PARTICLE KINETICS

KINETICS PROBLEMS

1] COORDINATE SYSTEM

2] FREE BODY DIAGRAMS

3] KINETICS EQUATIONS

3] KINEMATICS

4] SOLVE FOR...

→ SOLUTION
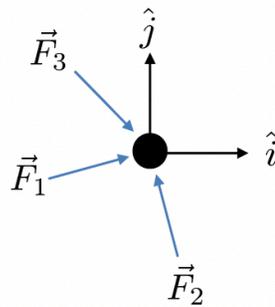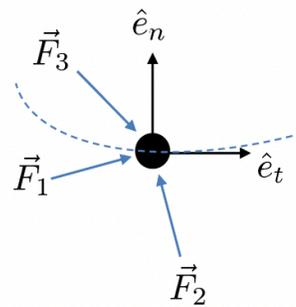
NEWTON'S LAWS:

Newton's Second Law of motion is now referenced to the appropriate description of motion:
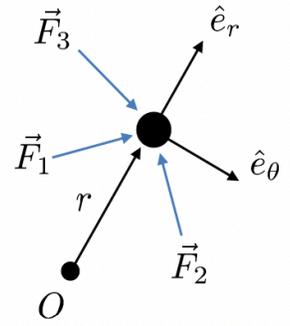
$$\Sigma \vec{F} = m\vec{a}$$

(a) Cartesian   (b) Path   (c) Polar

Cartesian:

$$\Sigma F_x = m\underbrace{\ddot{x}}_{a_x} \qquad\qquad \Sigma F_y = m\underbrace{\ddot{y}}_{a_y}$$

Path

$$\Sigma F_t = m\underbrace{\dot{v}}_{a_t} \qquad\qquad \Sigma F_n = m\underbrace{\frac{v^2}{\rho}}_{a_n}$$

Polar

$$\Sigma F_r = m\underbrace{(\ddot{r} - r\dot{\theta}^2)}_{a_r} \qquad \Sigma F_\theta = m\underbrace{(r\ddot{\theta} + 2\dot{r}\dot{\theta})}_{a_\theta}$$

Where does the "centrifugal force" term comes from?

D'Alembert proposed moving the inertial terms of the 2NL equation to the left, and treat it as a statics problem:

$$\Sigma F_n = m\frac{v^2}{\rho} = ma$$

$$\Rightarrow \Sigma F_n - m\frac{v^2}{\rho} = 0$$

$\underbrace{m\frac{v^2}{\rho}}$ "centrifugal force"

We engineers do not use centrifugal forces
May lead to sign and duplication errors.
"Centrifugal forces" are not real forces.

FBD is a representation of the body/particle of interest isolated from the "world" and being affected only by forces or moments connected to it

$$\Sigma F \quad , \quad \Sigma M$$

KINETIC DIAGRAMS

Representation of the inertial effects always pointing to the center of mass (or in a line of action passing through it)

$$m\vec{a} \quad , \quad I\vec{\alpha}$$

Mass times acceleration should be included in the free body diagram
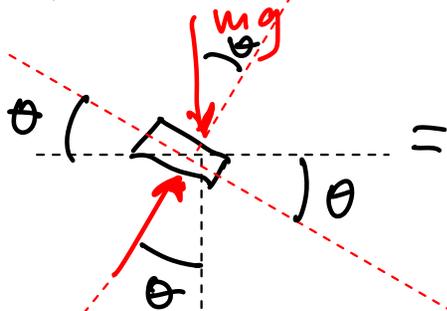
(A) True

(B) False

Example 4.A.4

Given: A collar P of mass m is free to slide along a smooth rod that is mounted at angle of $\theta = 36.87°$ in a frame. The frame is constrained to move along a horizontal surface, as shown, with a constant acceleration of a.

Find: Determine the value of a that is required for the collar to not slide along the rod as the frame accelerates to the right.
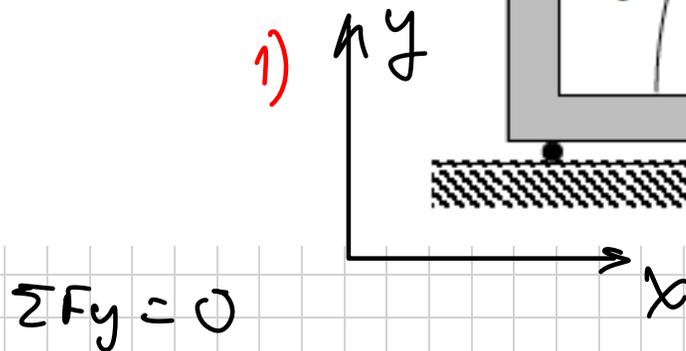
Force collar to move only in x => no vertical motion.

Solution

2) FBD

mg

θ

KD

θ

$= \qquad \longrightarrow m a_x$

3) $\Sigma Fx = m a_r$

$\Rightarrow N \sin \theta = m a_x$ ①

1) $\uparrow y$

a

P

θ

$\Sigma Fy = 0$

$N \cos \theta - mg = 0$ ②

$\longrightarrow x$

Solve ② and plug in ①

$$N = \frac{mg}{\cos\theta} \implies \frac{mg\sin\theta}{\cos\theta} = ma$$
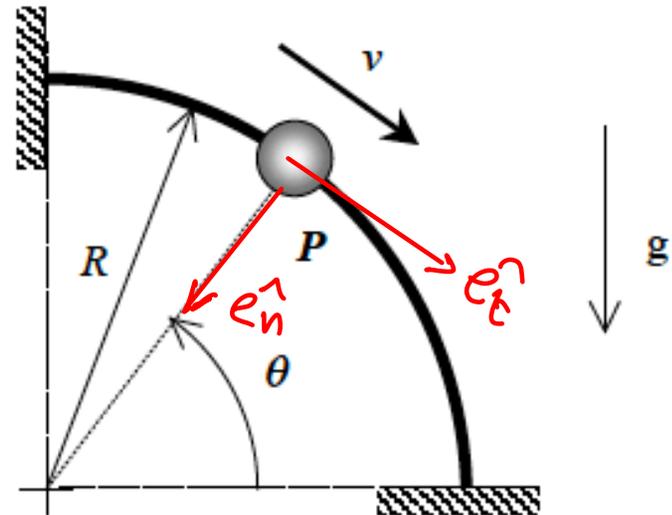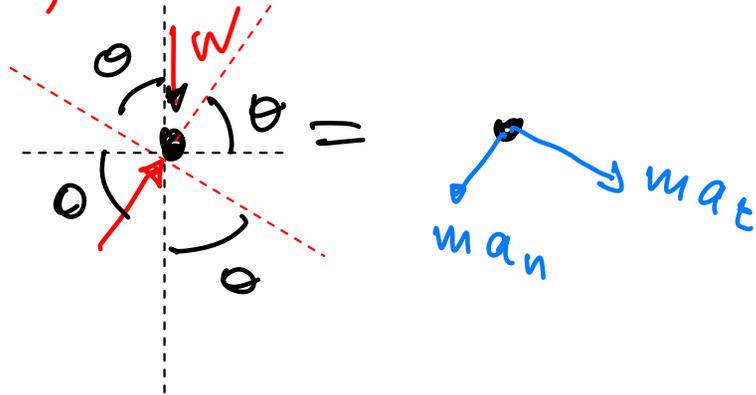
$$a = g\tan\theta$$

Example 4.A.7

Given: Particle P, having a weight of $W = 5$ lb, slides along a smooth, curved rod where $R = 3$ ft. At the position where $\theta = 53.13°$, the speed of P is known to be 20 ft/s.

Find: Determine:
(a) The normal force acting on P by the rod at the instant shown; and
(b) The rate of change of speed of P at the instant shown.

1) Coord system

2) FBD = KD

3) Kinetics

$$\Sigma F_n = m a_n = m \frac{v^2}{R}$$

$$-N + W \sin\theta = m \frac{v^2}{R} \quad \textcircled{1}$$

$$m = \frac{W}{g}$$

$$\Sigma F_t = m a_t = m \dot{v}$$

$$W \cos\theta = m \dot{v} \quad \textcircled{2}$$

$$m = \frac{W}{g}$$

4) Solve

$$\vec{N} = \left(w\sin\theta - \frac{w}{g}\frac{v^2}{R} \right) \hat{e}_n$$

$$\dot{v} = g\cos\theta$$