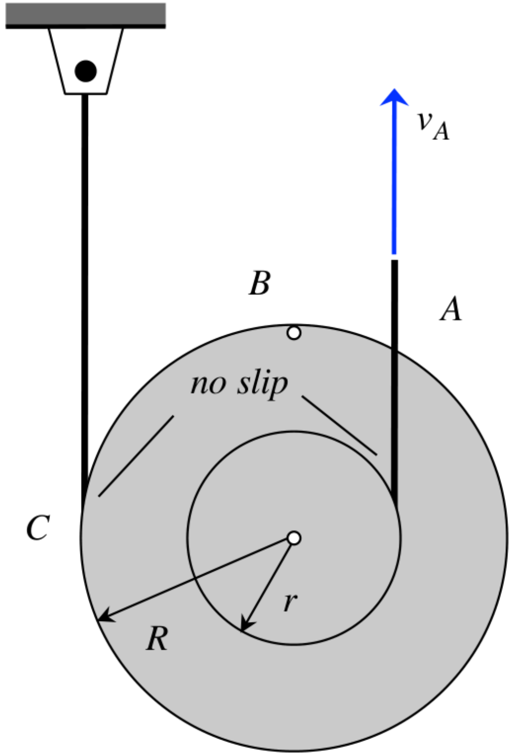
Given: A cable, wrapped around the inner radius of the pulley shown, is being raised at a rate of v_A. A second cable is wrapped around the outer radius of the same pulley with the upper end of this cable attached to ground. Assume that the pulley does not slip on either cable.

Find: Determine:

(a) The location of the instant center for the pulley; and

(b) The velocity of point B on the outer radius of the pulley when B is directly above the center O of the pulley. Sketch this velocity vector.

Use the following parameters in your analysis: $v_A = 3$ m/s, $r = 0.5$ ft and $R = 1$ ft.



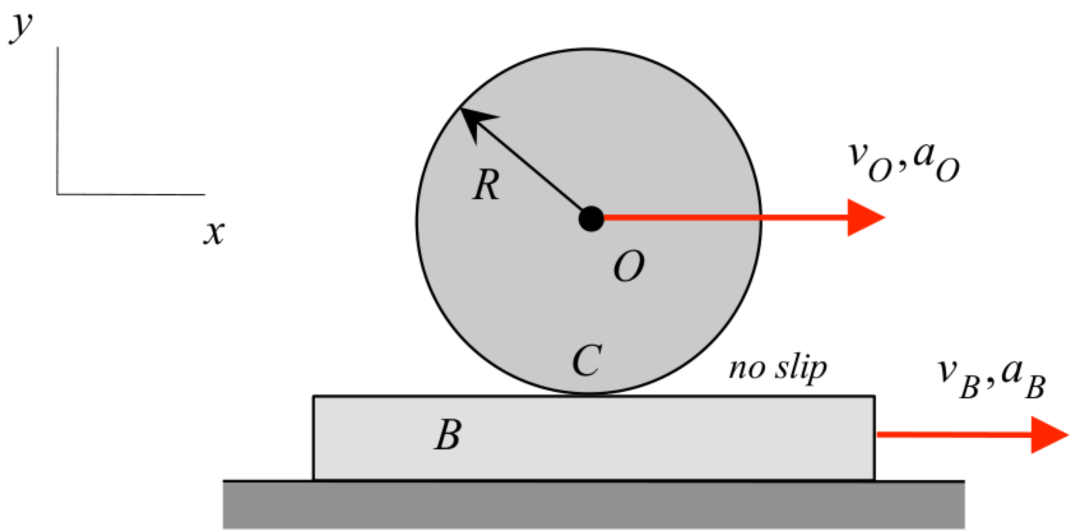
Sketch of Solution

How to use the “no slip” information?

Remember:

$$v_{Cx} = v_B$$

$$v_{Cy} = 0$$



Sketch of Solution

How to use the "no slip" information?

Remember:

$$v_{Cx} = v_B$$
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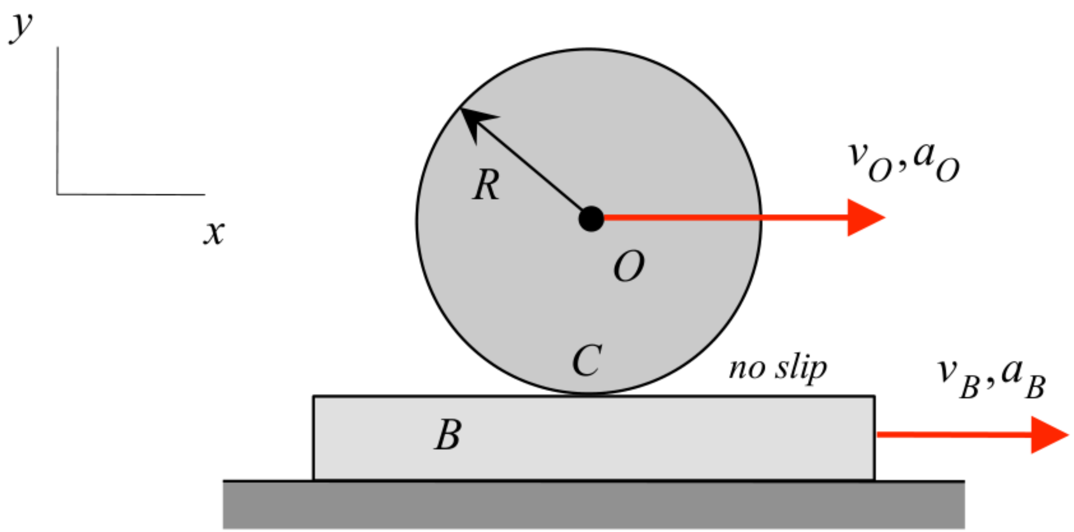
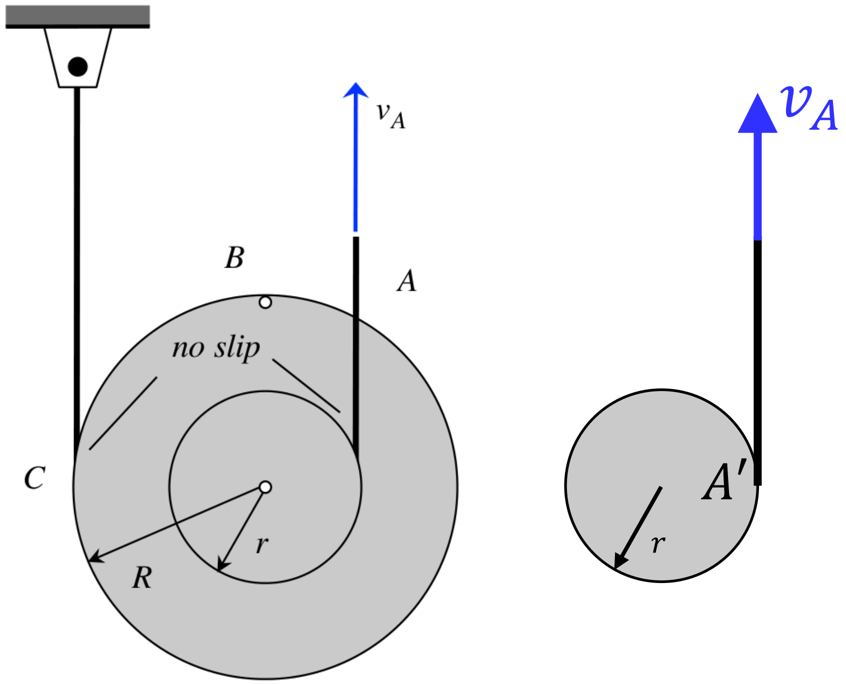


In Example 2.B.2:



Sketch of Solution

How to use the “no slip” information?

Remember:

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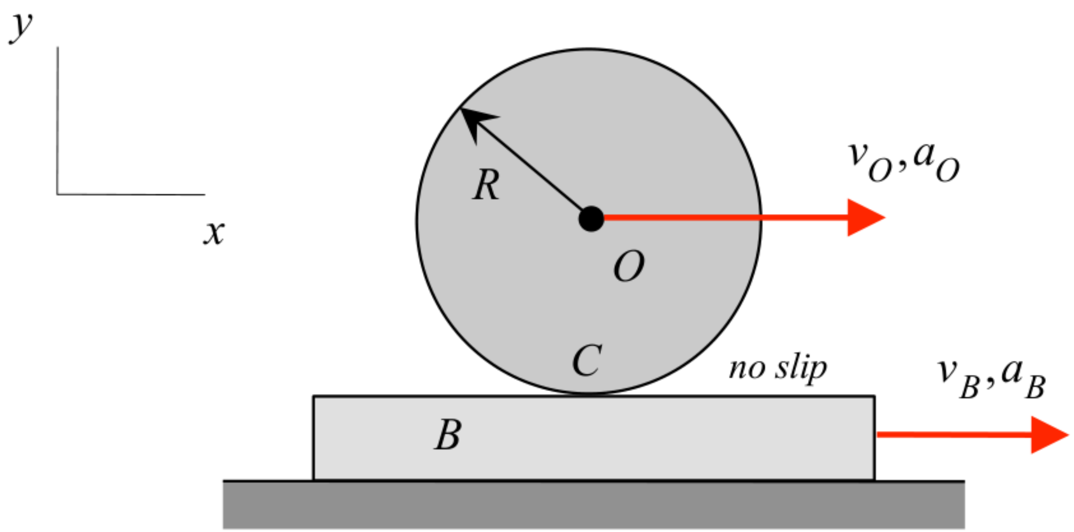
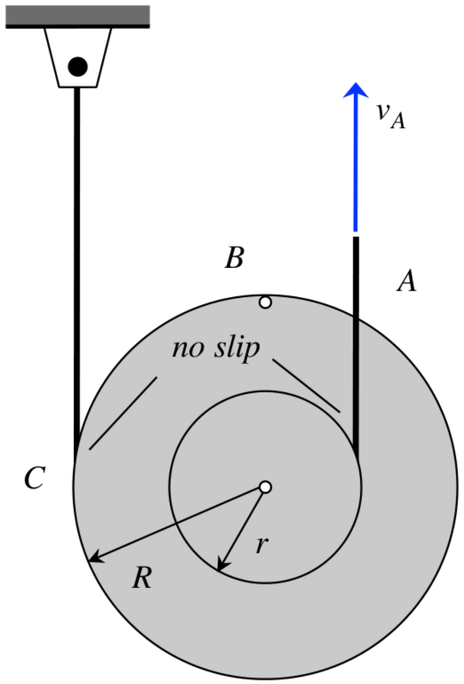
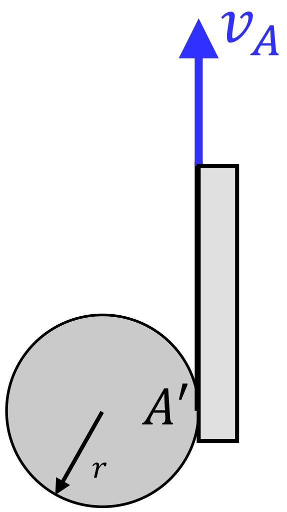


In Example 2.B.2:



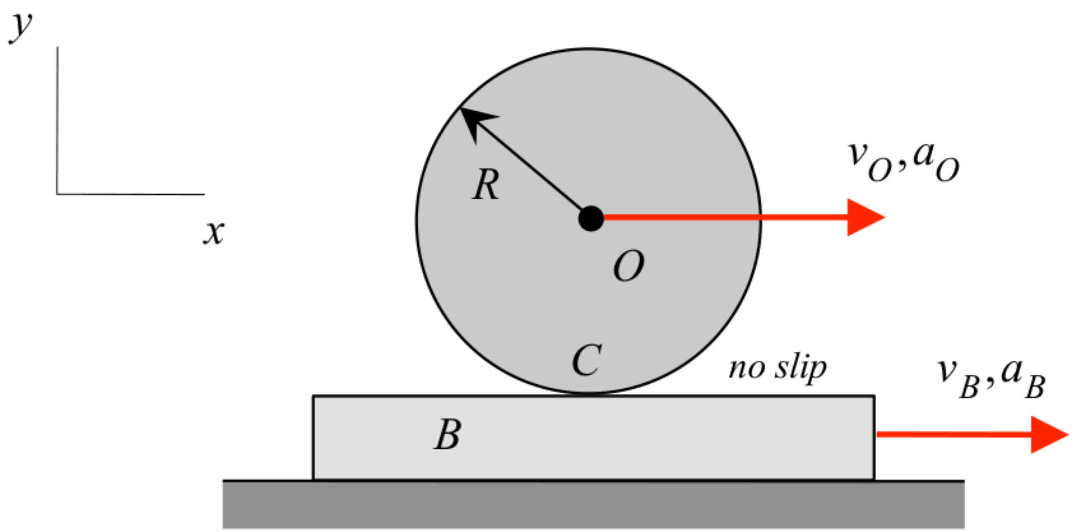


Sketch of Solution

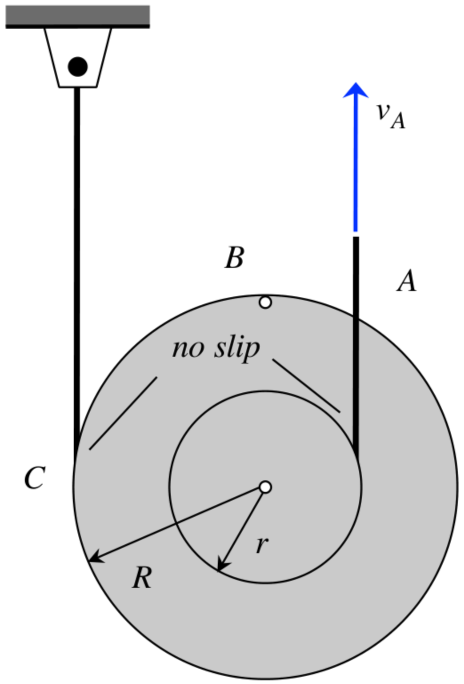
How to use the "no slip" information?

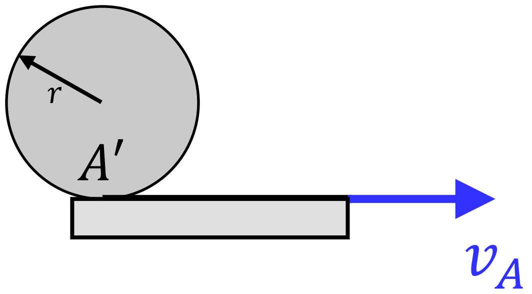
Remember:

$$v_{Cx} = v_B$$
$$v_{Cy} = 0$$



In Example 2.B.2:





$$v_{A'y} = v_A$$
$$v_{A'x} = 0$$

Sketch of Solution

For (a):

1. The ICR is somewhere along the line perpendicular to $\vec{v}_A$
2. The ICR has velocity $\vec{0}$
3. Point C is along the dotted line AND has $\vec{0}$
4. Thus, C is the ICR

For (b):

1. You know that $\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$
2. Use $\omega = \frac{|\vec{v}_A|}{|\vec{r}_{A/C}|}$ to get ω
3. Solve for $\vec{v}_B$

ME 274: Basic Mechanics II

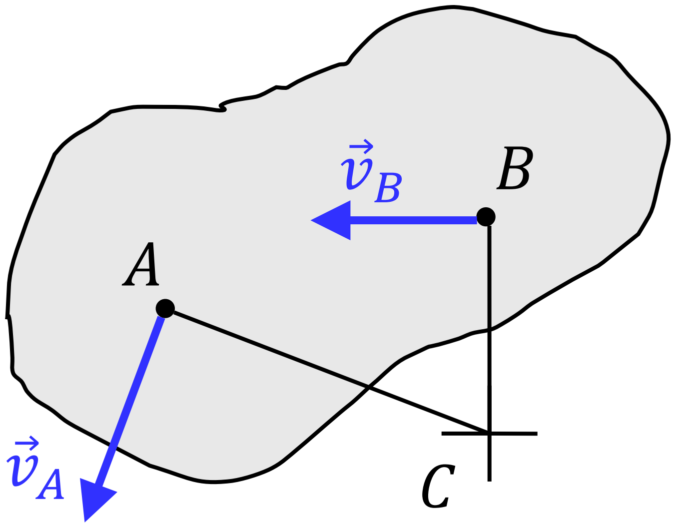
Week 4 *– Wednesday, February 4*

Particle kinematics: Instantaneous Centers of Rotation and Rigid Bodies Summary

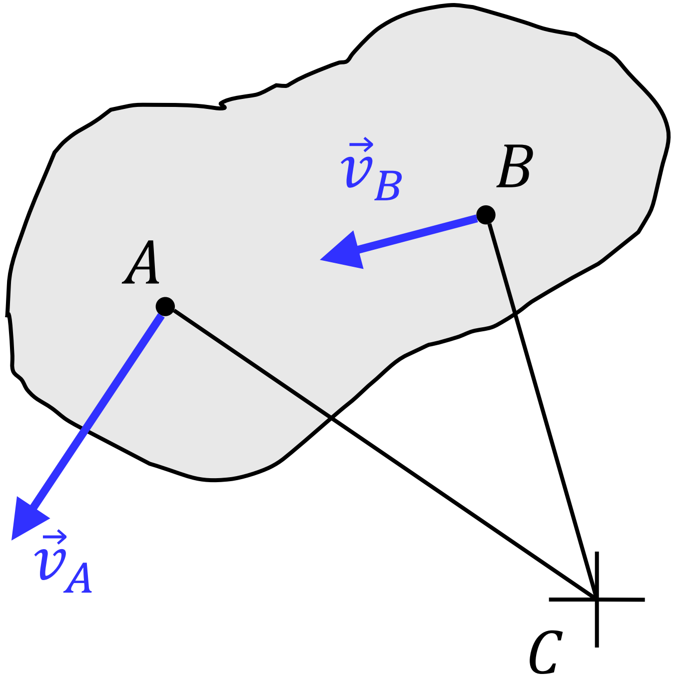
Instructor: Manuel Salmerón

Today’s Agenda

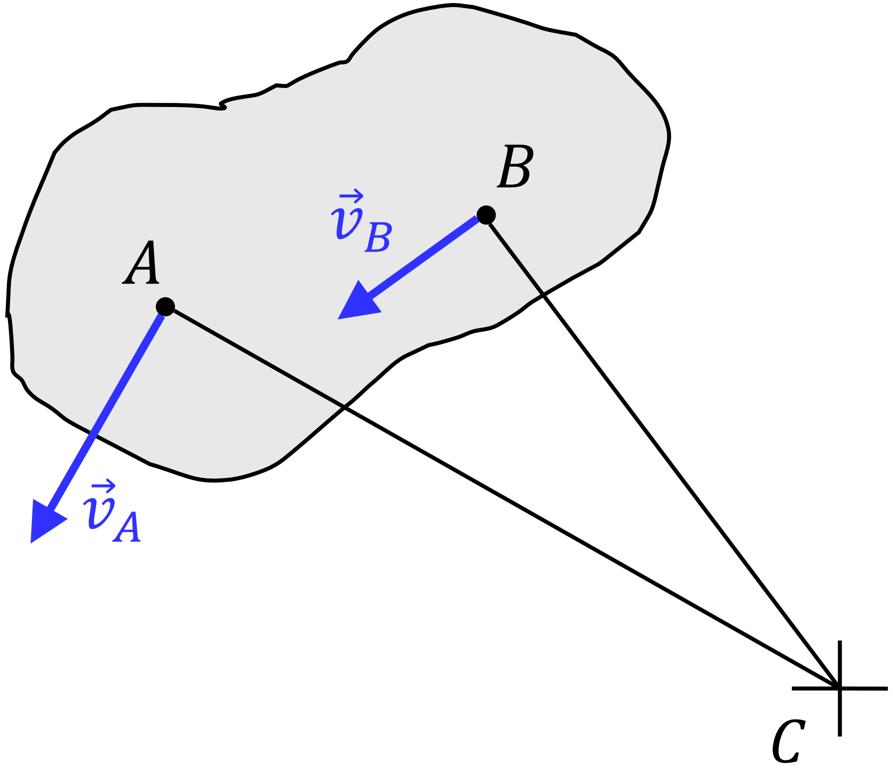
1. ICR for parallel paths
2. Example 2.C.2
3. Rigid bodies summary
4. Example 2.C.1

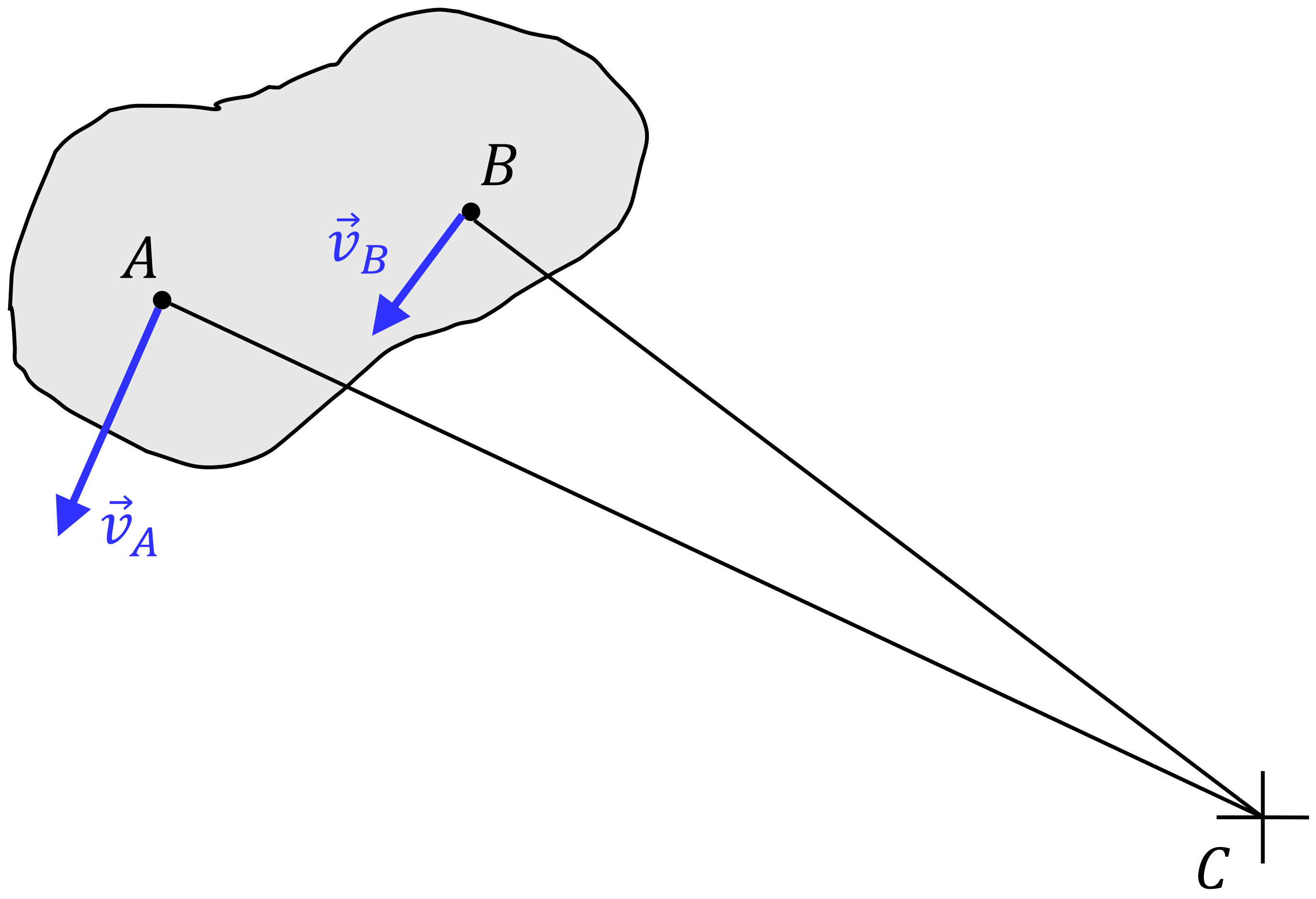


$\vec{v}_B$
B
A
$\vec{v}_A$
C

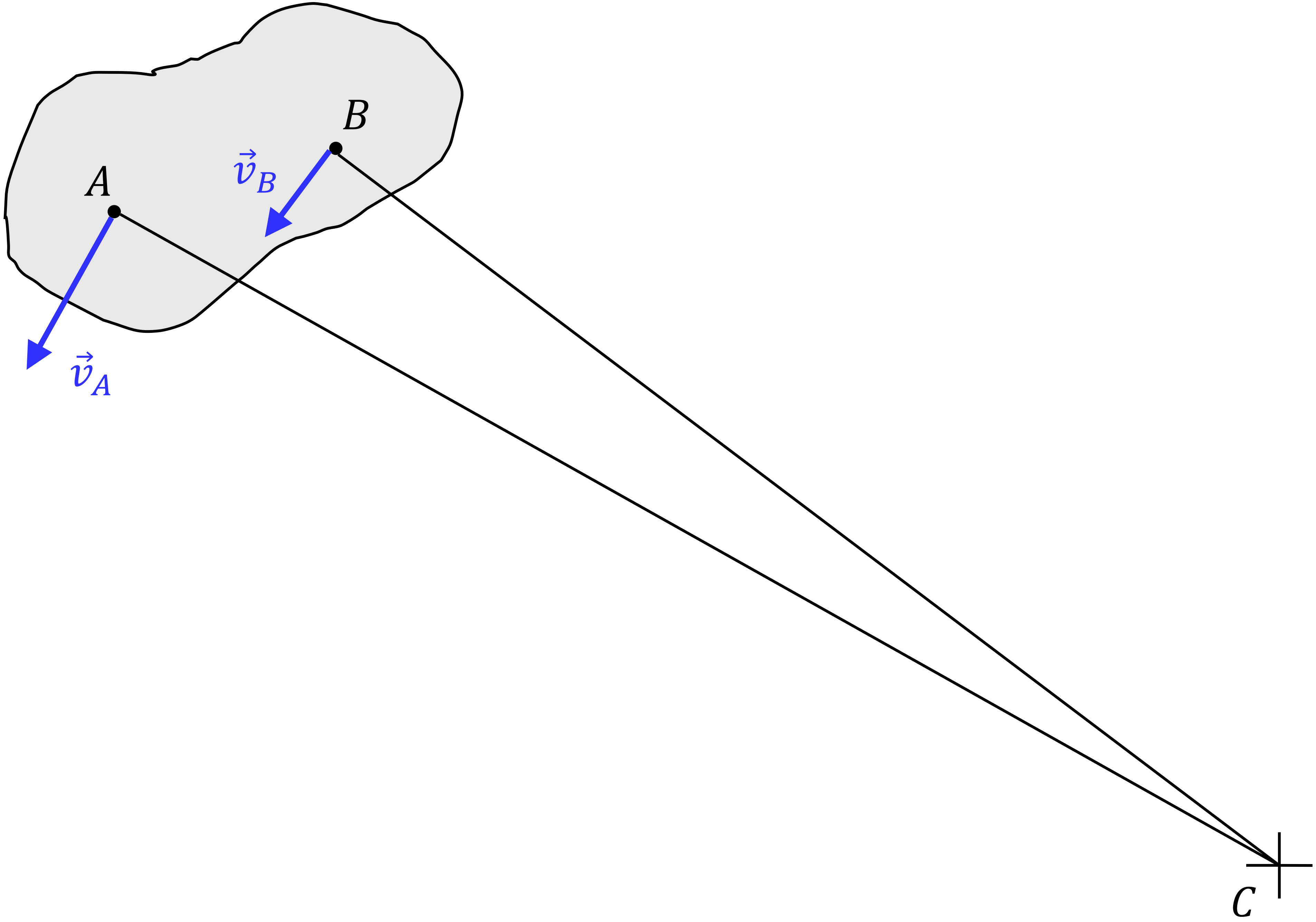


$\vec{v}_B$
B
A
$\vec{v}_A$
C

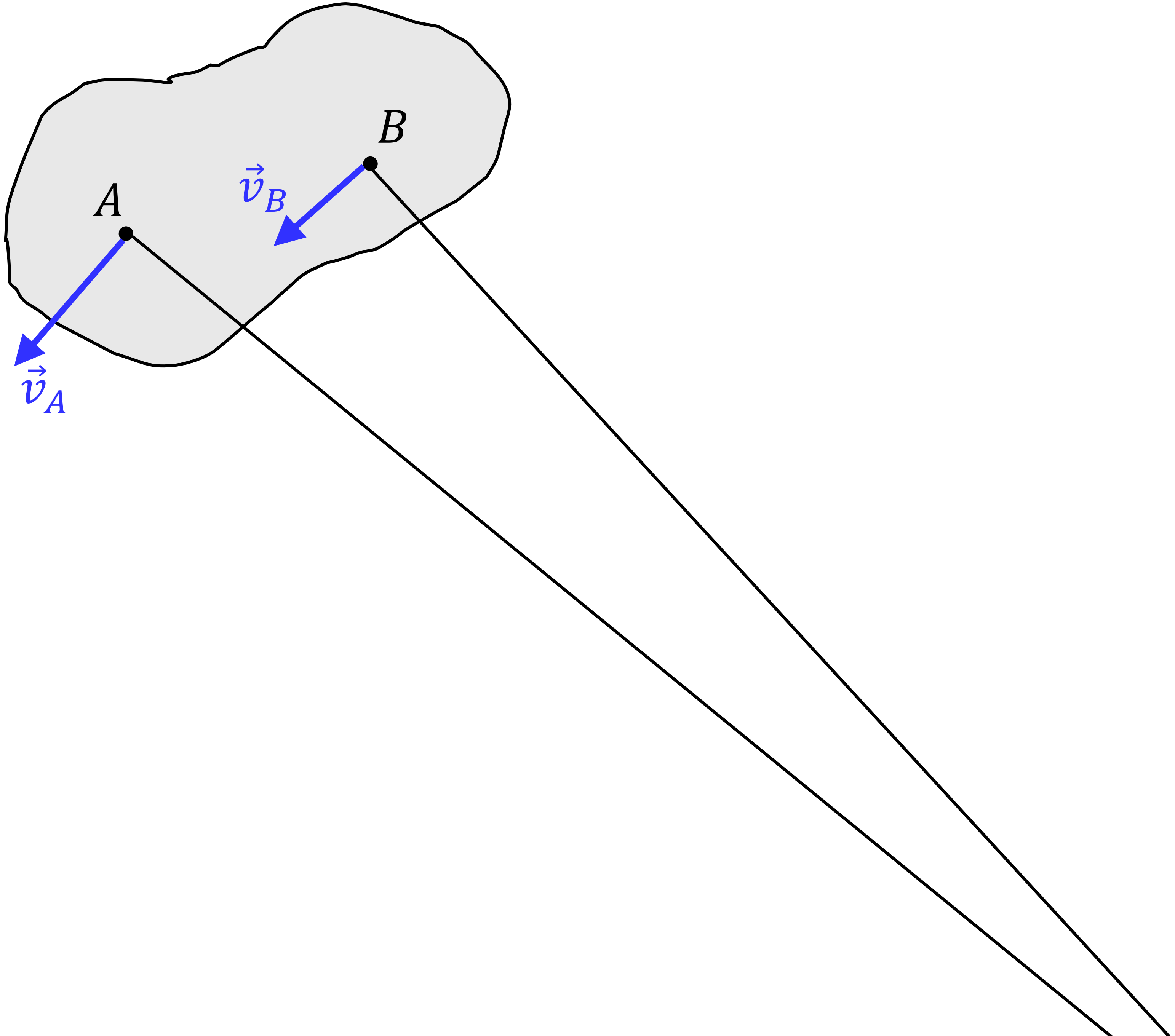

B
$\vec{v}_B$
A
$\vec{v}_A$
C



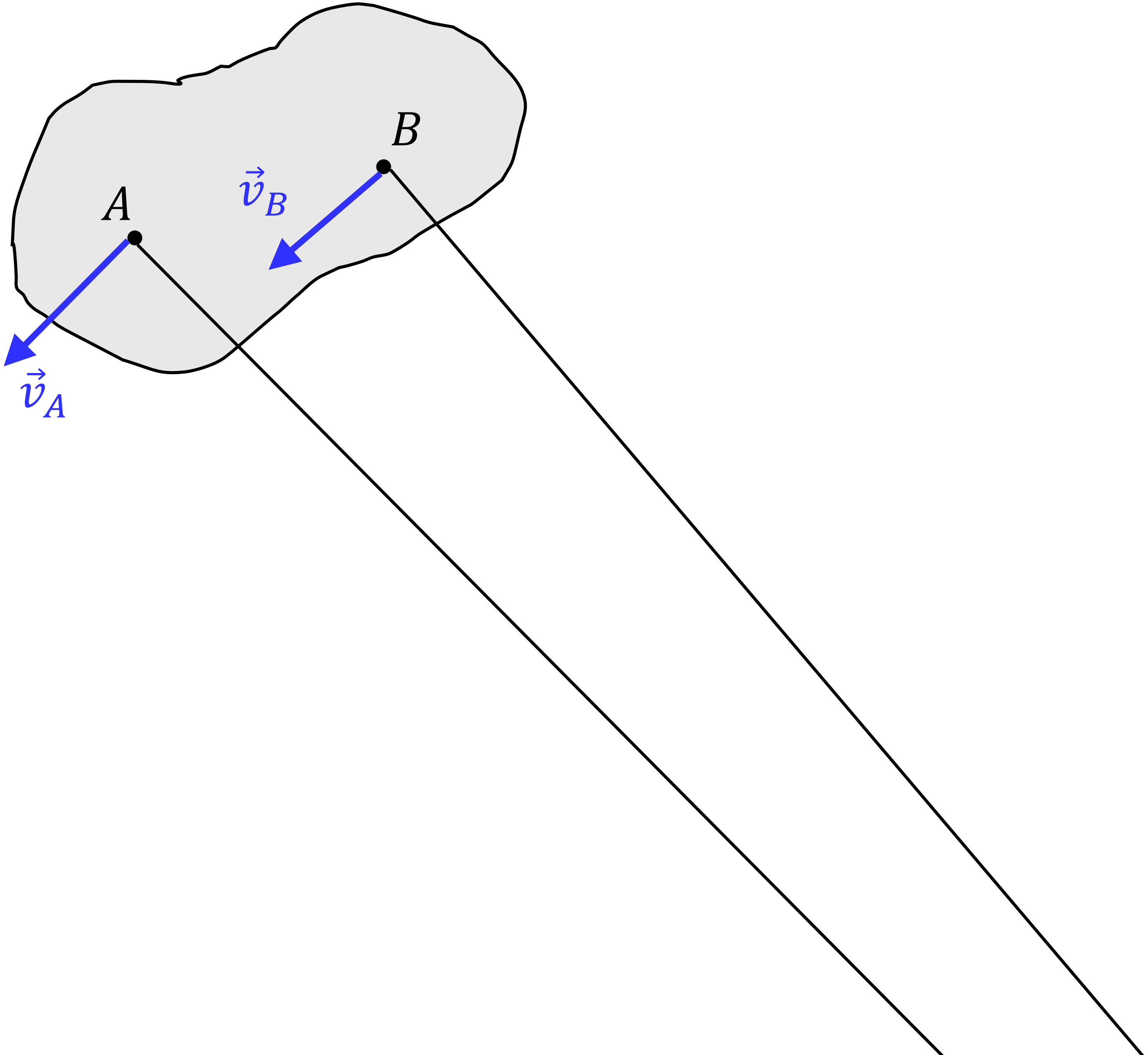
B
A
$\vec{v}_B$
$\vec{v}_A$
C

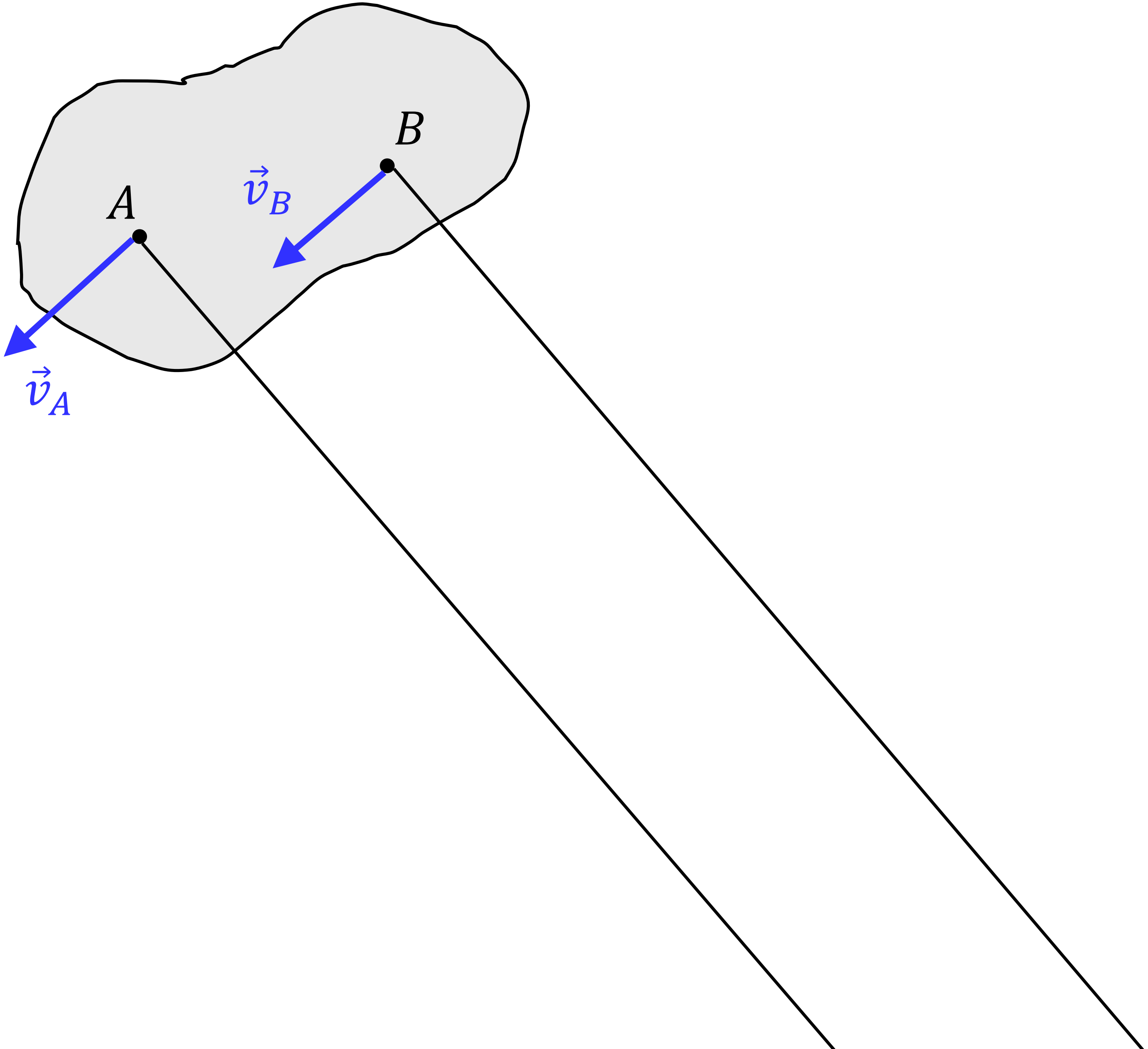


B
$\vec{v}_B$
A
$\vec{v}_A$
C

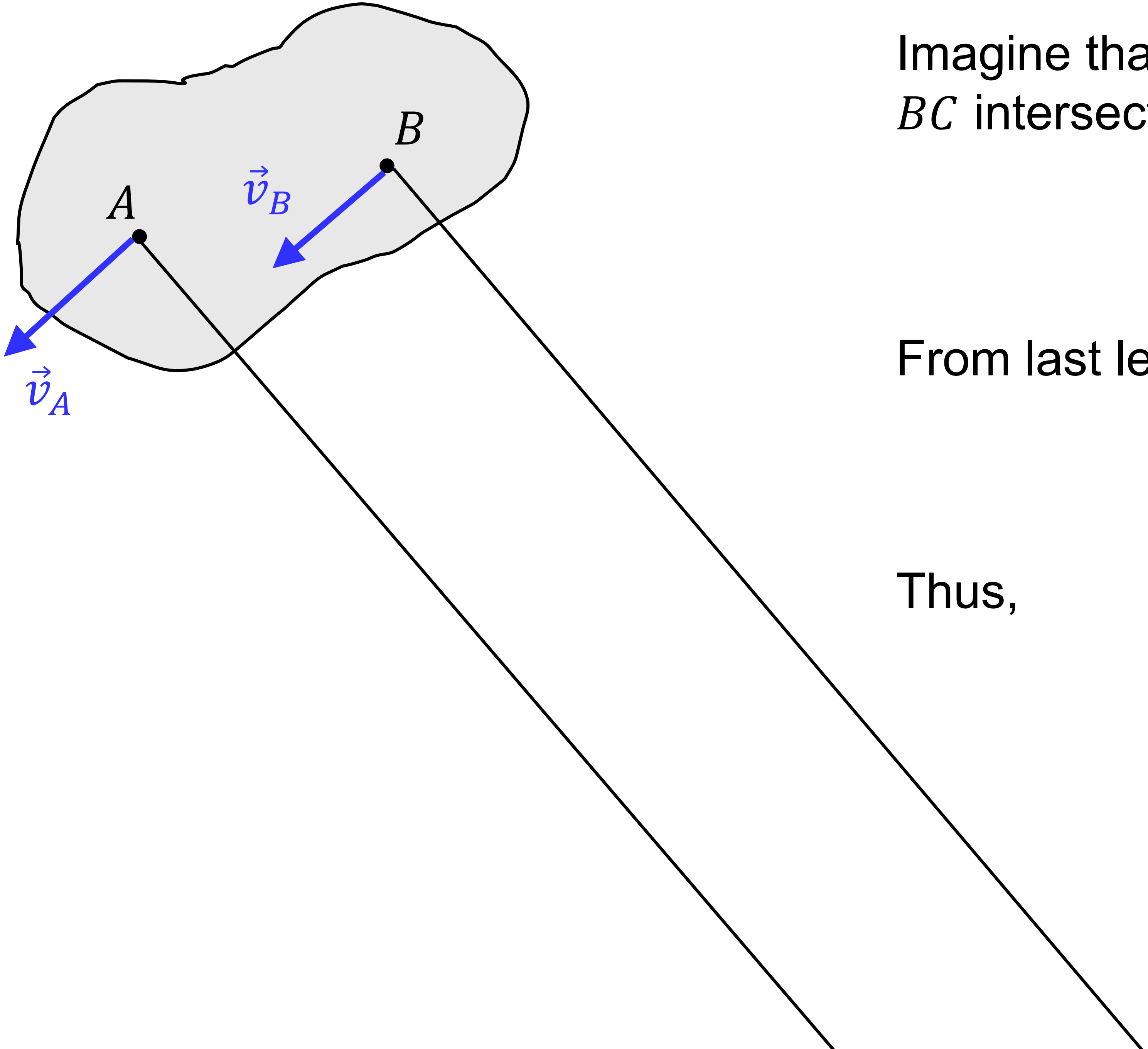


A
B
$\vec{v}_B$
$\vec{v}_A$


B
A
$\vec{v}_B$
$\vec{v}_A$



B
$\vec{v}_B$
A
$\vec{v}_A$



Imagine that the perpendicular lines AC and BC intersect at ∞ ($AC \parallel BC$):

$$|r_{A/C}| \to \infty$$

$$|r_{B/C}| \to \infty$$

From last lesson:

$$\omega = \frac{|v_A|}{|r_{A/C}|} = \frac{|v_B|}{|r_{B/C}|}$$

Thus,

$$\omega \to 0$$

(pure translation!)

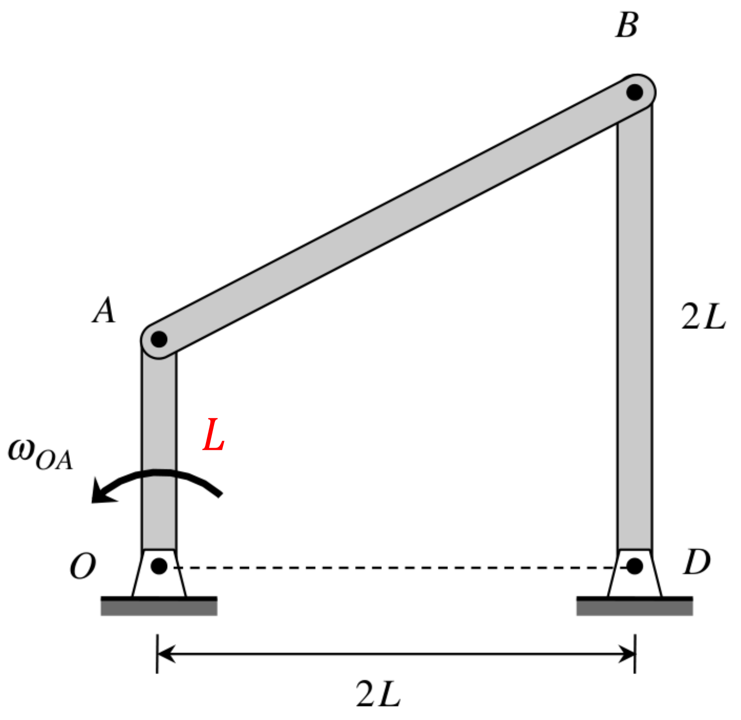
Example 2.C.2

Given: Link OA, of length L, in the mechanism shown below has a constant counterclockwise angular velocity of ω_{OA} as it moves into a vertical position. At this same instant, link BD is also vertically oriented, where pins O and D lie along the same horizontal line.

Find: Determine, at this instant in time:

(a) The angular velocity of link AB;
(b) The angular velocity of link BD;
(c) The angular acceleration of link AB; and
(d) The angular acceleration of link BD

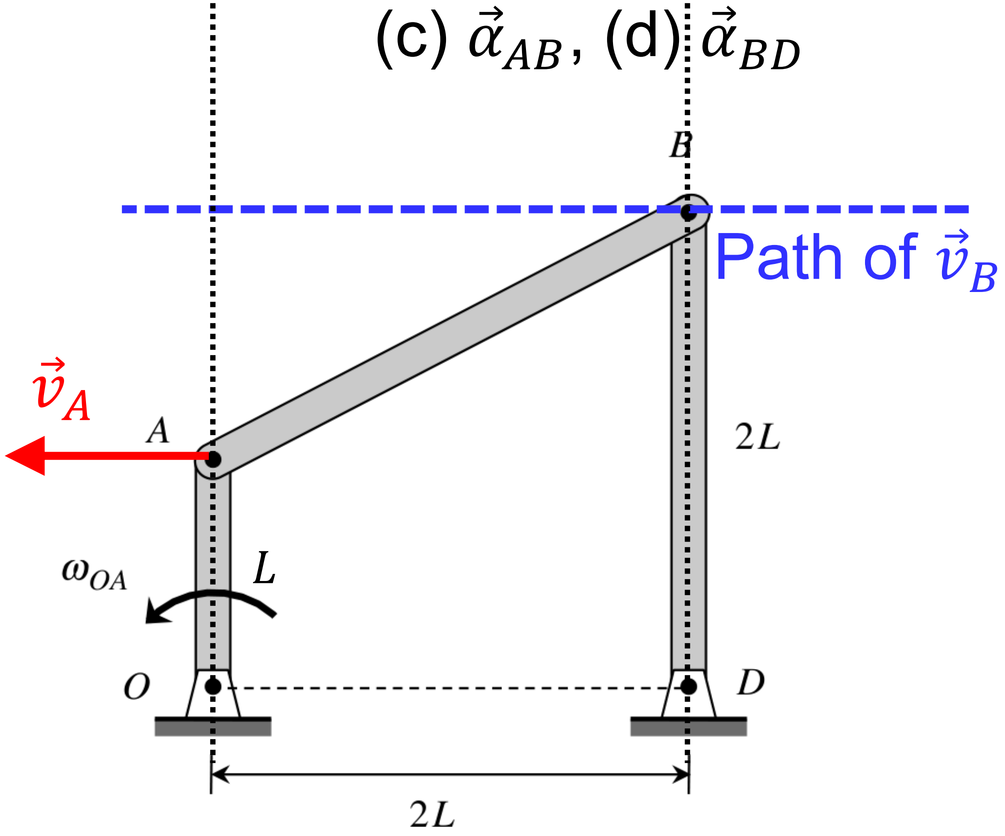
Use the following parameter in your analysis: $\omega_{OA} = 0.5$ rad/s.



Example 2.C.2

Given: ω_{OA} (CCW), L

Find: (a) $\vec{\omega}_{AB}$, (b) $\vec{\omega}_{BD}$, (c) $\vec{\alpha}_{AB}$, (d) $\vec{\alpha}_{BD}$



Solution:

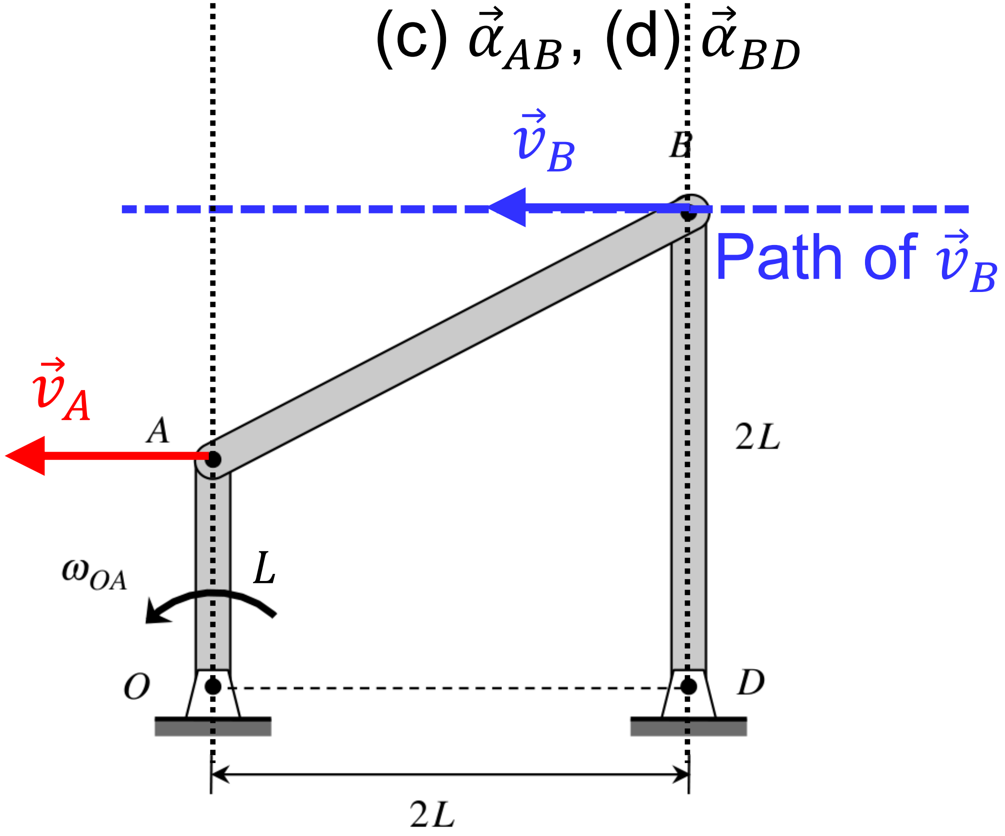
(a) Using instant centers of rotation:

- $\vec{v}_A$ can be deduced from $\vec{\omega}_{OA}$
- BD is pinned at D, so we can deduce the path of B
- $\vec{v}_A$ and $\vec{v}_B$ are parallel: $C = IC_{AB}$ is in ∞
- Thus: $\omega_{AB} = \frac{|v_A|}{|\vec{r}_{A/C}|} = \frac{|v_B|}{|\vec{r}_{B/C}|} \to 0$
- The angular velocity of AB is $\vec{\omega}_{AB} = \vec{0}$

Example 2.C.2

Given: ω_{OA} (CCW), L

Find: (a) $\vec{\omega}_{AB}$, (b) $\vec{\omega}_{BD}$,
(c) $\vec{\alpha}_{AB}$, (d) $\vec{\alpha}_{BD}$



Solution:

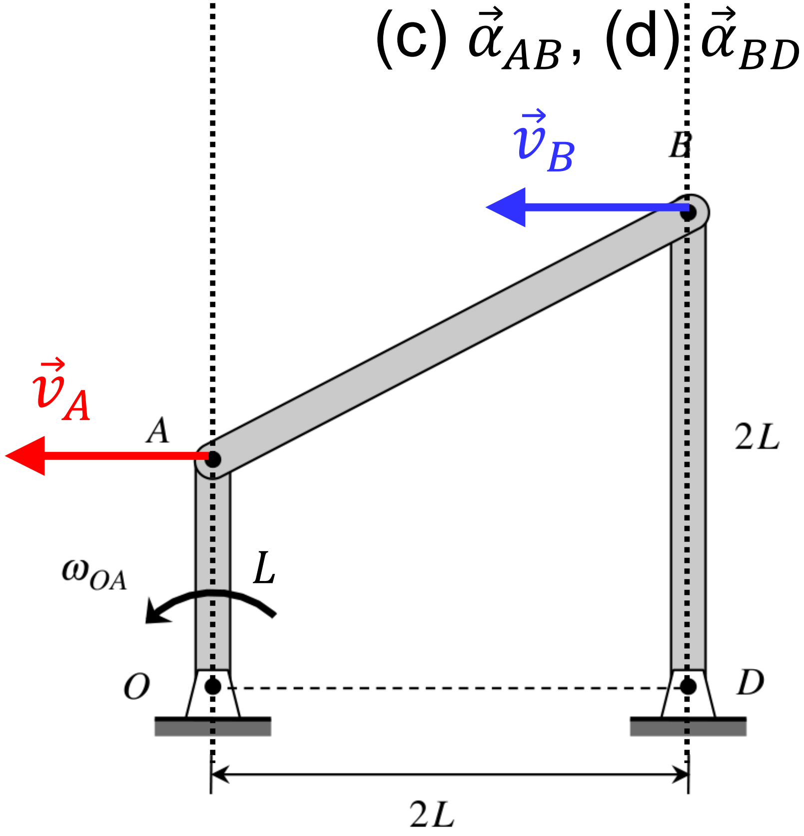
(b) We need information about B.

- Link AB: $\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A} = \vec{v}_A$
- Link OA: $\vec{v}_A = \vec{v}_O + \vec{\omega}_{OA} \times \vec{r}_{A/O} = -\omega_{OA} L \hat{\imath}$
- Thus: $\vec{v}_B = -\omega_{OA} L \hat{\imath}$
- Note that $IC_{BD} = D$
- Use $\omega_{BD} = \frac{|\vec{v}_B|}{|\vec{r}_{B/D}|} = \frac{\omega_{OA} L}{2}$
- You could have also computed $\vec{v}_B$ from link BD and then use coefficient balancing

Example 2.C.2

Given: ω_{OA} (CCW), L

Find: (a) $\vec{\omega}_{AB}$, (b) $\vec{\omega}_{BD}$,
(c) $\vec{\alpha}_{AB}$, (d) $\vec{\alpha}_{BD}$



Solution:

(c) **NEVER** use the ICR method for accelerations!

Proceed link by link:

$$\vec{a}_A = \vec{a}_O + \vec{\alpha}_{OA} \times \vec{r}_{A/O} - \omega_{OA}^2 \vec{r}_{A/O} = -\omega_{OA}^2 L\hat{j}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A} = -L\alpha_{AB}\hat{\imath} + (-\omega_{OA}^2 L + 2L\alpha_{AB})\hat{j}$$

$$\vec{a}_B = \vec{a}_D + \vec{\alpha}_{BD} \times \vec{r}_{B/D} - \omega_{BD}^2 \vec{r}_{B/D} = -2L\alpha_{BD}\hat{\imath} + \frac{\omega_{OA}^2}{2} L\hat{j}$$

Identity of vectors (coefficient balancing):

$(\hat{\imath})$: $-L\alpha_{AB} = -2\alpha_{BD}L$

$(\hat{j})$: $-\omega_{OA}^2 L + 2L\alpha_{AB} = -\frac{\omega_{OA}^2}{2} L$

Summary for Rigid Bodies

Some advice:

1. Go from simple to complex: $\vec{v}$ first, then $\vec{a}$ (most times)
2. For interconnected bodies:
   - Go body by body, link by link
   - “Store” information
3. Use all your RB knowledge to generate extra equations:
   - Constant velocity?
   - Constraints?
   - No slipping?
   - ICR?
4. Work with symbols, substitute at the very end: easier to “debug”.
   Exception: 0 values could simplify math and give you extra time.

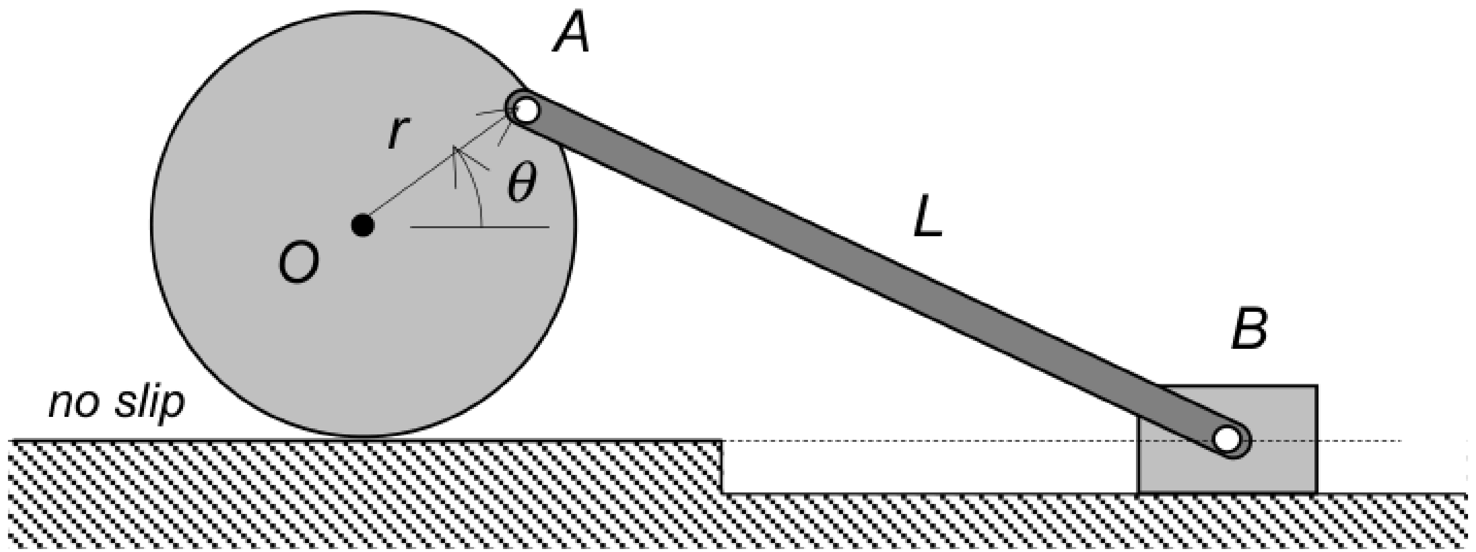
Example 2.C.1

Given: The wheel rolls without slipping in such a way that slider B moves to the left with a constant speed of $v_B = 5$ ft/s.

Find: Determine:

(a) The angular velocity of the wheel when $\theta = 0$; and
(b) The angular acceleration of the wheel when $\theta = 0$.

Use the following parameters in your analysis: $L = 2$ ft and $r = 0.5$ ft.



Attendance

1. Before start solving, describe your planned strategy
2. Checklist: units, vector directions (translational and rotational), signs, ICRs, numerical information is enough, etc.
3. After solving, describe the final strategy you followed.
4. Were there any changed between the planned and final strategy? Briefly describe them and the reason for the changes.

ME 274: Basic Mechanics II

Week 4 *– Friday, February 4*

Particle kinematics: 2D Rotating Reference Frames

Instructor: Manuel Salmerón

Announcements

Homework rubric change:

- 0.5 pt for clearly labeling/assigning pages
- 0.5 pt for proper format (Given / Find / Solution)

Announcements

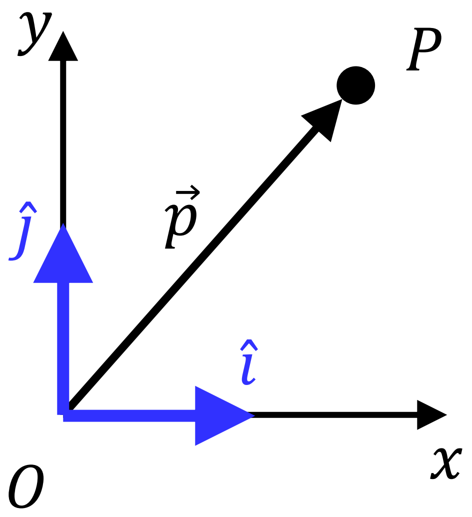
- *Exam details*
  - **Date/Time**: Thursday, 2/12, 8:00-9:30pm
  - **Locations**:
    WTHR 200 – 8:30 and 10:30 WL sections
    CL50 224 – 9:30 and 11:30 WL sections
    BHEE 129 – 12:30 and 1:30 WL sections
    PHYS 114 – 3:30 WL section
    MATH 175 – 4:30 WL section
    NU 112 – 10:30 Indy section
    NU 103 – 11:30 Indy section
  - **Material coverage**: Lectures 1-10 (Particle kinematics through planar rigid body kinematics)
  - *ME-approved calculators allowed.*
- ***Sample exams***: see ***Weekly Joys***. (Please note that Weekly Joys is run by ME undergraduate students, not by ME faculty.)
- ***Exam 1 review session:*** *7:00pm on Tuesday, February 10 on Zoom*
  - *Sample exam problems* for the exam review session
  - Video recording of review session
- ***Pi Tau Sigma review session:*** Thursday, February 5, 6:30-7:30pm, WTHR 172
- ***Exam 1 stats:***
- ***Exam 1 solution***

Students with DRC accommodations: HIKS G980
Topics: Chapters 1 and 2
Problems 1 and 2: similar to HW and in-class examples
Problem 3: conceptual questions (true/false, multiple choice)

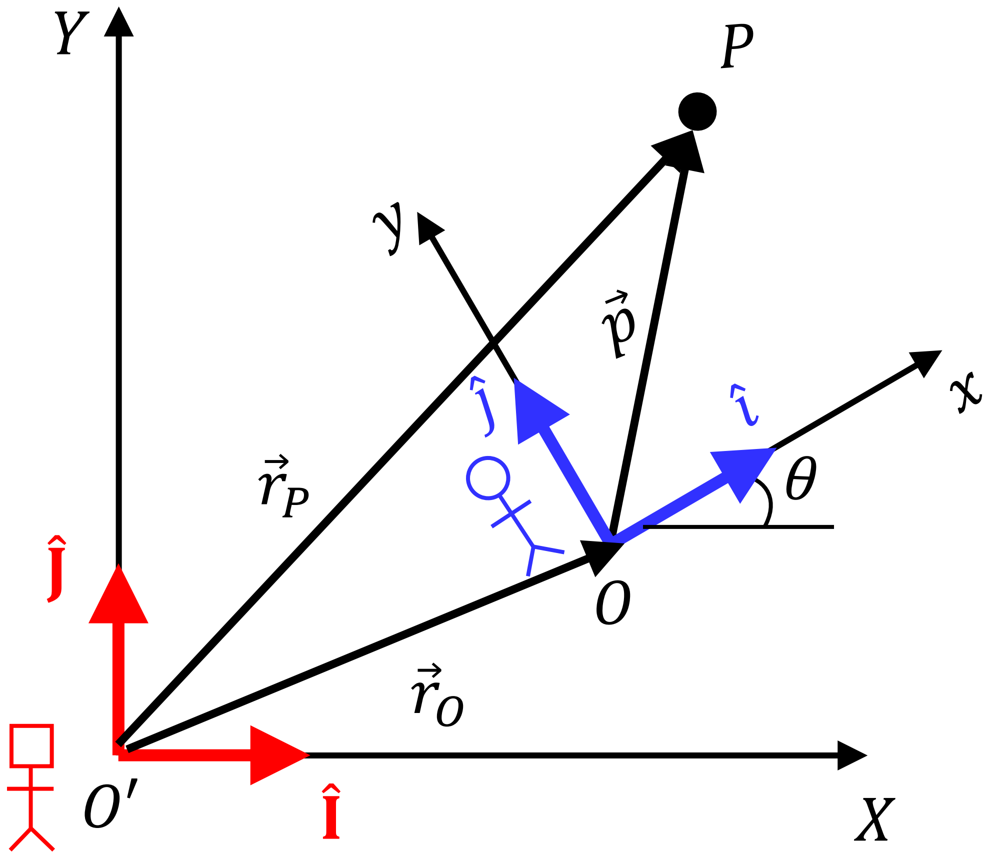
Today’s Agenda

1. 2D Rotating Reference Frames
2. Example 3.A.1
3. Example 3.A.2
4. Solving Steps

2D Rotating Reference Frames



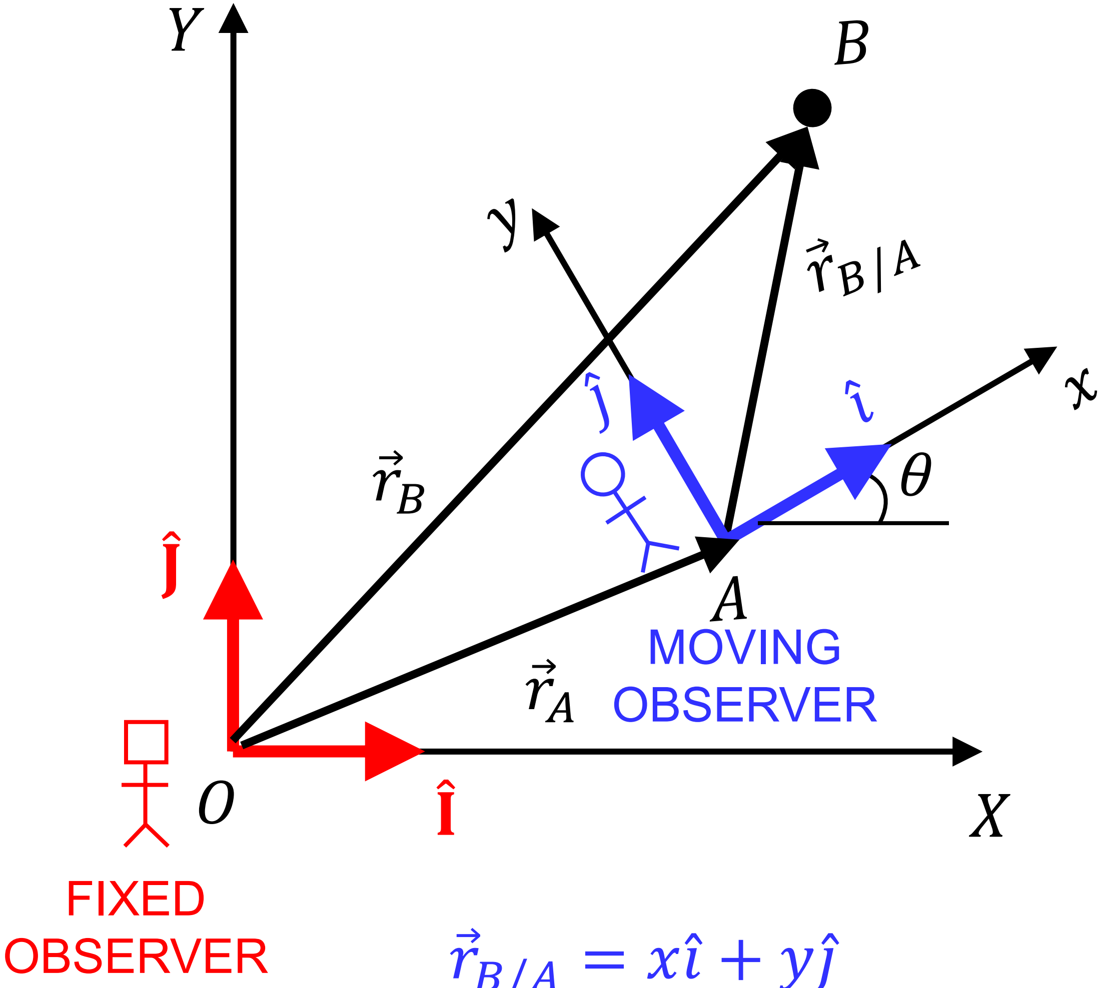
$$\vec{p} = x\hat{\imath} + y\hat{\jmath}$$



$$\vec{p} = x\hat{\imath} + y\hat{\jmath}$$

$$\vec{p} = \vec{r}_{P/O} = \vec{r}_P - \vec{r}_O$$

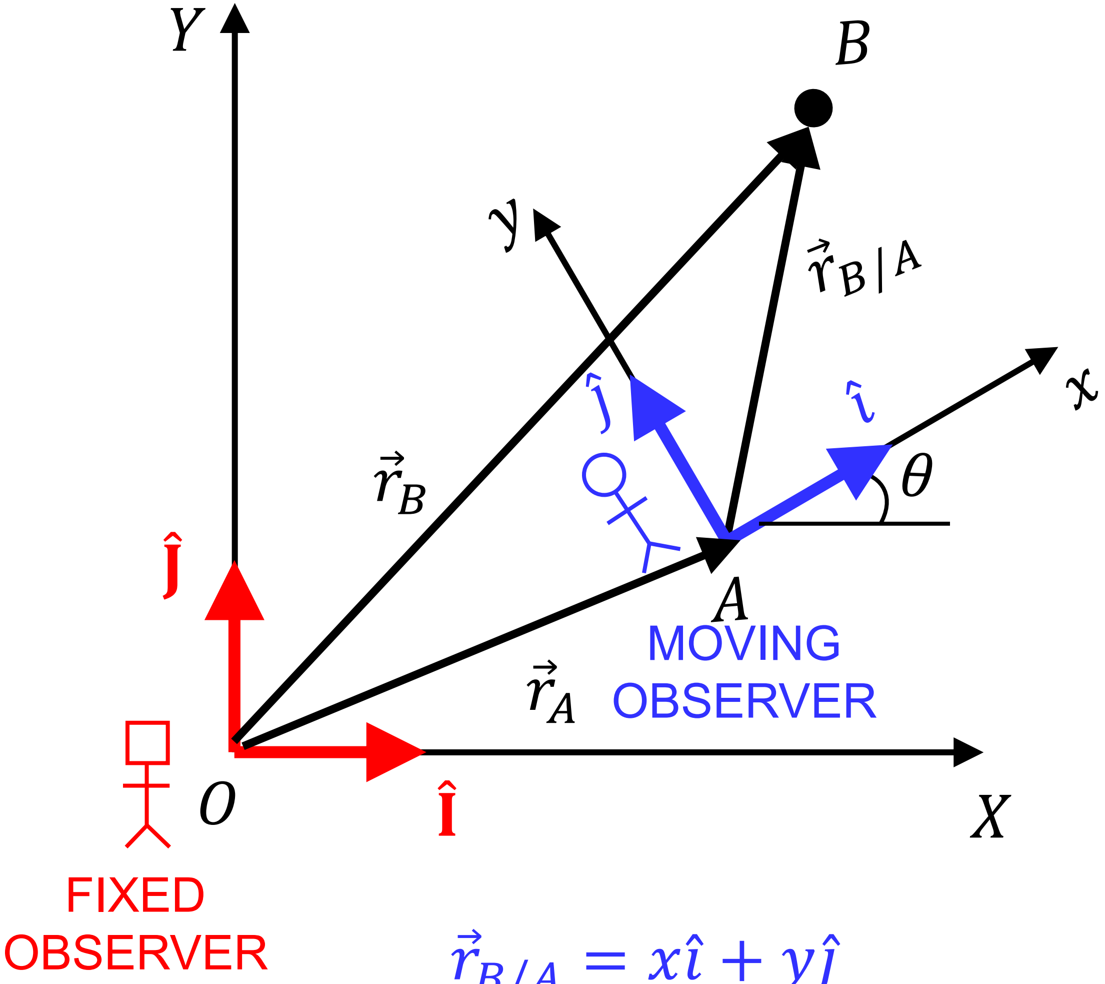
2D Rotating Reference Frames



$$\vec{r}_{B/A} = x\hat{\imath} + y\hat{\jmath}$$

$$\vec{r}_{B/A} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$$

2D Rotating Reference Frames



$$\vec{r}_{B/A} = x\hat{\imath} + y\hat{\jmath}$$

$$\vec{r}_{B/A} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$$

How does the velocity of B look from the perspective of the fixed observer?

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \frac{d}{dt}(x\hat{\imath} + y\hat{\jmath})$$

Express $\hat{\imath}$ and $\hat{\jmath}$ in terms of $\hat{I}$ and $\hat{J}$:

$$\hat{\imath} = \cos\theta\,\hat{I} + \sin\theta\,\hat{J}$$

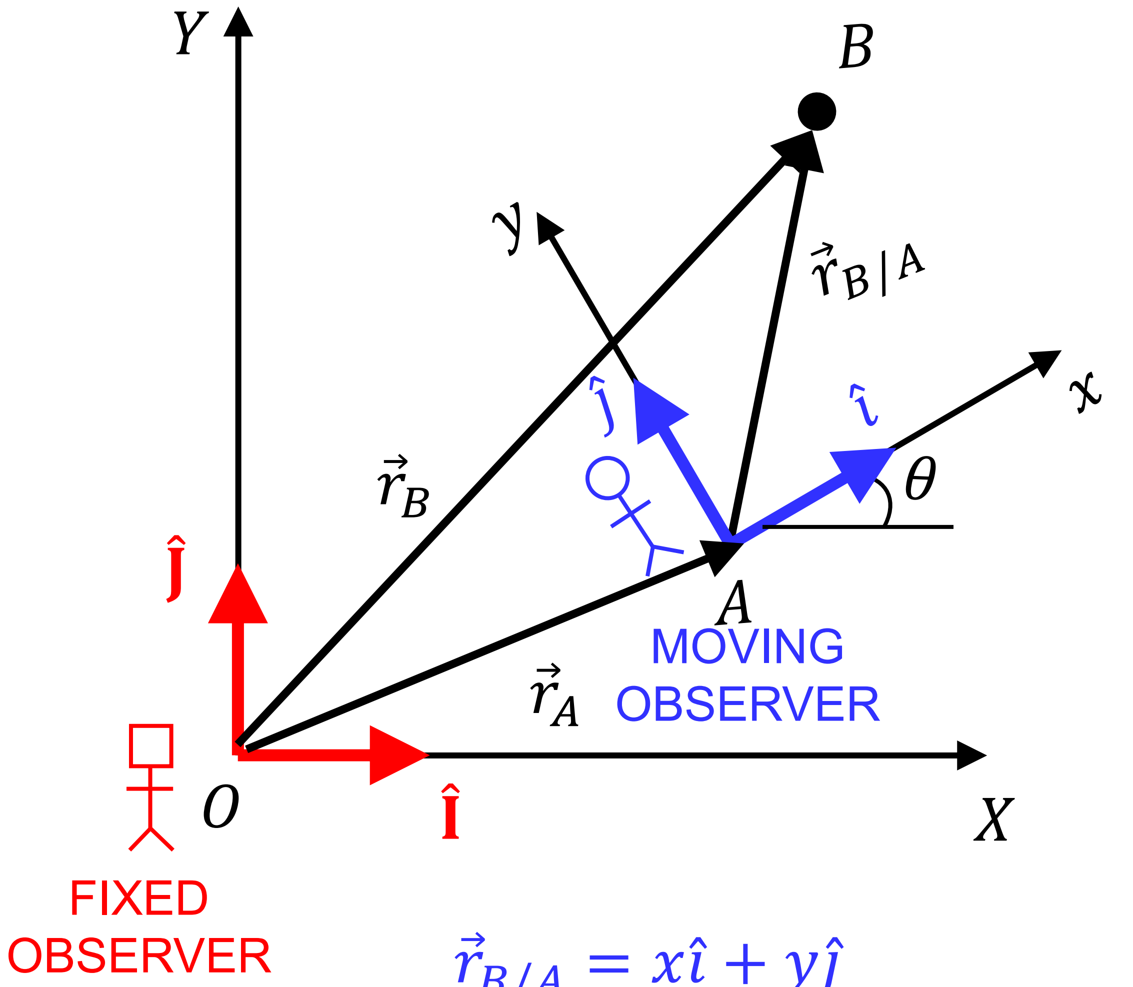
$$\hat{\jmath} = -\sin\theta\,\hat{I} + \cos\theta\,\hat{J}$$

θ depends on time, so $\hat{\imath}$ and $\hat{\jmath}$ depend on time!

$$\vec{v}_B = \vec{v}_A + \left(x\frac{d\hat{\imath}}{dt} + \hat{\imath}\dot{x} + y\frac{d\hat{\jmath}}{dt} + \hat{\jmath}\dot{y}\right)$$

(see full derivation in your lecture book)

2D Rotating Reference Frames



$$\vec{r}_{B/A} = x\hat{\imath} + y\hat{\jmath}$$

$$\vec{r}_{B/A} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$$

How does the velocity of B look from the perspective of the fixed observer?

$$\vec{v}_B = \vec{v}_A + \underbrace{(\dot{x}\hat{\imath} + \dot{y}\hat{\jmath})}_{\left(\vec{v}_{B/A}\right)_{rel}} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

Velocities seen from the FIXED OBSERVER perspective

Velocity of B seen from the MOVING OBSERVER perspective

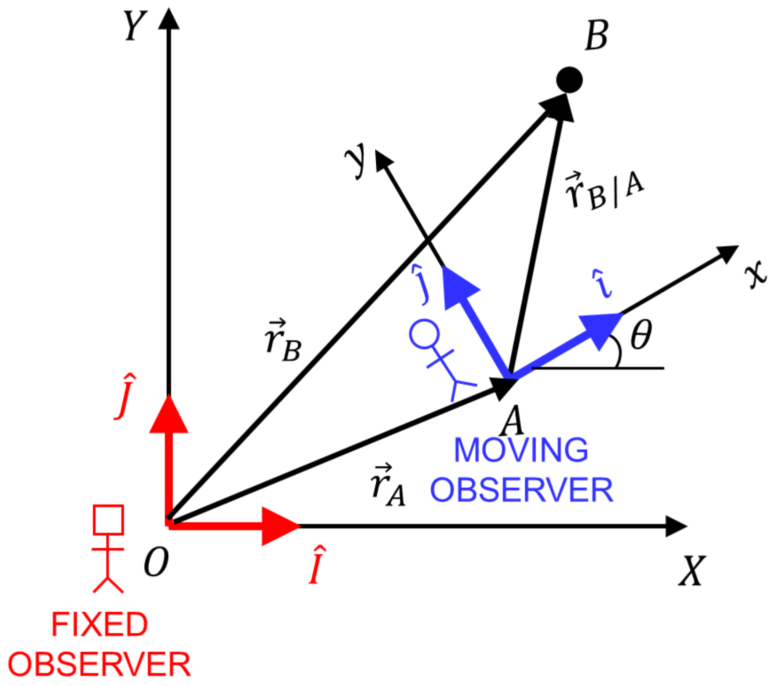
Rotation of the MOVING OBSERVER

2D Rotating Reference Frames

Formulas

$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$



- $\vec{v}_A$ and $\vec{v}_B$ are the velocities of points B and A as seen by a fixed observer.
- $\vec{a}_A$ and $\vec{a}_B$ are the accelerations of points B and A as seen by a fixed observer.
- $\vec{\omega}$ is angular velocity of the moving observer.
- $\vec{\alpha}$ is the angular acceleration of the moving observer.
- $\left(\vec{v}_{B/A}\right)_{rel}$ is the "velocity of point B as seen by the moving observer at A". In order to write down this term, you must clearly understand how the moving observer views the motion of B.
- $\left(\vec{a}_{B/A}\right)_{rel}$ is the "acceleration of point B as seen by the moving observer at A". In order to write down this term, you must clearly understand how the moving observer views the motion of B.

EXAMPLES

Examples

Solution to Example 3.A.1: https://www.youtube.com/watch?v=X5jL9qPhr6w

Solution to Example 3.A.2: https://www.youtube.com/watch?v=J2hHnY1SDG4