2/18/2026

C. A Review of Concepts Related to Chapter 3

The fundamental idea behind using the moving reference frame kinematics description is that it is often the case that the motion seen by a moving observer is often easier to describe than that of a fixed observer. Ultimately, we want velocity and acceleration described in terms of a fixed frame observation. The moving reference velocity and acceleration equations provide a means for "correcting" the moving frame observation to produce the fixed frame observation.

The critical components of the moving reference frame kinematics equations can be broken down into two parts:

- "How the observer moves": $\vec{\omega}$ and $\vec{\alpha}$ (the angular velocity and acceleration of the observer).
  (i.e. of moving ref. frame!)
- "What the observer sees": $\left(\vec{v}_{B/A}\right)_{rel}$ and $\left(\vec{a}_{B/A}\right)_{rel}$ (the velocity and acceleration of B as seen by the observer).

If you can keep these two sets of terms in the moving frame kinematics equations straight, you are off to a good start on this material.

The following is a set of conceptual/short answer/assessment questions related to the course material covered in Chapter 3. It is expected that you will use these questions to challenge your understanding of the fundamental concepts from the course as you learn the material and prepare for exams. Although these problems are conceptual in nature, you are not expected to be able to answer these questions by inspection. Rather, your answer will come from the use of a short analysis based on the fundamental equations covered. You can expect to see questions such as these on your exams this semester.

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Question C3.3

Sprinkler arm OA is pinned to a cart at point O. The cart moves to the right with a speed of v_{cart} with $\dot{v}_{cart} = 2$ ft/s^2 $=$ *constant*. Fluid flows through the sprinkler arm at a rate of $\dot{d}$ with $\ddot{d} =$ -3 ft/s^2 $=$ *constant*. The sprinkler arm is being raised at a constant rate of $\dot{\theta} = 4$ rad/s. An observer and xyz coordinate system are attached to the sprinkler arm, as shown in the figure below. The following equation is to be used to find the acceleration of a pellet P that flows with the fluid in the arm:

$$\vec{a}_P = \vec{a}_O + \left(\vec{a}_{P/O}\right)_{rel} + \vec{\alpha} \times \vec{r}_{P/O} + 2\vec{\omega} \times \left(\vec{v}_{P/O}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{P/O}\right]$$

Provide numerical values for the following terms when: $d = 3$ ft, $v_{cart} = 3$ ft/s, $\dot{d} = 5$ ft/s and $\theta = 90°$.

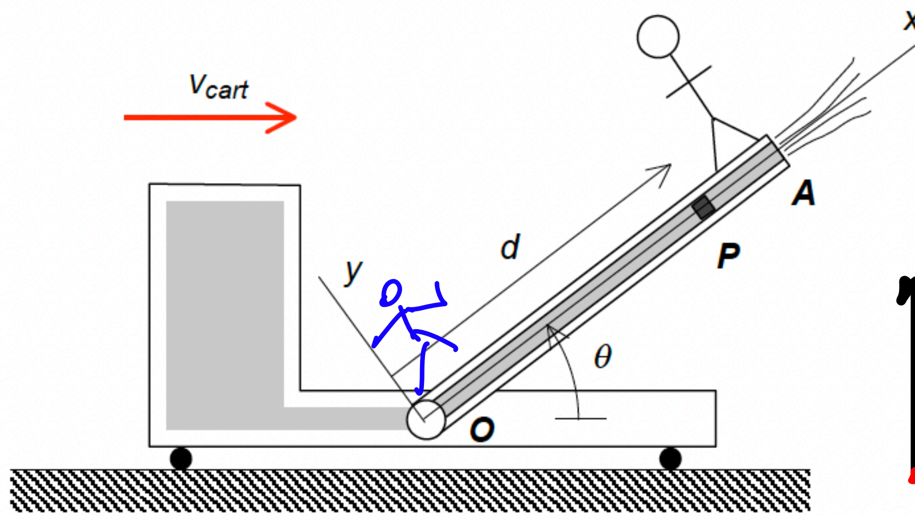
$\vec{a}_O =$ $2\,\hat{I}$

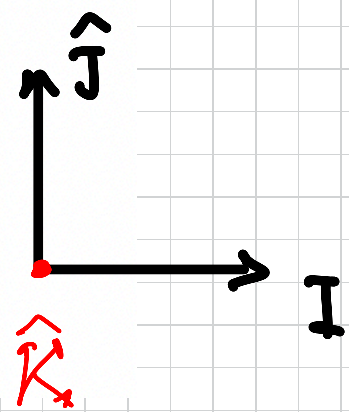
$\vec{\omega} =$ $\dot{\theta}\,\hat{K}$

$\vec{\alpha} =$ $\ddot{\theta}\,\hat{K} = 0\,\hat{K}$

$\left(\vec{v}_{P/O}\right)_{rel} =$ $\dot{d}\,\hat{i}$

$\left(\vec{a}_{P/O}\right)_{rel} =$ $\ddot{d}\,\hat{i}$





$(\vec{v}_{P/O})_{rel} = \dot{d}\,\hat{i} = 5\left(\cos\theta\,\hat{I} + \sin\theta\,\hat{J}\right)$ ft/s

$(\vec{a}_{P/O})_{rel} = \ddot{d}\,\hat{i} = -3\left(\cos\theta\,\hat{I} + \sin\theta\,\hat{J}\right)$

Question C3.4

The vertical shaft OA rotates about a fixed axis with a constant rate of $\Omega = 8$ rad/s. The arm AB is pinned to OA and is being raised at a constant rate of $\dot{\theta} = 10$ rad/s. An observer and xyz axes are attached to AB. The XYZ axes are stationary. What is the angular acceleration vector for arm AB when $\theta = 90°$?

$$\vec{\omega}_{TOT} = -\Omega \hat{J} + \dot{\theta} \hat{K}$$

$$\vec{\alpha}_{TOT} = \cancel{-\dot{\Omega} \hat{J}}^{\,0} - \Omega \cancel{\dot{\hat{J}}}^{\,0} \cancel{+ \ddot{\theta} \hat{k}}^{\,0} + \dot{\theta} \dot{\hat{k}}$$

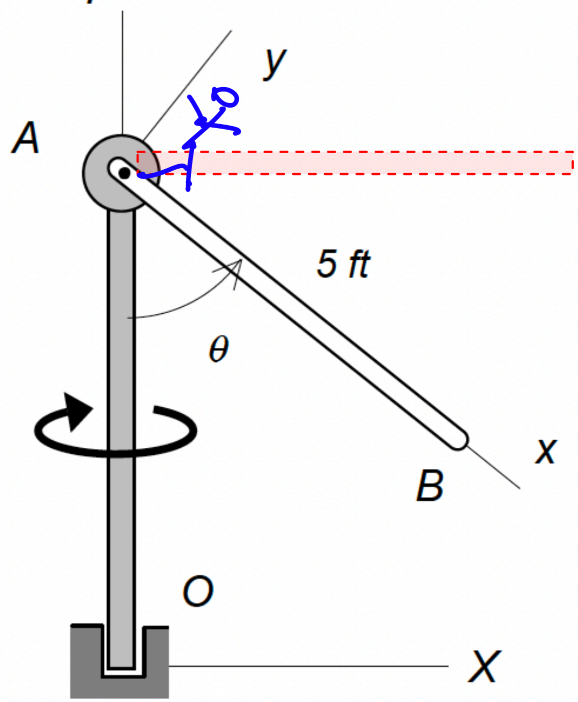
$$= \dot{\theta} (\vec{\omega}_{TOT} \times \hat{k})$$

$$= \dot{\theta} [(-\Omega \hat{J} + \dot{\theta} \hat{k})_{\Omega} \times \hat{k}]$$

$$\vec{\alpha}_{TOT} = -\dot{\theta} \Omega \hat{I}$$

$$\hat{k} = \hat{K}$$

$\hat{i}$ $\hat{j}$ k +



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$$\vec{a}_B = \cancel{\vec{a}_A}^{\,0} + \cancel{(\vec{a}_{B/A})_{rel}}^{\,0} + \vec{\alpha}_{TOT} \times \vec{r}_{B/A} + 2 \vec{\omega}_{TOT} \times \cancel{(\vec{v}_{B/A})_{rel}}^{\,0} + \vec{\omega}_{TOT} \times (\vec{\omega}_{TOT} \times \vec{r}_{B/A})$$

$$\vec{r}_{B/A} = L(\sin\theta\,\hat{I} - \cos\theta\,\hat{J})$$

Question C3.5

Arm AB rotates about a fixed vertical axis with a constant rate of ω_1. A ring, with its center at O and of radius r, rotates about arm AB with a constant rate of ω_2. A particle P moves along the ring with $\dot{\theta} = constant$. Let the XYZ axes be fixed, and the xyz axes be attached to the ring. At the position shown, $\theta = 90°$ and the xyz axes are aligned with the XYZ axes. It is desired to use the following equation to determine the acceleration of P for the position shown:

$$\vec{a}_P = \vec{a}_O + (\vec{a}_{P/O})_{rel} + \underset{TOT}{\vec{\alpha}} \times \vec{r}_{P/O} + 2\underset{TOT}{\vec{\omega}} \times (\vec{v}_{P/O})_{rel} + \underset{TOT}{\vec{\omega}} \times [\underset{TOT}{\vec{\omega}} \times \vec{r}_{P/O}]$$

$\hat{i} \to \hat{j} \to \hat{k}$

Provide expressions for the following terms appearing in this equation.

$\vec{a}_O = \cancel{\vec{a}_A} + \cancel{\vec{\alpha}_{AB}} \times \vec{r}_{O/A} + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{O/A})$

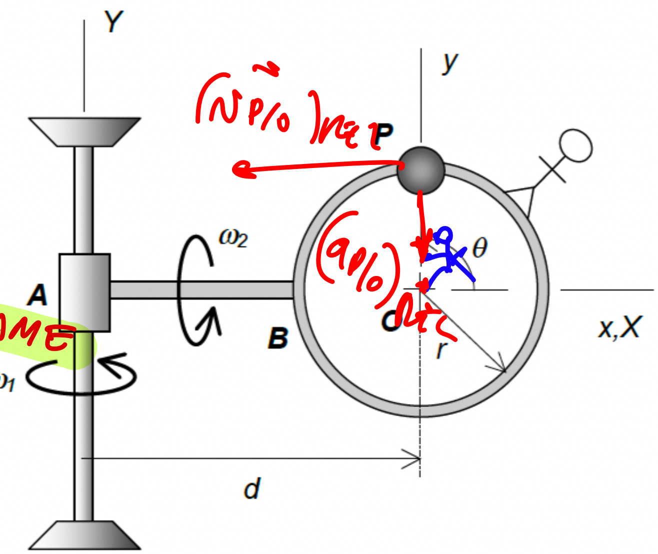
$\omega_1 \hat{J} \times (\omega_1 \hat{J} \times d\hat{I})$

$\vec{\omega} = \omega_1 \hat{J} + \omega_2 \hat{i}$ ← Total angular velocity of the moving ref. frame

$\vec{\alpha} = -\omega_2 \omega_1 \hat{K}$

$(\vec{v}_{P/O})_{rel} = -\dot{\theta} r \hat{I}$

$(\vec{a}_{P/O})_{rel} = -\dot{\theta}^2 r \hat{J}$



$\vec{\alpha}_{TOT} = \cancel{\dot{\omega}_1} \hat{J} + \omega_1 \cancel{\dot{\hat{J}}} + \cancel{\dot{\omega}_2} \hat{i} + \omega_2 \dot{\hat{i}}$

$= \omega_2 [\vec{\omega}_{TOT} \times \hat{i}] = \omega_2[(\omega_1 \hat{J} + \omega_2 \hat{i}) \times \hat{i}]$

$\hat{i} = \hat{I}$

$\vec{\alpha}_{TOT} = -\omega_2 \omega_1 \hat{K}$ ← TOTAL ANG. ACCEL OF MOVING REF. FRAME

Note: If we wanted to place the moving reference frame in the rotating particle P, this would add the $\dot{\theta}\hat{R}$ term to the $\vec{\omega}_{TOT}$. However, it's not recommended because in such case the observer would not see any relative motion $\Rightarrow$ would yield a trivial equation